Gambler's Ruin Simulation

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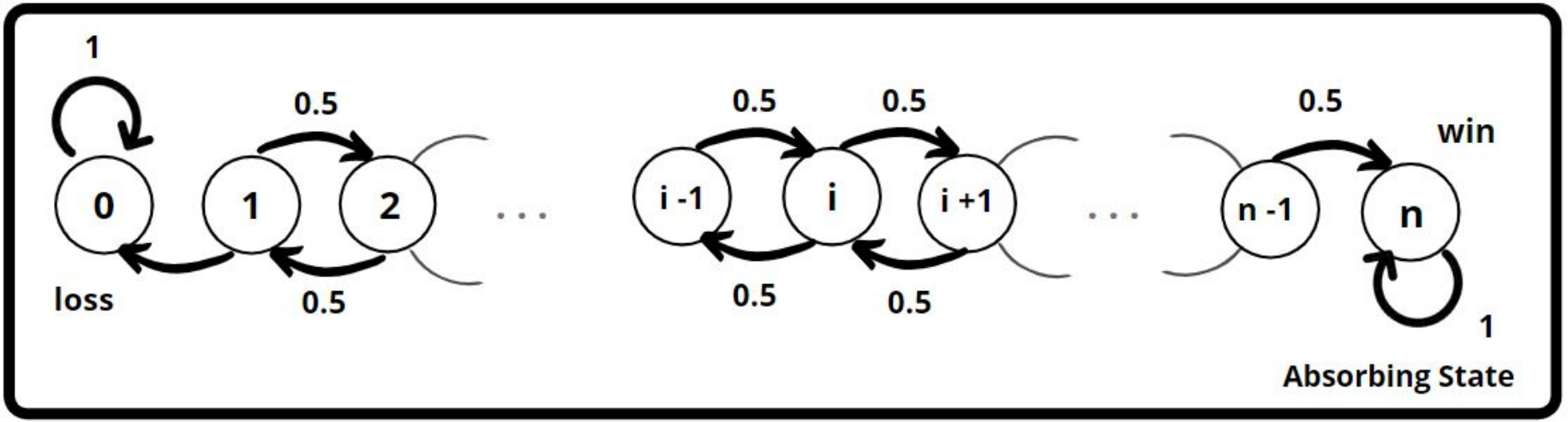
The Setup

A Gambler has $i dollars

Goal: win $N or go bankrupt, i.e. end up with $0.

The gambler bets $1 in each round and can either play a fair game, i.e. *probability of each outcome is same,* or an unfair game.

This problem is like a Random Walk and can be represented as a Markov Chain.



Markov Chain representation for a fair chance

There are 4 questions to be answered:

1. What is the probability that the Gambler ends up having $N?
2. What is the expected wealth of the Gambler at the end?
3. How long does the Gambler expect to stay in the game?
4. What happens in the case of unfavourable odds?

Probability that Gambler ends up having $N

In a fair game,

Boundary conditions: for i = 0, $P_i = 0$ and for i = N, $P_i = 1$,

For 0 < i < N, $P_i = 0.5P_{i+1} + 0.5P_{i-1}$

This difference equation is solved to get $P_i = i/N$

$if N \to \infty, i \to 0$

Expected wealth with gambler in the end: $0.(1 - P_i) + N.P_i = N * \frac{i}{N} = i$

How long Gambler stays in the game?

μ_i = expected number of plays, starting with i

$$1 < i < N, \mu_i = 1 + 0.5\mu_{i+1} + 0.5\mu_{i-1}$$

$$\mu_i = i(N - i)$$

In general, $\mu_i = 1 + \Sigma_j p_{ij}\mu_j$

In case of unfavourable odds

When $p \neq 0.5,$ and the probability of each transition is ‘p’ or (1 - p) then,

