Notes for Capstone: Discrete Math

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5. Sequences, Mathematical Induction, and Recursion

5.1. Sequences

Definition 5.1.1 (Sequence)**.** A sequence is a function whose domain is either all the integers between two given integers or all the integers greater than or equal to a given integer.

My definition: A sequence is a list of numbers. The list can be finite or infinite. The list can be indexed by the natural numbers or the integers.

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