Practice Questions for the ECE 661 Spring 2025 Final Exam

Introduction to Machine Learning for Engineers
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The number of practice questions included in this document may not be representative of the length of the exam. These are solely intended for you to gain more experience in answering questions similar to what you will see on the exam.

1 True or False

Problem 1: *[1 points]* The reason why Naïve Bayes is naïve is that it assumes a prior on the label.

○ True

○ False

Problem 2: *[1 points]* Training a logistic regression model for binary classification is equivalent to training a two-layer neural network with one linear layer and one softmax layer.

○ True

○ False

Problem 3: *[1 points]* In decision tree algorithms, including attributes that have no direct effect on the target variable can be beneficial for the robustness of the learned model because they add complexity that can capture unforeseen patterns.

○ True

○ False

Problem 4: *[1 points]* The K-means clustering algorithm is guaranteed to converge if it does not encounter any ties in the distance of a point from the cluster centers.

○ True

○ False

Problem 5: *[1 points]* Suppose we want to use a Gaussian Mixture Model (GMM) to solve a clustering problem and that we decide to use the EM (Expectation-Maximization) algorithm to solve for the GMM parameters. It is guaranteed that the solution of the EM algorithm converges to parameters that maximize the GMM likelihood function.

○ True

○ False

Problem 6: *[1 points]* Consider using synchronous distributed SGD (stochastic gradient descent) to train a linear regression model, where the data is shuffled and uniformly distributed across m workers. The expected error (i.e., ℓ_2 loss) after t iterations of synchronous distributed SGD with m workers and a batch size of b per worker is equivalent to the expected error of single-worker mini-batch SGD with a batch size of mb after t iterations.

○ True

○ False

Problem 7: *[1 points]* A Markov Decision Process (MDP) becomes equivalent to the multi-armed bandit formulation when we set the discount factor to $\gamma = 0$, there is a single state without any state transitions, and the reward function is only action dependent (i.e. the reward is given by $r(a)$ where a represents the action).

○ True

○ False

2 Multiple Choice

Problem 8: *[2 points]* You are building a k-nearest neighbors model to predict students' final grades based on their study hours, attendance rate, midterm grades, and assignment grades. From earlier knowledge, you know that assignment grades are the most important indicator. After training your model with a certain dataset, you realize that it performs well on the training set but poorly on the test set. Which of the following steps is most likely to improve the generalization performance of your k-nearest neighbors model?

○ Increase the value of k.

○ Decrease the value of k.

○ Remove the assignment grades feature.

○ Replace the Euclidean distance (L2-norm) with the Manhattan distance (L1-norm).

Problem 9: *[2 points]* Suppose you train a neural network on a training dataset $\mathcal{D}_T$, using a validation dataset $\mathcal{D}_V$ to determine the optimal value of the model hyperparameters. However, you observe that while the trained neural network has low training error, it has high testing error on the test dataset. Which of the following mistakes in your training procedure might explain these results?

○ You accidentally omitted half of the training data.

○ You accidentally used the same training and validation datasets (i.e., $\mathcal{D}_V$ was mistakenly set to be the same as $\mathcal{D}_T$).

○ You used a neural network with too many layers.

○ All of the above.

Problem 10: *[2 points]* Suppose you are given a dataset with two classes and are asked to train a classification model on this dataset. You are told to expect that the classification boundary will be a linear one, so you are trying to decide between using logistic regression or support vector machines (SVMs). Which of the following characteristics of the dataset would indicate that SVM will train a more accurate model using fewer computational resources?

○ Your dataset has a very high number of features.

○ The data points corresponding to the two classes are linearly separable.

○ One of the features has a range of values from -1000000 to 1000000, but another has a range of values from -1 to 1.

○ Your dataset has a very high number of data points.

