All Definitions and Theorems from 4.1-4.4 (aka stuff you should memorize)

- An integer n is **even** if, and only if, there exists an integer k such that $n = 2k$.
- An integer n is **odd** if, and only if, there exists an integer k such that $n = 2k + 1$.
- An integer n is **prime** if, and only if, $n > 1$ and for all positive integers r and s, if $n = rs$, then either r or s equals 1 (and the other equals n).
- An integer n is **composite** if, and only if, $n > 1$ and there exist positive integers r and s such that $n = rs$ and neither r nor s equals 1 (and neither equal n).
- A real number r is **rational** if, and only if, there exist integers p and q, where $q \neq 0$, such that $r = \frac{p}{q}$.
- **Theorem (Zero Product Property):** If neither of two real numbers is zero, then their product is also not zero.
- **Theorem:** The sum of any two rational numbers is rational.
- **Theorem:** The product of any two rational numbers is rational.
- **Theorem:** Given any two rational numbers r and s, there exists another rational number between r and s.
- If n and d are integers and $d \neq 0$, then $\boldsymbol{d}$ **divides** $\boldsymbol{n}$ if, and only if, there exists an integer k such that $n = dk$.
- **Theorem:** For all integers a and b, if a and b are positive and a divides b, and $a \leq b$.
- **Theorem:** The only divisors of 1 are 1 and -1.
- **Theorem (Transitivity of Divisibility):** For all integers a, b, and c, if $a|b$ and $b|c$, then $a|c$.
- **Theorem:** Any integer $n > 1$ is divisible by a prime number.
- **Theorem (Unique Factorization of Integers):** Given any integer $n > 1$, there exist a positive integer k, distinct prime numbers $p_1, p_2, \dots, p_k$, and positive integers $e_1, e_2, \dots, e_k$ such that
$$n = p_1^{e_1} p_2^{e_2} p_3^{e_3} \cdots p_k^{e_k},$$
and any other expression for n as a product of prime numbers is identical to this except, perhaps, for the order in which the factors are written.
- **Theorem (Quotient-Remainder Theorem):** Given any integer n and positive integer d, there exist unique integers q and r such that
$$n = dq + r \qquad \text{and} \qquad 0 \leq r < d$$
- Given an integer n and a positive integer d, $\boldsymbol{n \ div \ d}$ **=** the integer quotient obtained when n is divided d**,** and, $\boldsymbol{n \ mod \ d}$ **=** the nonnegative integer remainder obtained when n is divided by $\boldsymbol{d}$**.**
- **Theorem:** Any two consecutive integers have opposite parity.
- For any real number x, the **absolute value of** $\boldsymbol{x}$, is defined as follows:
$$|x| = \begin{cases} x & if\ x \geq 0 \\ -x & if\ x > 0 \end{cases}$$
- **Theorem (Triangle Inequality):** For all real numbers x and y, $|x + y| \leq |x| + |y|$.