Capstone: Discrete Mathematics 2023-2024

Lecture Notes

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1 Speaking Mathematically

1.1 Variables

1.2 The Language of Sets

1.3 The Language of Relations and Functions

2 The Logic of Compound Statements

2.1 Logical Form and Logical Equivalence

2.2 Conditional Statements

2.3 Valid and Invalid Arguments

2.4 Application: Digital Logic Circuits

2.5 Application: Number Systems and Circuits for Addition

3 The Logic of Quantified Statements

3.1 Predicates and Quantified Statements I

3.2 Predicates and Quantified Statements II

3.3 Statements with Multiple Quantifiers

3.4 Arguments with Quantified Statements

4 Elementary Number Theory and Methods of Proof

4.1 Direct Proof and Counterexample I: Introduction

4.2 Direct Proof and Counterexample II: Rational Numbers

4.3 Direct Proof and Counterexample III: Divisibility

4.4 Direct Proof and Counterexample IV: Division into Cases and the Quotient-Remainder Theorem

4.5 Direct Proof and Counterexample V: Floor and Ceiling

4.6 Indirect Argument: Contradiction and Contraposition

4.7 Indirect Argument: Two Classical Theorems

4.8 Application: Algorithms

5 Sequences, Mathematical Induction, and Recursion

5.1 Sequences

5.2 Mathematical Induction I

5.3 Mathematical Induction II

5.4 Strong Mathematical Induction and the Well-Ordering Principle for the Integers

5.5 Application: Correctness of Algorithms

5.6 Defining Sequences Recursively

5.7 Solving Recurrence Relations by Iteration

5.8 Second-Order Linear Homogeneous Recurrence Relations with Constant Coefficients

5.9 General Recursive Definitions and Structural Induction

6 Set Theory

6.1 Set Theory: Definitions and the Element Method of Proof

6.2 Properties of Sets

6.3 Disproofs, Algebraic Proofs, and Boolean Algebras

6.4 Boolean Algebras, Russell's Paradox, and the Halting Problem