Problem 11: *[2 points]* You want to perform K-means clustering on a dataset of N social network users to identify groups of similar users. You are not sure what is the right value of K, so you decide to plot the final value of the cost function J when using K clusters versus K for all K in the range 1 to N. Recall that

$$J = \sum_{n=1}^{N} \sum_{k=1}^{K} r_{nk} \|\boldsymbol{x}_n - \boldsymbol{\mu}_k\|^2.$$

where $\boldsymbol{x}_n$ is the feature vector of the n-th sample, $\boldsymbol{\mu}_k$ is the k-th cluster mean and r_{nk} is 1 if sample n is assigned to cluster k and 0 otherwise.
Which of the following statements is TRUE?

○ J decreases or stays the same as K increases. The optimal K is the smallest possible value.

○ J first decreases and then increases with K. The optimal K is the one that minimizes J.

○ J first increases and then decreases with K. The optimal K is the one that maximizes J.

○ None of the above.

Problem 12: *[2 points]* Assume you are given a sample training dataset which consists of real numbers $(x_i, y_i) \in \mathbb{R}^2$, for $i = 1, 2, ..., N$. To understand the data you plotted those points with x values corresponding to the horizontal axis and y values corresponding to the vertical axis, as seen in Figure 1. You will fit a curve to those points using a nonlinear regression model. Which of the following basis functions would result in a model with the least training error?

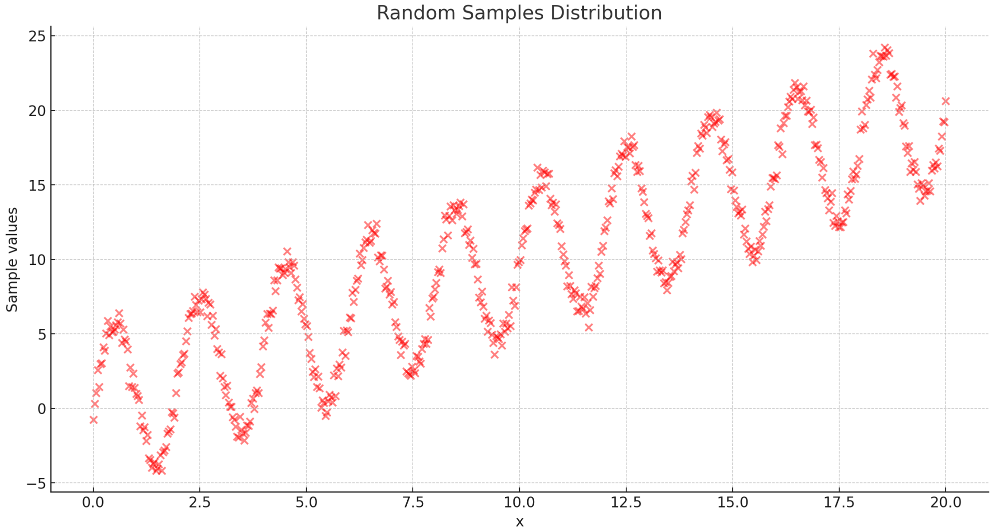


Figure 1: Random samples plot

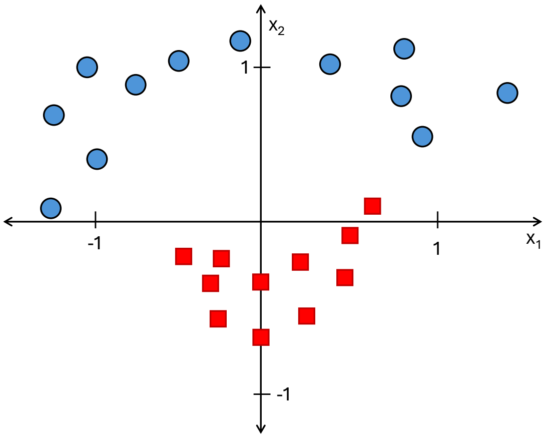
○ $\boldsymbol{\phi}(x) = \begin{bmatrix} x & \cos(2\pi x) \end{bmatrix}^T$

