Normalized Subband Adaptive Filter Algorithms for Acoustic Echo Cancellation Using Set Membership

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Abstract

This report presents the implementation and evaluation of set-membership improved normalized subband adaptive filter (SM-INSAF) algorithms for acoustic echo cancellation (AEC). Based on the concepts of the INSAF framework and the set-membership filtering theory, the SM-INSAF and its smoothed variant (SSM-INSAF) are developed to enhance steady-state performance and reduce computational complexity. Simulation results under different echo path conditions (sparse and dispersive) demonstrate that the proposed algorithms outperform traditional INSAF approaches in low signal-to-noise ratio (SNR) environments.

1. Introduction

Acoustic echo cancellation (AEC) is a crucial challenge in modern telecommunication systems, especially in hands-free telephony and teleconferencing. Conventional adaptive filters like the normalized least mean square (NLMS) algorithm often suffer from slow convergence when dealing with coloured inputs such as speech [1]. To address these issues, subband adaptive filters (SAFs) and their variants have been proposed. The improved normalized subband adaptive filter (INSAF) utilizes past weight vectors to enhance robustness in noisy environments. However, further improvements are necessary to balance convergence speed, steady-state error, and computational efficiency. This work implements set-membership concepts into INSAF, yielding SM-INSAF and SSM-INSAF algorithms [1].

2. Algorithm Description [1]

The SM-INSAF algorithm modifies the INSAF by introducing a constraint set based on output error bounds. The weight update occurs only when the subband error exceeds a predefined threshold, reducing unnecessary computations. Furthermore, the SSM-INSAF variant applies smoothing to the subband error estimates to stabilize the step size variation, leading to improved steady-state performance.

2.1 Signal Preparation

Sampling Rate and Duration:
I set the sampling rate to fs=8000Hz and the signal duration to T=5 seconds, resulting in L=fs×T=40000 samples.

Input Generation:
The input signal u(n) is generated as a first-order autoregressive (AR(1)) process, filtered from white Gaussian noise with a pole at 0.9.

Echo Path Construction:

- **Sparse System**: Only a few taps have significant values. (e.g., taps at indices 30, 90, 250, 400).
- **Dispersive System**: A long FIR filter with energy spread across many taps (based on a lowpass FIR filter).

Desired Signal:
The clean echo signal is generated by convolving u(n) with the echo path w_{true} , and additive white Gaussian noise is added to reach a target SNR of 20 dB.

$$d(n)=u(n)Tw_{true}+v(n)$$

2.2 Subband Decomposition

Filter Bank Design:
A cosine-modulated filter bank is designed using a prototype FIR lowpass filter (Hamming windowed) to split signals into N_{sub} subbands.

Subband Signal Generation:
The input u(n)and desired d(n) signals are filtered into N_{sub} subbands without down sampling.

Noise Subband Estimation:
An independent noise sequence is filtered similarly to estimate subband noise powers, required for threshold calculation.

Subband error:

$$\varepsilon_{i,D}(k) = d_{i,D}(k) - u_i^T(k)\,\bar{w}(k)$$

2.3 Set-Membership Conditions

Error Bound Calculation:
For each subband i, the error threshold γi is determined as:

$$\gamma_i = t_{\text{bound}} \times \sqrt{\sigma_{\eta,i}}$$

where $\sigma_{\eta,i}$ is the noise variance in the i-th subband.

Adaptive Step-Size Control:

- **SM-INSAF**:
  Update occurs only if $|\epsilon_i(k)|>\gamma_i$. The adaptive step size is computed as:

$$\mu_i(k) = 1 - \frac{\gamma_i}{|\epsilon_i(k)|}$$

- **SSM-INSAF**:
  A smoothed error $\sigma_{\epsilon,i}(k)$ is used:

$$\sigma_{\epsilon,i}(k) = \beta\, \sigma_{\epsilon,i}(k-1) + (1-\beta)\, |\epsilon_i(k)|$$

  Update happens only if both $|\epsilon_i(k)|$and $\sigma_{\epsilon,i}(k)$ exceed γ_i.