$$P_i = pP_{i+1} + (1-p)P_{i-1}$$

$$P_i = \frac{1 - r^i}{1 - r^N} \qquad \text{where,} \qquad r = \frac{1-p}{p}$$

$$\mu_i = \frac{r+1}{r-1}(i - N * \frac{1 - r^i}{1 - r^N})$$

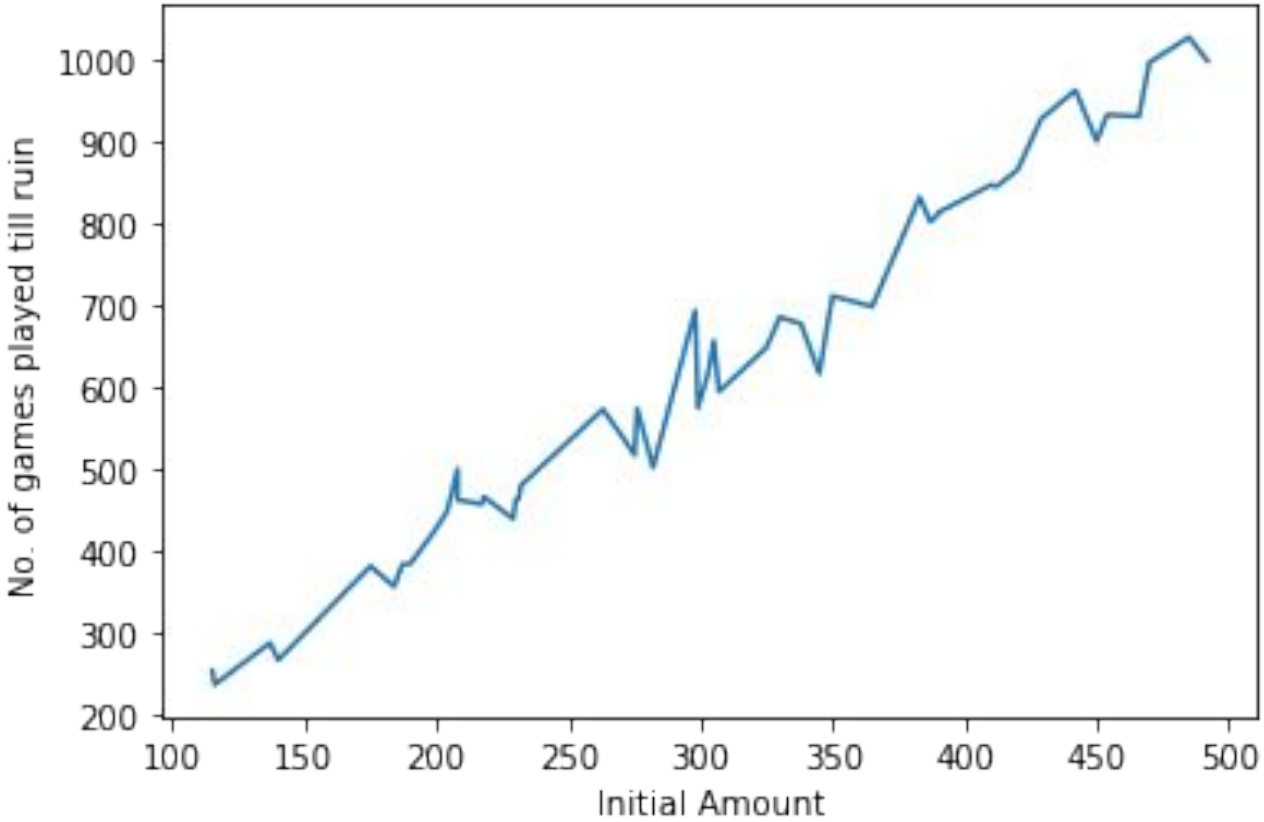


