Synthesis of Layout Programs

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1 Abstract

A layout program is used to generate coordinate points to draw a pictorial representation of a data structure. These drawing are required to have certain aesthetic properties that represent the relations between elements of the data structure. Different algorithms have been developed to generate layouts for common data structures like trees and graphs, which have certain aesthetic properties. This work describes the use of program synthesis to generate layout programs for tree data structures using partial programs and aesthetic verifying predicates.

2 Introduction

Graphical representations of data structures are used to demonstrate the concepts of algorithms in introductory computer science courses. Preparing the graphical representations is a challenging task and it prevents instructors from using such tools. One of the most challenging task for the development of graphical representations is the layout of objects. The programs for drawing layouts can be difficult to write for most programmers. There are libraries that provide general purpose layouts, but they might not be suitalbe for the user. The problem of writing suitable layout programs can be solved by using modern program synthesis tools that can generate programs from high level specifications in the form of constraints.

In this report, we demonstrate high-level specification and synthesis of two types of layout programs: One-dimentional arrays and Trees.

The array layout requires linear arrangement of data points which are spaced eqally apart. Spacing between data points and size of each shape representing a data point can be parametrized.

Tree layout is two-dimentional and can take several forms. The layout we consider here, is a heirarchical top-down representation in which the root is that the top and each level is drawn vertically below and parallel to the previous one.

A layout program takes as input a data-structure and returns a list of coordinates in the two-dimentional plane that represent the layout of visual representation of the data-structure.

3 Program Synthesis

Program synthesis deals with automated generation of programs in a given language using high-level specifications. The goal of program synthesis is to generate programs that are correct by construction and require minimal specification from the user.

Several types of specification have been developed which are suitable for different kind of problems and use-cases. Programs can be specified using input-output examples[1], traces, partial programs [2], reference programs [2], natural language [3] or complete formal specification[4] using logic formulas.

Program synthesis is a second order search problem. The goal is to search for a program in a given program space restricted by the constraints given in the specification.

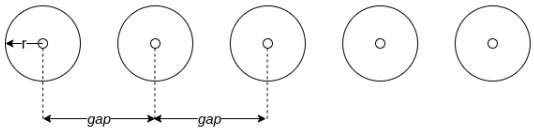
In this work, sketch based program synthesis is used to generate layout programs using partial programs.

4 Layout Algorithms

4.1 Array Layout

Array layout is a one dimentional diagram. We generally draw arrays horizontally, in which case the y-coordinate remains constant and we only need to find the the x-coordinates for all all data points. The contraints for an array layout can describes as follows:

$$\begin{aligned} \forall i, i \in N \wedge (0 \leq i < array.length - 1) \\ \implies x_{i+1} - x_i = gap \end{aligned} \tag{1}$$



The above constraint states that each pair of adjacent points are spaced equally apart and the that the separation between each adjacent pair should be equal to the given *gap* parameter.

4.1.1 Synthesis of Array Layout Program

Inputs to the program that generates the list of coordinates for an array layout are the size of the array and gap between each element.

The precondition for this function is that the gap should be greater than the diameter of each item, which would ensure that there is enough space for the items.

```
int[N] arrayLayout(int N, int gap){
  int[N] coords;
  for (int i=0; i<N; i++) {
    coords[i] = {| ?? (+|*) gap (+|*) i |};
  }
  return coords;
}
```

The predicate *equidistant(coords, gap)* listed below checks if the gap between each element is consistent.

```
bit equidistant([int N], int[N] coords, int gap) {
  for(int i=0; i<N-1; i++) {
    if (coords[i+1] - coords[i] != gap) {
      return false;
    }
  }
  return true;
}

harness void main(){
  int N = 5;
  int gap = 10;
  int radius = 3;
  assert(gap > 2*radius);
  int[N] res = arrayLayout(N, gap);
  assert equidistant(res, radius, gap);
}
```

4.2 Tree Layout

A *Tree*[5] is formally a finite, directed, connected, acyclic graph in which every node has at most one predecessor and exactly one distinguished node, the root, has none

The conventional way of representing trees is to start with the root at the top of the figure, and drawing the children below the root, such that the root is centered between the children and arrows are pointing from root to each child. This representation clearly highlights the hierarchical nature of trees.