○ $\boldsymbol{\phi}(x) = \begin{bmatrix} 1 & \cos(\pi x) \end{bmatrix}^T$

○ $\boldsymbol{\phi}(x) = \begin{bmatrix} x & \sin(\pi x) \end{bmatrix}^T$

○ $\boldsymbol{\phi}(x) = \begin{bmatrix} 1 & \sin(2\pi x) \end{bmatrix}^T$

Problem 13: *[2 points]* Support vector machines (SVMs) are a historically popular classification algorithm due to their ability to learn nonlinear decision boundaries. Consider the dataset shown in the figure below. Each data point has two-dimensional input $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and the labels are indicated by blue circles (label 0) and red squares (label 1).



Which of the following nonlinear basis functions $\phi(\mathbf{x})$, when used to learn a SVM model, will NOT learn a decision boundary that perfectly separates the two classes?

○ $\phi(\mathbf{x}) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

○ $\phi(\mathbf{x}) = \begin{bmatrix} x_1^2 \\ x_2^2 \\ 1 \end{bmatrix}$

○ $\phi(\mathbf{x}) = \begin{bmatrix} x_1 x_2 \\ x_1^2 x_2^2 \end{bmatrix}$

○ This is a trick question. Learning nonlinear decision boundaries in SVM requires specifying a kernel function. We cannot specify a nonlinear basis function.

Problem 14: *[2 points]* Consider a distributed SGD implementation with m workers with a mini-batch size b per worker. Suppose that each worker takes exponential time rate μ (and mean $1/\mu$) to finish its mini-batch gradient computation, and that these times are independent across workers and mini-batches. What are the expected times taken by synchronous and asynchronous SGD to finish processing N mini-batches of data?

[Hint:] The expected minimum and maximum of n independent exponential random variables with rate μ are $\frac{1}{n\mu}$ and $\frac{\log n}{\mu}$ respectively.

○ Sync SGD $= \frac{N \log m}{\mu}$, Async SGD $= \frac{N}{m\mu}$

○ Sync SGD $= \frac{N\mu}{\log m}$, Async SGD $= \frac{Nm}{\mu}$

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○ Sync SGD $= \frac{Nm \log m}{\mu}$, Async SGD $= \frac{N}{m\mu}$

3 Descriptive

Problem 15: *[5 points]* Consider a dataset containing the following three 2-dimensional points along with their corresponding labels (y-values) as shown in Figure 2:

- Point A: $(x_1 = 1, x_2 = 1)$, $y = 1$
- Point B: $(x_1 = 4, x_2 = 4)$, $y = 2$
- Point C: $(x_1 = 7, x_2 = 1)$, $y = 3$

Suppose that you perform k-Nearest Neighbors regression for predicting the y-value, $\hat{y}$, with $k = 1$ and with the Euclidean distance (i.e., ℓ_2 distance) metric. For any new point (x_1, x_2) we want to find the decision regions for all possible $\hat{y}$ in terms of x_1 and x_2.

When answering the questions below, **show all your work** and explicitly state each decision region for each possible $\hat{y}$ (there are three possible values, $\hat{y} = 1, 2, 3$) using x_1 and x_2 values and '$=, <, >, +, -$'. For example, the decision rule for declaring $\hat{y} = 1$ can be of the form $\{ax_1 + bx_2 \geq c$ and $dx_1 + ex_2 \geq f\}$ for some scalars a, b, c, d, e, f. You can break ties arbitrarily.

a. *[2 points]* Draw the decision boundaries on Figure 2.

b. *[1 points]* Specify the decision region in which we predict label $\hat{y} = 1$ for a point (x_1, x_2).

c. *[1 points]* Specify the decision region in which we predict label $\hat{y} = 2$ for a point (x_1, x_2).

d. *[1 points]* Specify the decision region in which we predict label $\hat{y} = 3$ for a point (x_1, x_2).