Figure 1: Initial amount vs Number of games played till ruin

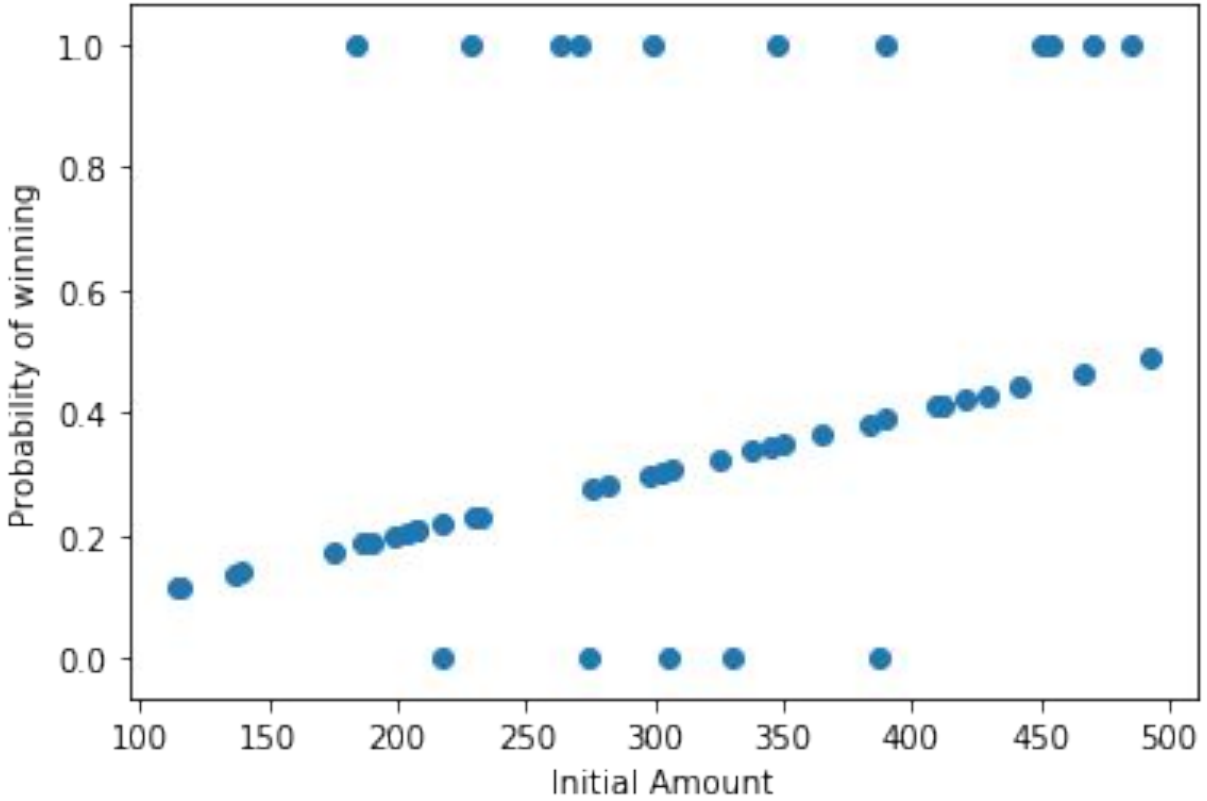


Figure 2: Initial amount vs Probability of winning the game

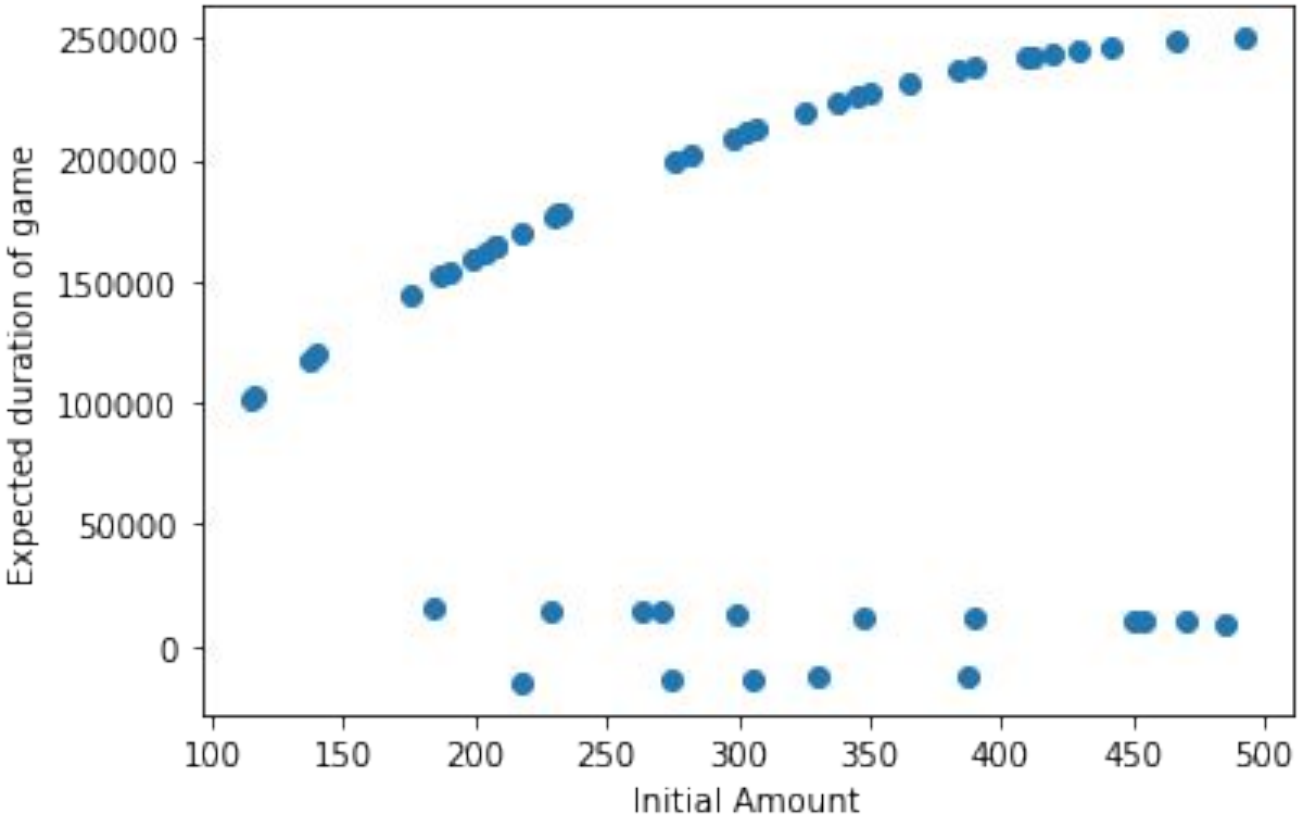
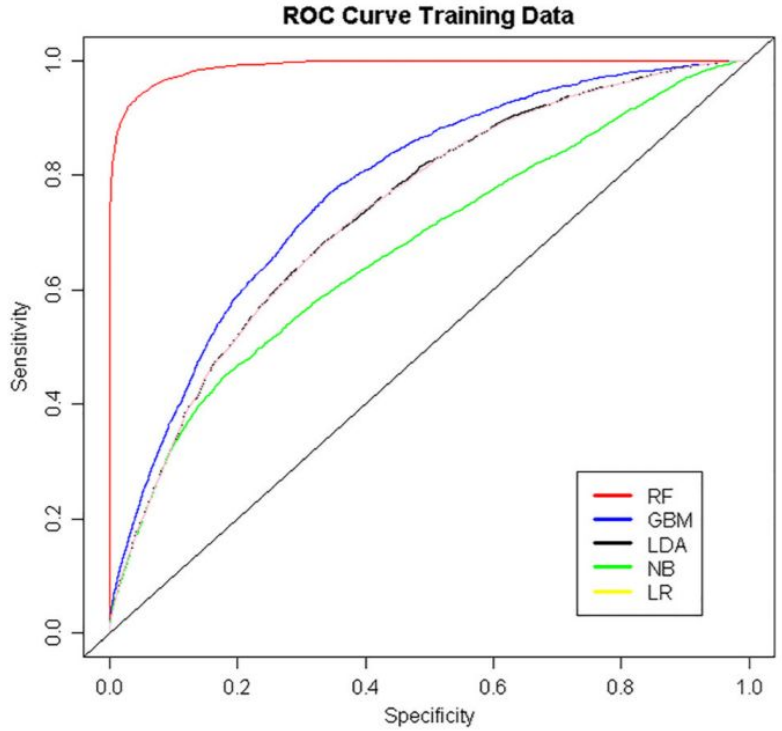