2.4 Adaptive Filtering Structure

Past Weight Reuse:
A moving average over the past P weight vectors is used to form the reference:

$$\bar{w}(k) = \frac{1}{P}\sum_{p=0}^{P-1} w(k-p)$$

Subband Update Equation:
For each eligible subband i,

$$\Delta w \leftarrow \Delta w + \mu_0\, \mu_i(k) \frac{\epsilon_i(k) u_i(k)}{\|u_i(k)\|_2^2 + \delta}$$

Weight Update:

$$w(k+1) = \bar{w}(k) + \Delta w$$

Efficiency Consideration:
When no subband violates the threshold, no update is performed, significantly saving computation.

2.5 Simulation Settings

Sparse Echo Path:

- N_{sub}=16, step size equals subband number.

- P=2 past weights, β=0.7.

Dispersive Echo Path:

- N_{sub}=8.
- P=2 past weights, β=0.9.

Parameter Sensitivity Tests:

- Comparisons over varying N_{sub} , step size, and number of reused weights P.

3. NUMERICAL EXPERIMENTS

3.1 Noise Echo Paths

In practical acoustic echo cancellation (AEC) tasks, the unknown system to be identified can exhibit different structural properties. Two common types of echo paths are encountered in real environments (**Figure 1**):

- Sparse Echo Path: Only a few filter coefficients are significantly non-zero, typically corresponding to strong early reflections. This sparsity is common in small rooms with prominent primary echoes.
- Dispersive Echo Path: Many filter coefficients are active with relatively small but distributed energy, representing reverberant environments where multiple reflections with similar strengths occur over time.

These distinct conditions necessitate adaptive filtering algorithms that can perform well in both scenarios. In the following subsections, we evaluate the SM-INSAF and SSM-INSAF algorithms under each type of echo path.

3.2 Sparse System

Figure 2 shows the NMSD performance comparison of SM-INSAF, SSM-INSAF, and Full-INSAF algorithms under a sparse echo path. The results demonstrate that both set-membership algorithms achieve significantly better steady-state accuracy than Full-INSAF. In particular, the SSM-INSAF exhibits the lowest NMSD and smoothest curve, thanks to its error smoothing mechanism. Moreover, the Full-INSAF, which updates at every iteration, converges slowly and suffers from higher residual error levels.

3.3 Dispersive System

Figure 3 illustrates the normalized mean square deviation (NMSD) performance of SM-INSAF, SSM-INSAF, and Full-INSAF algorithms under a dispersive echo path scenario. All algorithms exhibit stable convergence, but key differences are observable:

- The **Full-INSAF** converges more slowly and settles at a higher steady-state error, as expected due to its constant step size and full update at each iteration.

- The **SM-INSAF** improves convergence and achieves lower residual error by using a selective update mechanism.
- The **SSM-INSAF** performs best, with the lowest steady-state NMSD and a relatively smooth transition. This confirms that smoothing the subband error yields more stable adaptation in dense systems.

3.4 Effect of Subband Number, Step Size ρ, and Memory Depth

Figure 4 presents a comparative study of SM-INSAF performance under different algorithmic parameters. The normalized mean square deviation (NMSD) is plotted against input samples, clearly showing how each parameter influences convergence rate and steady-state error:

- **Subplot (a)**: Increasing the number of subbands ($N = 2, 4, 8$) significantly improves convergence and final accuracy. A higher number of subbands allows better whitening of input signals, benefiting adaptation. However, improvement tends to saturate beyond $N = 8$.
- **Subplot (b)**: Smaller step sizes (e.g., step = 4) yield slower convergence but better final accuracy. Larger step sizes (e.g., step = 16) accelerate early adaptation but suffer from higher steady-state errors due to over-updating.
- **Subplot (c)**: Reusing more past weight vectors ($P = 1, 2, 3$) enhances steady-state performance by smoothing fluctuations, though the marginal benefit reduces with each additional vector. Notably, $P = 2$ appears to balance performance and complexity well.

