Deutsch Algorithm

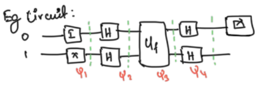
→ we have Query gate Circuit

→ Input is the "function" which is to be 'Queried'.

→ Efficiency is measured in the number of Queries required.

→ Dont worry about the function involved. this somewat theoretical concept on proving superposition, & Interferance.

Eg Circuit:



$\psi_1 = |1\rangle|0\rangle$

1st qubit ; 0th qubit

$$\psi_2 = |-\rangle|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$= \frac{1}{2}(|0\rangle - |1\rangle)|0\rangle + \frac{1}{2}(|0\rangle - |1\rangle)(|1\rangle)$$

Apply U_f.

$$\left.\begin{matrix} f(0) \\ f(1) \end{matrix}\right\} = |0 \oplus a\rangle - |1 \oplus a\rangle = (-1)^a(|0\rangle - |1\rangle)$$

Here, we have know the phase-kick back.

for any state 'b' if we apply the function 'c'

c → should be unitary

$$|b \oplus c\rangle = X^c|b\rangle$$

this can be verified with two values of c.

$$|b \oplus 0\rangle = X^0|b\rangle = |b\rangle$$

$$|b \oplus 1\rangle = X^1|b\rangle = |\neg b\rangle.$$

that is

$$X|b\rangle = -|b\rangle \rightarrow \text{Eigen vector}$$

(− → Eigen value)

this Negative is known as "phase kick back"

Because this Negative sign indicate phase shift.

Now after applying U_f.

$$\psi_3 = \frac{1}{2}(|0 \oplus f(0)\rangle - |1 \oplus f(0)\rangle)|0\rangle$$

$$+ \frac{1}{2}(|0 \oplus f(1)\rangle - |1 \oplus f(1)\rangle)|1\rangle$$

Depending first Qubit we are apply that function to the second Qubit.

Now from the phase kick back

$$\psi_3 = \frac{1}{2}(-1)^{f(0)}(|0\rangle - |1\rangle)|0\rangle$$

$$+ \frac{1}{2}(-1)^{f(1)}(|0\rangle - |1\rangle)|1\rangle$$

$$= \boxed{\frac{|0\rangle - |1\rangle}{\sqrt{2}}} \quad \frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}}$$

$$= |-\rangle \quad (-1)^{f(0)} \frac{|0\rangle + (-1)^{f(0) \oplus f(1)}|1\rangle}{\sqrt{2}}$$

If $f(0) \oplus f(1) = 0$

$$\psi_3 = (-1)^{f(0)}|-\rangle|+\rangle$$

$f(0) \oplus f(1) = 1$

$$\psi_3 = (-1)^{f(0)}|-\rangle|-\rangle$$

After applying final Hadamard

$$\psi_4 = (-1)^{f(0)}|1\rangle|0\rangle$$