Figure 3: Initial amount vs Expected duration of the game

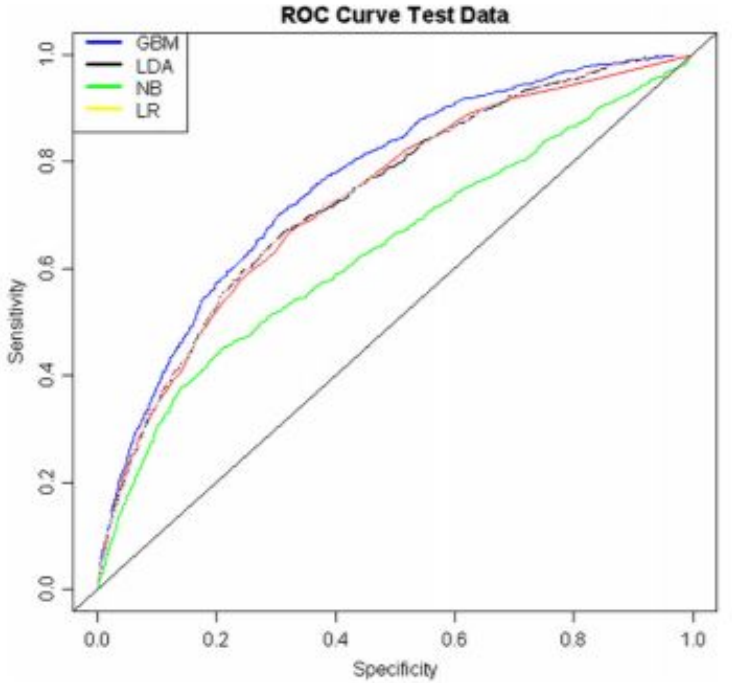
Table 1 Descriptive statistics of the training dataset ($N = 56{,}632$)

	Mean	SD
Amount bet	11,638	54,286
Amount lost	2399	31,812
Number of days	16	17
Global monthly limit	5100	5909
Maximum monthly limit	10%	30%
PlayScan red	5%	22%
Age	41	13
Female	28%	45%
Limit feedback	14%	35%
Limit changed	6.7%	25.1%

RF	GBM	LDA	NB	LR
Limit	Feedback	Feedback	Feedback	Theoretical loss
Theoretical loss	Limit	Limit increases	Limit increases	Number of days
Loss	Amount bet	Limit increases	Limit	Amount bet VLT
Amount bet	Theoretical loss	Limit decreases	Limit decreases	Amount bet casino
Amount won	Limit increases	Number of days	Number of days	Casino playing



Receiver operating curve of the training data (N = 56,632)



Receiver operating curve of the test data (N = 14,157)

ROC curve for training and testing data

THANK YOU.