Lab Writeup

-The following will discuss and compare the results between the library and user-defined values of the functions sine, cosine, tangent and e^x.

1) Tangent

x	Tan	Library	Difference
-	---	-------	----------
-1.5718	999.99966688	999.99966667	0.0000002094
-1.3754	-5.05374640	-5.05374640	0.0000000000
-1.1791	-2.42105852	-2.42105852	0.0000000000
-0.9827	-1.49985045	-1.49985045	0.0000000000
-0.7864	-1.00200200	-1.00200200	0.0000000000
-0.5900	-0.66962607	-0.66962607	0.0000000000
-0.3937	-0.41538562	-0.41538562	-0.0000000000
-0.1973	-0.19995214	-0.19995214	0.0000000000
-0.0010	-0.00100000	-0.00100000	0.0000000000
0.1953	0.19787301	0.19787301	0.0000000000
0.3917	0.41304247	0.41304247	0.0000000000
0.5880	0.66673314	0.66673314	0.0000000000
0.7844	0.99800200	0.99800200	-0.0000000000
0.9807	1.49337077	1.49337077	0.0000000000
1.1771	2.40740158	2.40740158	-0.0000000000
1.3734	5.00119677	5.00119677	-0.0000000000
1.5698	999.99966646	999.99966667	-0.0000002044

For the tangent test above, the difference between the library and the Pade approximation used is nearly non-existent at least to the degree of 10^-10. There is some inaccuracy as reflected by the difference calculated near x = -pi/2 and x=pi/2 where the difference is on the order of 10^-7. This indicates that there was a slight difference between the library and the approximation near the vertical asymptotes. This is expected as tan(x) has vertical asymptotes at pi/2 and -pi/2 and so the aggressive change in the function value near the asymptotes, at which it's undefined, may not be accounted for as well in the approximation.

<u>Potential Causes of Error:</u>

- The aggressive/steep change in the function near the asymptotes may not be accounted for in approximation.
- Not in enough taylor terms calculated, given the domain, for the series that the Pade approximant is based on.

2) Sine

```
x       Sin             Library         Difference
-       ---             -------         ----------
-6.2832 0.10503870      0.00000000      0.1050386974
-6.0868 0.26715334      0.19509032      0.0720630157
-5.8905 0.43122601      0.38268343      0.0485425810
-5.6941 0.58764012      0.55557023      0.0320698862
-5.4978 0.72786095      0.70710678      0.0207541677
-5.3014 0.84460858      0.83146961      0.0131389683
-5.1051 0.93200442      0.92387953      0.0081248826
-4.9087 0.98568478      0.98078528      0.0048995012
-4.7124 1.00287581      1.00000000      0.0028758076
-4.5160 0.98242487      0.98078528      0.0016395919
-4.3197 0.92478538      0.92387953      0.0009058520
-4.1233 0.83195330      0.83146961      0.0004836848
-3.9270 0.70735562      0.70710678      0.0002488415
-3.7306 0.55569315      0.55557023      0.0001229160
-3.5343 0.38274149      0.38268343      0.0000580556
-3.3379 0.19511642      0.19509032      0.0000260947
-3.1416 0.00001110      -0.00000000     0.0000110990
-2.9452 -0.19508588     -0.19509032     0.0000044372
-2.7489 -0.38268178     -0.38268343     0.0000016539
-2.5525 -0.55556966     -0.55557023     0.0000005691
-2.3562 -0.70710660     -0.70710678     0.0000001785
-2.1598 -0.83146956     -0.83146961     0.0000000503
-1.9635 -0.92387952     -0.92387953     0.0000000125
-1.7671 -0.98078528     -0.98078528     0.0000000026
-1.5708 -1.00000000     -1.00000000     0.0000000005
-1.3744 -0.98078528     -0.98078528     0.0000000001
-1.1781 -0.92387953     -0.92387953     0.0000000000
-0.9817 -0.83146961     -0.83146961     0.0000000000
-0.7854 -0.70710678     -0.70710678     0.0000000000
-0.5890 -0.55557023     -0.55557023     0.0000000000
-0.3927 -0.38268343     -0.38268343     -0.0000000000
-0.1963 -0.19509032     -0.19509032     0.0000000000
0.0000  0.00000000      0.00000000      0.0000000000
0.1963  0.19509032      0.19509032      0.0000000000
0.3927  0.38268343      0.38268343      -0.0000000000
0.5890  0.55557023      0.55557023      -0.0000000000
0.7854  0.70710678      0.70710678      -0.0000000000
0.9817  0.83146961      0.83146961      -0.0000000000
1.1781  0.92387953      0.92387953      -0.0000000000
1.3744  0.98078528      0.98078528      -0.0000000001
1.5708  1.00000000      1.00000000      -0.0000000005
1.7671  0.98078528      0.98078528      -0.0000000026
1.9635  0.92387952      0.92387953      -0.0000000125
2.1598  0.83146956      0.83146961      -0.0000000503
2.3562  0.70710660      0.70710678      -0.0000001785
2.5525  0.55556966      0.55557023      -0.0000005691
2.7489  0.38268178      0.38268343      -0.0000016539
2.9452  0.19508588      0.19509032      -0.0000044372
```