Several aesthetic requirements have been identified for such tree drawings, that ensure certain properties about the tree are visible in the drawings while also following constraints of space availability. Table 1 describes some of the aesthetic requirements. Along with these aesthetics the tree drawings should also be as narrow as possible.

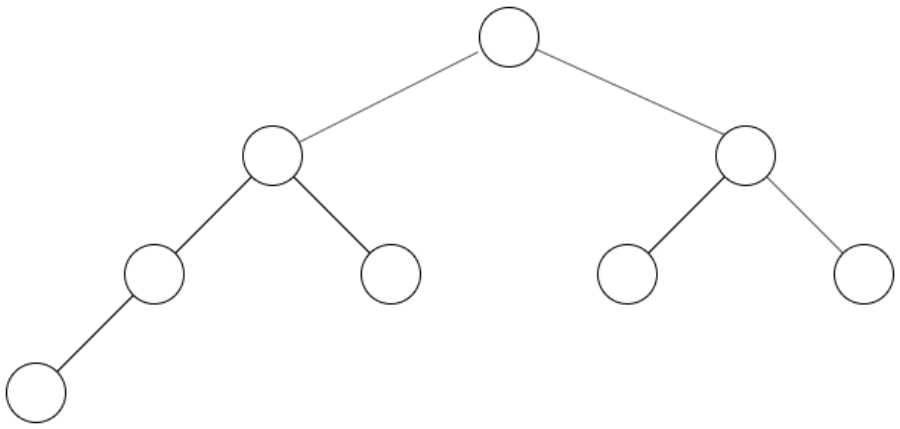


Figure 1: Tree drawing generated using Knuth's algorithm. Such drawings contain single node in each column, hence the width is maximum

Algorithms for tree layout generation confirm to some or all of these aesthetic requirements. The first tree drawing algorithm developed by Donald Knuth [6] generates drawings that follow the first three aesthetic rules, but generate drawings that are as wide as the number of nodes. Shannon and Wetherell[5] developed an improved algorithm that generates trees narrower than the knuth's algorithm. Reingold Tilford [7] present an algorithm that generates narrowest possible drawings while confirming to all the aesthetics.

Aesthetic	**Description**
Parallel Levels	Nodes of a tree at the same height should lie along a straight line, and the straight lines defining the levels should be parallel
Relative Placement	In a binary tree, each left child should be placed left of its parent and each right child right of its parent
Centering	A parent should be centered over its children
Symmetry	A tree and its mirror image should produce drawings that are reflections of one another; moreover, a sub-tree should be drawn the same way regardless of where it occurs in the tree.

Table 1: List of Aesthetic Requirements for a tree layout

Aesthetic	Knuth	Wetherell 1	Wetherell 2	Reingold Tilford
Parallel Levels	True	True	True	True
Relative Placement	True	True	True	True
Centering	True	True	False	True
Symmetry	False	False	False	True

Table 2: Algorithms and Aesthetics

The above mentioned aesthetics are some examples of the types of constraints possible on a layout algorithm. A different application may require some other aesthetic or physical constraints. A different algorithms would be needed for these constraints. The process of developing these algorithms can be simplified by writing partial programs that define high-level strategies and predicates that define the constaints. The complete implementation can then be generated using program synthesis solvers.

4.2.1 Aesthetics as Constraints

The aesthetics are encoded as predicates that check if the aesthetic requirement is satisfied by a given algorithm or not.