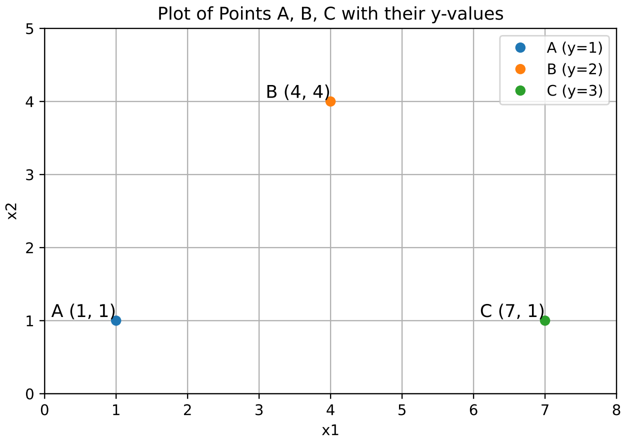


Figure 2: Dataset for Problem 17.

Problem 16: *[5 points]* Suppose you are given a dataset $\mathcal{D}$ of N data points, where N is an even number and $\mathcal{D} = \left\{ (-1)^j \left\lceil \frac{j}{2} \right\rceil \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix},\ j = 1, 2, \ldots, N \right\}$. Each data point therefore has three input features. Note that the notation $\lceil \alpha \rceil$ refers to the smallest integer greater than or equal to α. For example, $\left\lceil \frac{1}{2} \right\rceil = 1$.

a. *[2 points]* Let $N = 2$, so that $\mathcal{D} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$. Suppose you run PCA (principal component analysis) to find the top principal component on this dataset. What principal component do you find? Show your work or explain your answer.

b. *[2 points]* Now suppose that $N = 4$, and that you again run PCA to find the top principal component on $\mathcal{D}$. What principal component do you find? Is it the same as the one you found in part (a) for $N = 2$? Briefly explain your answer.

c. *[1 points]* You are now asked to find the top *two* principal components for the dataset in part (b). Will this lead to a decrease in the average reconstruction error over the data points in $\mathcal{D}$, compared to the reconstruction error with the single principal component you found in part (b)? Briefly explain your answer.

Recall that the *reconstruction error* of a data point $\mathbf{x}$ is the Euclidean distance between $\mathbf{x}$ and its reconstructed data point $\tilde{\mathbf{x}}$, where $\tilde{\mathbf{x}}$ is the representation of $\mathbf{x}$ in terms of the principal components.

Problem 17: *[6 points]* Suppose we have a neural network for performing nonlinear regression which consists of three linear layers, where all but the last layer have a sigmoid activation and the last layer uses a linear activation. Letting $\mathbf{x}_0 \in \mathbb{R}^{d_0}$ denote the d_0-dimensional input to the network and $\hat{y}$ the scalar output, we can write

$$\begin{aligned} \mathbf{x}_1 &= A_1\mathbf{x}_0 + b_1\mathbf{1}, \quad \tilde{\mathbf{x}}_1 = \sigma(\mathbf{x}_1) \\ \mathbf{x}_2 &= A_2\tilde{\mathbf{x}}_1 + b_2\mathbf{1}, \quad \tilde{\mathbf{x}}_2 = \sigma(\mathbf{x}_2) \\ \hat{y} = x_3 &= A_3\tilde{\mathbf{x}}_2 + b_3. \end{aligned}$$

