Study Guide for Test 2

1. Prove that if a, b and c are integers such that a divides b and a divides c, then a divides $b+c$.
2. Show that for a real number x, we get $-2 \leq x < 1$ if, and only if, $\frac{2x+1}{x-1} \leq 1$.
3. Prove that if the product xy is not rational, then x or y must be irrational.
4. Show that $\sqrt{3}$ is irrational.
5. Prove that $\sqrt{2}+\sqrt{3}$ is not rational.
6. Prove that

$$\lfloor n/3 \rfloor = \begin{cases} n/3 & \text{if } n \bmod 3 = 0 \\ (n-1)/3 & \text{if } n \bmod 3 = 1 \\ (n-2)/3 & \text{if } n \bmod 3 = 2 \end{cases}$$

7. Prove that $\lceil n/2 \rceil = \frac{n+1}{2}$.
8. Show that the square of an integer cannot be of the form $3k+2$, where k is an integer.
9. Prove the lower triangle inequality: for all real numbers x and y,

$$||x| - |y|| \leq |x-y|.$$

10. Prove that for all real numbers z and w,

$$|(1+z)(1+w)-1| \leq (1+|z|)(1+|w|)-1.$$

11. Prove or provide a counterexample: there are no positive integers x and y such that $x^2 - y^2 = 10$.
12. Prove that for all real numbers x, we have $x \leq -5$ if, and only if, $1 \leq \frac{2x+3}{x-2} \leq 2$.
13. Prove that if ab is even, then a or b is even.
14. Prove that if $a+b$ is odd, then a is odd or b is odd.
15. Prove that for all prime numbers p, either $p=2$ or p is odd.
16. Prove there are no integers x and y such that x is a prime greater than 5 and $x = 6y+3$.
17. Prove that for all integers n, if n is a multiple of 3, then n can be written as the sum of three consecutive integers. (Note: you proved the converse of this for homework).
18. Prove that for all integers a and b, if a^2+b^2 is odd, then a or b is odd.
19. Prove that for all integers n, n is even if, and only if, $3n$ is even.
20. Prove that for all integers a and positive integers n, $(a-1) \mid (a^n - 1)$.
21. Prove that the sum of two prime numbers, each larger than 2, is not a prime number.
22. Prove that if x is an irrational number and $x > 0$, then $\sqrt{x}$ is also irrational.
23. Prove that if x is an irrational number, then $1/x$ is also irrational.
24. Prove that if $x+y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

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25. Prove that $(n+1)^3 \geq 3^n$ if n is a positive integer with $n \leq 4$.
26. Prove for all integers x and y, $x = y$ if, and only if, $xy = \frac{(x+y)^2}{4}$.
27. Prove that $\log(7)$ is irrational.
28. Prove that given n is an integer, if $n^2 - (n-2)^2$ is not divisible by 8, then n is even.
29. Let x and y be real numbers. Show that if $x \neq y$, then $2x+4 \neq 2y+4$.
30. Let n be an integer. Prove that if $3n$ is odd, then n is odd.
31. Let $x \in \mathbb{N}$. Prove that if x is odd, then $\sqrt{2x}$ is not an integer.
32. Let x and y be real numbers. Show that if $x \neq y$ and $x, y \geq 0$, then $x^2 \neq y^2$.
33. Prove that if the product of two integers x and y is odd, then both integers are odd.
34. Let n be an odd integer. Prove that $n^3 - n$ is divisible by 24.
35. Prove that every integer greater than 11 can be expressed as the sum of two composite integers. (Hint: can each of $n-4$, $n-6$, and $n-8$ be prime?)