3.1416	-0.00001110	-0.00000000	-0.0000110990
3.3379	-0.19511642	-0.19509032	-0.0000260947
3.5343	-0.38274149	-0.38268343	-0.0000580556
3.7306	-0.55569315	-0.55557023	-0.0001229160
3.9270	-0.70735562	-0.70710678	-0.0002488415
4.1233	-0.83195330	-0.83146961	-0.0004836848
4.3197	-0.92478538	-0.92387953	-0.0009058520
4.5160	-0.98242487	-0.98078528	-0.0016395919
4.7124	-1.00287581	-1.00000000	-0.0028758076
4.9087	-0.98568478	-0.98078528	-0.0048995012
5.1051	-0.93200442	-0.92387953	-0.0081248826
5.3014	-0.84460858	-0.83146961	-0.0131389683
5.4978	-0.72786095	-0.70710678	-0.0207541677
5.6941	-0.58764012	-0.55557023	-0.0320698862
5.8905	-0.43122601	-0.38268343	-0.0485425810
6.0868	-0.26715334	-0.19509032	-0.0720630157
6.2832	-0.10503870	0.00000000	-0.1050386974

For the sine test results above, it's quite evident that when the simulation moves from -2pi, which is furthest from zero on the domain, to 2pi, which is equally far, the accuracy of the approximant, at first, increases as indicated by the difference value getting closer to zero. However, once the simulation reaches x=0 and goes towards 2pi, the difference increase again as we get further from zero. As a result, values near zero on the domain have the better accuracy compared to values further from it.

<u>Potential Causes of Error:</u>

- Pade approximant for sine is based on a series centered around 0. As a result, any values further from 0 on the simulation domain from -2pi to 2pi will be relatively less accurate than those closer.
- Not in enough taylor terms calculated, given the domain, for the series that the Pade approximant is based on.
- Since the simulation domain is larger than that of the tan function, the pade approximant would have to be determined for taylor series of more terms would be helpful.

3) Cos

x	Cos	Library	Difference
-	---	-------	----------
-6.2832	0.72764144	1.00000000	-0.2723585573
-6.0868	0.78416194	0.98078528	-0.1966233430
-5.8905	0.78442229	0.92387953	-0.1394572410
-5.6941	0.73439688	0.83146961	-0.0970727331
-5.4978	0.64087026	0.70710678	-0.0662365216
-5.3014	0.51132312	0.55557023	-0.0442471158
-5.1051	0.35378699	0.38268343	-0.0288964397
-4.9087	0.17667019	0.19509032	-0.0184201345
-4.7124	-0.01144113	-0.00000000	-0.0114411282
-4.5160	-0.20200097	-0.19509032	-0.0069106486
-4.3197	-0.38673367	-0.38268343	-0.0040502348
-4.1233	-0.55786776	-0.55557023	-0.0022975303
-3.9270	-0.70836459	-0.70710678	-0.0012578109
-3.7306	-0.83213199	-0.83146961	-0.0006623824
-3.5343	-0.92421379	-0.92387953	-0.0003342565
-3.3379	-0.98094619	-0.98078528	-0.0001609112
-3.1416	-1.00007351	-1.00000000	-0.0000735083
-2.9452	-0.98081695	-0.98078528	-0.0000316664
-2.7489	-0.92389230	-0.92387953	-0.0000127667
-2.5525	-0.83147438	-0.83146961	-0.0000047726
-2.3562	-0.70710842	-0.70710678	-0.0000016354
-2.1598	-0.55557074	-0.55557023	-0.0000005063
-1.9635	-0.38268357	-0.38268343	-0.0000001390
-1.7671	-0.19509036	-0.19509032	-0.0000000330
-1.5708	-0.00000001	0.00000000	-0.0000000066
-1.3744	0.19509032	0.19509032	-0.0000000010
-1.1781	0.38268343	0.38268343	-0.0000000001
-0.9817	0.55557023	0.55557023	-0.0000000000
-0.7854	0.70710678	0.70710678	-0.0000000000
-0.5890	0.83146961	0.83146961	-0.0000000000
-0.3927	0.92387953	0.92387953	0.0000000000
-0.1963	0.98078528	0.98078528	0.0000000000
0.0000	1.00000000	1.00000000	0.0000000000
0.1963	0.98078528	0.98078528	0.0000000000
0.3927	0.92387953	0.92387953	0.0000000000
0.5890	0.83146961	0.83146961	-0.0000000000
0.7854	0.70710678	0.70710678	-0.0000000000
0.9817	0.55557023	0.55557023	-0.0000000000
1.1781	0.38268343	0.38268343	-0.0000000001
1.3744	0.19509032	0.19509032	-0.0000000010
1.5708	-0.00000001	-0.00000000	-0.0000000066
1.7671	-0.19509036	-0.19509032	-0.0000000330
1.9635	-0.38268357	-0.38268343	-0.0000001390
2.1598	-0.55557074	-0.55557023	-0.0000005063
2.3562	-0.70710842	-0.70710678	-0.0000016354
2.5525	-0.83147438	-0.83146961	-0.0000047726
2.7489	-0.92389230	-0.92387953	-0.0000127667
2.9452	-0.98081695	-0.98078528	-0.0000316664
3.1416	-1.00007351	-1.00000000	-0.0000735083
3.3379	-0.98094619	-0.98078528	-0.0001609112