For a binary tree, the *parallel levels* aesthetic is described in equation 2. This particular implementation assumes that all lines on which nodes at a level are placed should be horizontal (parallel to x-axis). This is a stronger constraint than allowing lines to parallel at any angle, but for this application, it should be acceptable because most tree drawings have horizontal level-lines.

$$\forall node, node \in nodeset(tree) \wedge \neg(node.left = null) \wedge \neg(node.right = null) \implies node.left.y = node.right.y \tag{2}$$

```
bit parallelLevels(Tree root) {
  if ((root.left != null) && (root.right != null)) {
    if (root.left.y != root.right.y) {
      return false;
    }
    parallelLevels(root.right);
    parallelLevels(root.left);
  }
  return true;
}
```

The *Relative Placement* aesthetic states that the left child should be places left of its parent and the right child should be places right of its parent. Equation 3 describes this aesthetic.

$$\begin{aligned} &\forall node, node \in nodeset(tree) \\ \implies &((node.left.x < node.x) \wedge (node.x < node.right.x)) \end{aligned} \tag{3}$$

```
bit relativePlacement(Tree root) {
  if (root.left != null) {
    if (root.left.x > root.x || root.left.x == root.x) {
      return false;
    }
    relativePlacement(root.left);
  }
  if (root.right != null) {
    if (root.right.x < root.x || root.right.x == root.x) {
      return false;
    }
    relativePlacement(root.right);
  }
  return true;
}
```

The centering aesthetic is described by equation 4.

$$\begin{aligned} &\forall node, node \in nodeset(tree) \wedge \neg(node.left = null) \wedge \neg(node.right = null) \\ &\implies node.x = (node.left.x + node.right.x)/2 \end{aligned} \tag{4}$$

```
bit centering(Tree root) {
  if ((root.left != null) && (root.right != null)) {
    if (root.center.x != (root.left.x + root.right.x) / 2) {
      return false;
    }
    centering(root.right);
    centering(root.left);
  }
  return true;
}
```

The predicates described above act as specifications for the layout program, which is written as a partial program with holes. Provided with the partial program and the constraints, the synthesis solver generates a program which satisfies the given constraints, if any such solution is possible.

```
generator int treeLayout([int N_NODES], TreeNode root,
int i, int depth, ref int[3] nexts, ref int next_x) {
  if (root == null) {
```

```
    return i;
  }
  int j = treeLayout(root.left, i, depth+1, nexts, next_x);
  point = {root.val, {| nexts[depth] | next_x |}, depth};
  root.x = point[0];
  root.y = point[1];
  {| nexts[depth] | next_x |} += 1;
  int k = treeLayout(root.right, j+1, depth+1, nexts, next_x);
  return k;
}
```

The above listing describes a partial program that performs inorder traversal on a tree and assigns x and y values to each node. Exact eqauations for assigning x and y coordinates are derived by the synthesis solver using the constraints. The constraints which are defined as predicates can applied to the partial program using assertions as described below.

```
harness void main() {

  // setup ...

  // partial program generator
  treeLayout(root, 0, nexts, next_x);

  // layout constraints
  assert(parallelLevels(root));
  assert(centering(root));
  assert(relativePosition(root));
}
```

The above specification will generate a layout program that satisfied the assertions and is derived from the stucture provided in the partial program.

Im addition to the predicates described above, the solver can also generate program using input-output examples as constraints.

5 Conclusion and Future Work

This report described the application of sketch based program synthesis to generate data structure layout programs. A summary of aesthetics and several algorithms that genrerate tree layouts was discribed. The method to specify tree layout programs using logical constraints that is discussed in this report is a basis to develop new algorithms using some other aesthetics and can be extended to other data strcutures.

In future work, I plan to use this method to develop an application where a user can describe the constraints using tree drawings as examples to generate layouts programs. An interactive environment where a user can easily specify

the aesthetics should be more useable than writing predicates and partial programs. The extent to which the writing of speficiation and partial programs can be eleminated in favor of input-output example and what kinds of algorithms can be generated from them remains to be seen.

References

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