From all subplots, it's evident that choosing the right combination of parameters is crucial depending on whether the goal is fast convergence (e.g., for real-time applications) or minimum residual error (e.g., for high-fidelity echo cancellation).

3.5 Effect of Subband Step Size μ

Figure 4 shows the performance comparison of SSM-INSAF with varying global step sizes μ. Three values were tested: 0.1 (conservative), 0.3 (moderate), and 0.5 (aggressive). The figure clearly illustrates the trade-off between convergence speed and steady-state accuracy:

- **μ_0=0.5**: The algorithm converges rapidly in early iterations but plateaus at a higher normalized mean square deviation (NMSD), indicating degraded steady-state accuracy and higher fluctuations.
- **μ_0=0.3**: Offers a balance between speed and precision, with relatively fast

convergence and improved steady-state error. This setting was used in most simulations throughout the report.

- **μ0=0.1**: Delivers the lowest steady-state NMSD, demonstrating high accuracy, but at the cost of significantly slower initial convergence. Suitable when precise echo suppression is preferred over quick adaptation.

This comparison highlights the importance of step size tuning in achieving optimal performance under various acoustic conditions.

4. Conclusions

This project investigated the implementation and performance of SM-INSAF and its smoothed variant, SSM-INSAF, in the context of acoustic echo cancellation. Building upon the INSAF framework, the incorporation of set-membership filtering principles effectively reduced the computational burden by avoiding unnecessary updates, while maintaining or even improving convergence behavior. Simulation results under both sparse and dispersive echo path conditions confirmed that SM-INSAF and SSM-INSAF outperform the traditional INSAF algorithm, especially in low SNR scenarios. Notably, SSM-INSAF achieved superior steady-state accuracy due to its smoothing mechanism for error estimation. Additionally, parameter studies revealed that the number of subbands, adaptive step size, and memory depth significantly affect the trade-off between convergence speed and final error level. Overall, the set-membership approach demonstrates strong potential for efficient and accurate echo path identification in practical AEC systems.

5. References

[1] Yi Yu, Haiquan Zhao, Badong Chen, "Set-membership improved normalised subband adaptive filter algorithms for acoustic echo cancellation," IET Signal Processing, vol. 12, no. 1, pp. 42-50, 2018.