3.5343	-0.92421379	-0.92387953	-0.0003342565
3.7306	-0.83213199	-0.83146961	-0.0006623824
3.9270	-0.70836459	-0.70710678	-0.0012578109
4.1233	-0.55786776	-0.55557023	-0.0022975303
4.3197	-0.38673367	-0.38268343	-0.0040502348
4.5160	-0.20200097	-0.19509032	-0.0069106486
4.7124	-0.01144113	0.00000000	-0.0114411282
4.9087	0.17667019	0.19509032	-0.0184201345
5.1051	0.35378699	0.38268343	-0.0288964397
5.3014	0.51132312	0.55557023	-0.0442471158
5.4978	0.64087026	0.70710678	-0.0662365216
5.6941	0.73439688	0.83146961	-0.0970727331
5.8905	0.78442229	0.92387953	-0.1394572410
6.0868	0.78416194	0.98078528	-0.1966233430
6.2832	0.72764144	1.00000000	-0.2723585573

For the cosine test above, the results are very similar to that of the sine test. As x moves further from zero, regardless of it becoming more positive or negative, the difference between the approximation and library value increases. Since this type of behavior is also present in the sine simulation, this inaccuracy could be attributed to Pade approximant as well.

Potential Causes of Error:

- Pade approximant for cosine is based on a series centered around 0. As a result, any values further from 0 on the simulation domain from -2pi to 2pi will be relatively less accurate than those closer.
- Not in enough taylor terms calculated, given the domain, for the series that the Pade approximant is based on.
- Since the simulation domain is larger than that of the tan function, the pade approximant would have to be determined for taylor series of more terms would be helpful.

4) Exponential function(e^x)

x	Exp	Library	Difference
1.0000	2.71828183	2.71828183	-0.0000000000
1.1000	3.00416602	3.00416602	-0.0000000000
1.2000	3.32011692	3.32011692	-0.0000000000
1.3000	3.66929667	3.66929667	-0.0000000000
1.4000	4.05519997	4.05519997	-0.0000000000
1.5000	4.48168907	4.48168907	-0.0000000000
1.6000	4.95303242	4.95303242	-0.0000000001
1.7000	5.47394739	5.47394739	-0.0000000000
1.8000	6.04964746	6.04964746	-0.0000000001
1.9000	6.68589444	6.68589444	-0.0000000000
2.0000	7.38905610	7.38905610	-0.0000000000
2.1000	8.16616991	8.16616991	-0.0000000001
2.2000	9.02501350	9.02501350	-0.0000000000
2.3000	9.97418245	9.97418245	-0.0000000001
2.4000	11.02317638	11.02317638	-0.0000000000
2.5000	12.18249396	12.18249396	-0.0000000000
2.6000	13.46373803	13.46373804	-0.0000000001
2.7000	14.87973172	14.87973172	-0.0000000000
2.8000	16.44464677	16.44464677	-0.0000000001
2.9000	18.17414537	18.17414537	-0.0000000001
3.0000	20.08553692	20.08553692	-0.0000000000
3.1000	22.19795128	22.19795128	-0.0000000001
3.2000	24.53253020	24.53253020	-0.0000000001
3.3000	27.11263892	27.11263892	-0.0000000000
3.4000	29.96410005	29.96410005	-0.0000000001
3.5000	33.11545196	33.11545196	-0.0000000001
3.6000	36.59823444	36.59823444	-0.0000000000
3.7000	40.44730436	40.44730436	-0.0000000001
3.8000	44.70118449	44.70118449	-0.0000000002
3.9000	49.40244911	49.40244911	-0.0000000000
4.0000	54.59815003	54.59815003	-0.0000000001
4.1000	60.34028760	60.34028760	-0.0000000002
4.2000	66.68633104	66.68633104	-0.0000000000
4.3000	73.69979370	73.69979370	-0.0000000001
4.4000	81.45086866	81.45086866	-0.0000000002
4.5000	90.01713130	90.01713130	-0.0000000000
4.6000	99.48431564	99.48431564	-0.0000000001
4.7000	109.94717245	109.94717245	-0.0000000002
4.8000	121.51041752	121.51041752	-0.0000000000
4.9000	134.28977968	134.28977968	-0.0000000001
5.0000	148.41315910	148.41315910	-0.0000000001
5.1000	164.02190730	164.02190730	-0.0000000000
5.2000	181.27224188	181.27224188	-0.0000000001
5.3000	200.33680997	200.33680997	-0.0000000001
5.4000	221.40641620	221.40641620	-0.0000000000
5.5000	244.69193226	244.69193226	-0.0000000001
5.6000	270.42640743	270.42640743	-0.0000000001
5.7000	298.86740097	298.86740097	-0.0000000002