Appendix

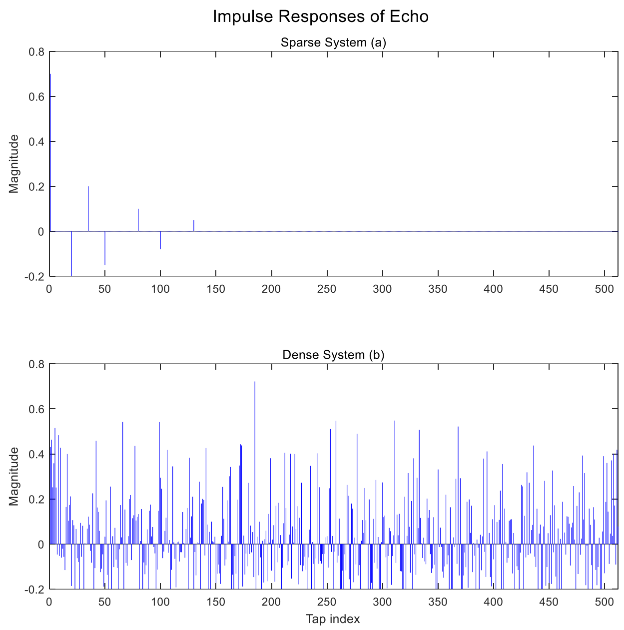

Fig. 1 Two acoustic echo paths (a) Sparse case, (b) Dispersive case

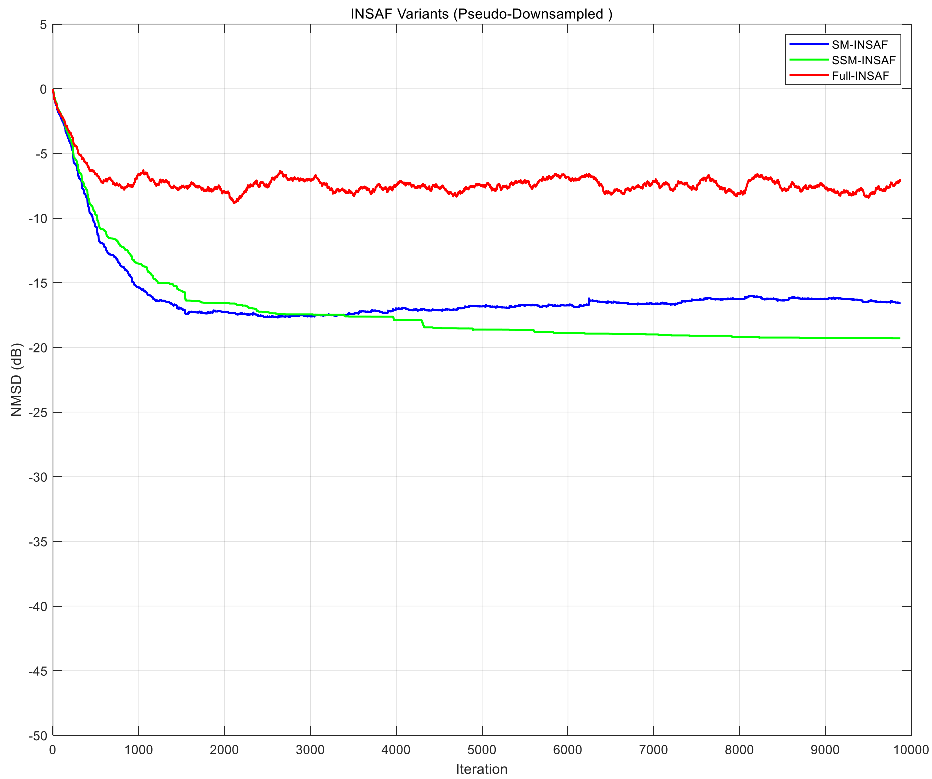

Fig. 2 NMSD curves of various SAF algorithms for a dispersive echo path

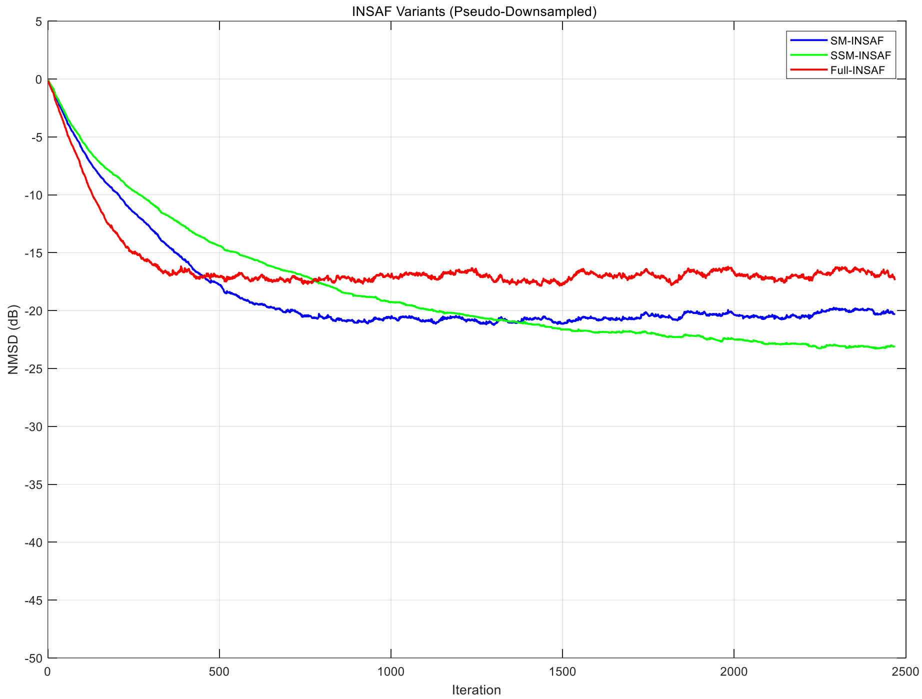