5.8000	330.29955991	330.29955991	-0.0000000001
5.9000	365.03746787	365.03746787	-0.0000000001
6.0000	403.42879349	403.42879349	-0.0000000002
6.1000	445.85777008	445.85777008	-0.0000000001
6.2000	492.74904109	492.74904109	-0.0000000001
6.3000	544.57191013	544.57191013	-0.0000000002
6.4000	601.84503787	601.84503787	-0.0000000001
6.5000	665.14163304	665.14163304	-0.0000000001
6.6000	735.09518924	735.09518924	-0.0000000002
6.7000	812.40582517	812.40582517	-0.0000000001
6.8000	897.84729165	897.84729165	-0.0000000001
6.9000	992.27471560	992.27471561	-0.0000000001
7.0000	1096.63315843	1096.63315843	-0.0000000002
7.1000	1211.96707449	1211.96707449	-0.0000000001
7.2000	1339.43076439	1339.43076439	-0.0000000001
7.3000	1480.29992758	1480.29992758	-0.0000000002
7.4000	1635.98443000	1635.98443000	-0.0000000001
7.5000	1808.04241446	1808.04241446	-0.0000000001
7.6000	1998.19589510	1998.19589510	-0.0000000002
7.7000	2208.34799189	2208.34799189	-0.0000000001
7.8000	2440.60197762	2440.60197762	-0.0000000001
7.9000	2697.28232827	2697.28232827	-0.0000000001
8.0000	2980.95798704	2980.95798704	-0.0000000002
8.1000	3294.46807528	3294.46807528	-0.0000000001
8.2000	3640.95030733	3640.95030733	-0.0000000001
8.3000	4023.87239382	4023.87239382	-0.0000000002
8.4000	4447.06674770	4447.06674770	-0.0000000001
8.5000	4914.76884030	4914.76884030	-0.0000000001
8.6000	5431.65959136	5431.65959136	-0.0000000002
8.7000	6002.91221726	6002.91221726	-0.0000000003
8.8000	6634.24400628	6634.24400628	-0.0000000001
8.9000	7331.97353916	7331.97353916	-0.0000000001
9.0000	8103.08392758	8103.08392758	-0.0000000002
9.1000	8955.29270348	8955.29270348	-0.0000000001
9.2000	9897.12905874	9897.12905874	-0.0000000001
9.3000	10938.01920816	10938.01920817	-0.0000000002
9.4000	12088.38073022	12088.38073022	-0.0000000001
9.5000	13359.72682966	13359.72682966	-0.0000000001
9.6000	14764.78156558	14764.78156558	-0.0000000002
9.7000	16317.60719801	16317.60719802	-0.0000000003
9.8000	18033.74492783	18033.74492783	-0.0000000001
9.9000	19930.37043823	19930.37043823	-0.0000000001
10.0000	22026.46579481	22026.46579481	-0.0000000002

For the exponential test above, the results are fairly steady like the tangent function test However, for each value there is a steady error of either -0.0000000001 or -0.0000000002. Since this test isn't based on a Pade approximant but an actual series that expands until the value of the taylor term(x^n/n!) goes below 10^-9 for enough taylor terms,n, for each x. Since the computation for each x goes to the order of 10^-9, the approximation is good enough to the point where there is no perceivable difference till the order of 10^-9.

Causes of error: Since the number of taylor terms calculated stop when the taylor term(x^n/n!) goes below 10^-9, the accuracy of the values past the of order 10^-9 will not be accurate.