Fig. 3 NMSD curves of various SAF algorithms for a sparse echo path

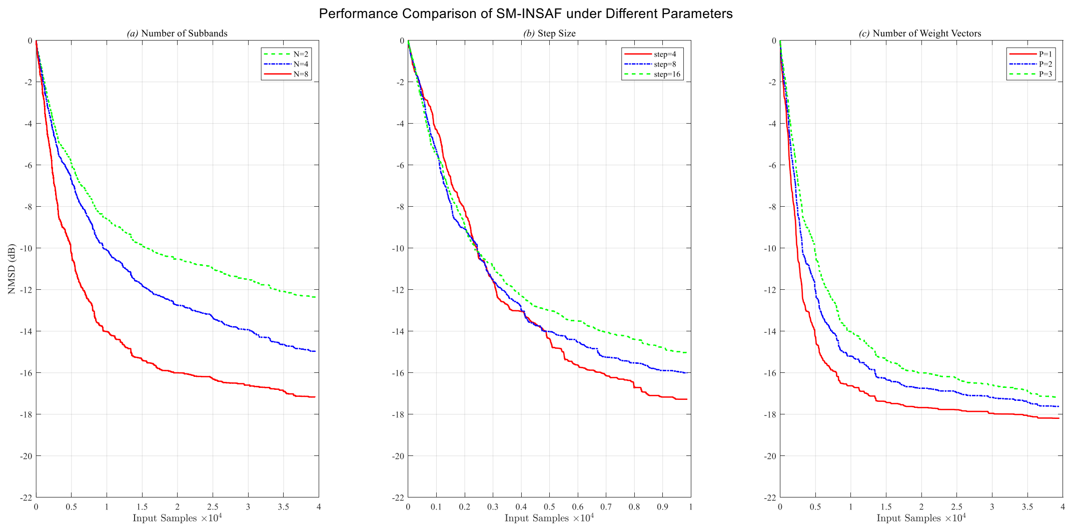

Fig. 4 NMSD curves of the SM-INSAF versus various parameters (a) Different N (b) Different ρ (c) Different P

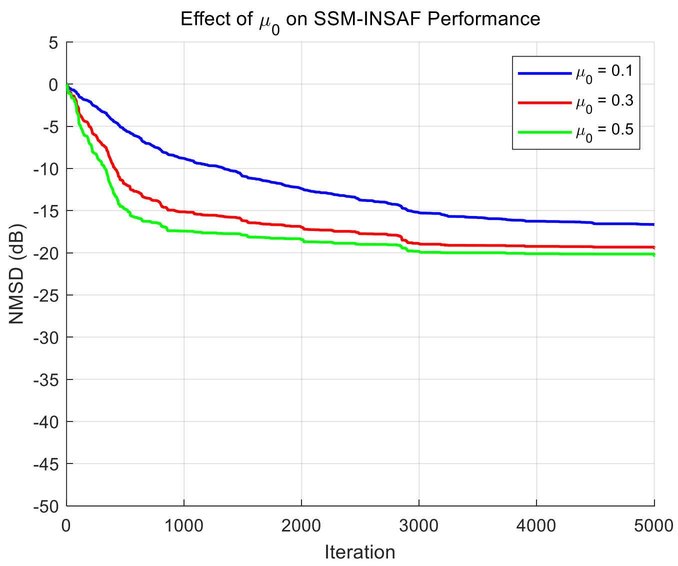

Fig. 5 NMSD curves of the SM-INSAF versus various step size μ0