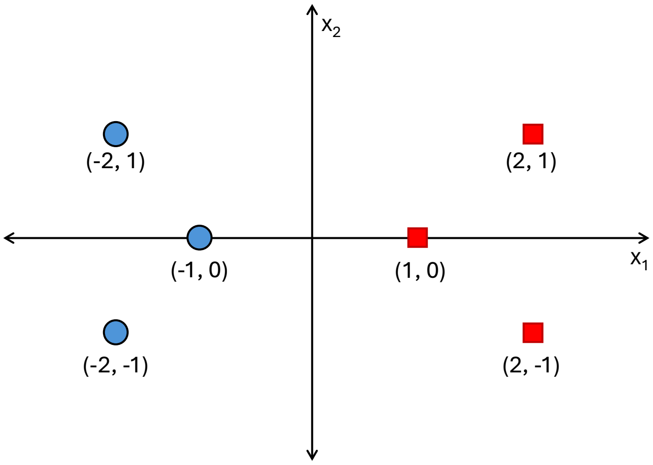
Here our model parameters are the bias terms $b_1, b_2, b_3 \in \mathbb{R}$ and the matrices $A_1 \in \mathbb{R}^{d_1 \times d_0}$, $A_2 \in \mathbb{R}^{d_2 \times d_1}$, $A_3 \in \mathbb{R}^{1 \times d_2}$, where the hidden states $\mathbf{x}_1 \in \mathbb{R}^{d_1}$ and $\mathbf{x}_2 \in \mathbb{R}^{d_2}$. We use $\mathbf{1}$ to denote a vector of all ones; thus, the bias term in each layer is added to each entry of the output vector of this layer.
Recall that the sigmoid function $\sigma(a) = (1 + e^{-a})^{-1}$ and that $\sigma'(a) = \sigma(a)(1 - \sigma(a))$; when we write $\sigma(\mathbf{a})$ for a vector $\mathbf{a}$, we apply the sigmoid function to each element of $\mathbf{a}$. For example, $\sigma\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} \sigma(a_1) \\ \sigma(a_2) \end{bmatrix}$.
Normally, we optimize the parameters of our neural network $A_1, A_2, A_3, b_1, b_2, b_3$ in order to minimize a given loss function. Now, suppose we are instead trying to attack the neural network in order to cause it to obtain the *wrong* answer by choosing the input $\mathbf{x}_0$ that *maximizes* the loss.

a. *[3 points]* Suppose we use the l_2 loss function $\mathcal{L}(\hat{y}, y^*) = ||\hat{y} - y^*||_2^2$, where y^* is the ground truth label. Express $\frac{\partial \mathcal{L}}{\partial \mathbf{x}_0}$ in terms of y^*, parameters $A_1, A_2, A_3, b_1, b_2, b_3$, and intermediate values $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, x_3, \tilde{\mathbf{x}}_1, \tilde{\mathbf{x}}_2$. Remember to show your work.

b. *[1 points]* Use your answer to part (a) to write the expression for a gradient update with a learning rate of λ for $\mathbf{x}_0$ that will *maximize* the loss $\mathcal{L}(\hat{y}, y^*) = ||\hat{y} - y^*||_2^2$ when performed repeatedly.

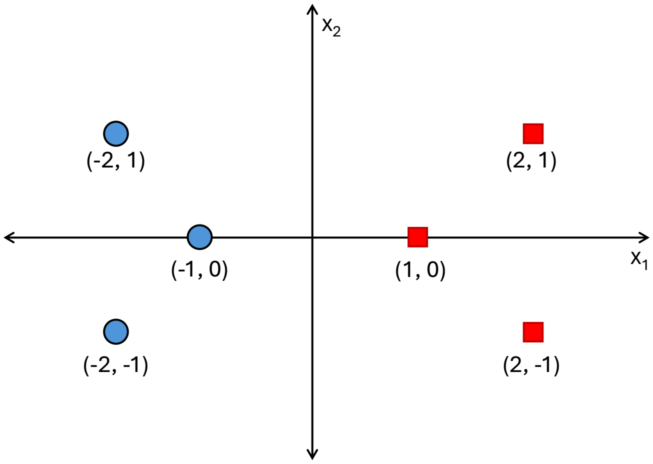
c. *[2 points]* Assuming a constant learning rate that is appropriately small, is the loss value associated with the gradient-based optimization procedure you described in part (b) guaranteed to converge? Why or why not?

Problem 18: *[6 points]* Recall that ensemble methods aim to train multiple decision trees and combine their results to produce a classifier. In this problem, we will apply ensemble methods to the dataset of six points shown in the figure below. Each dataset has a two-dimensional input $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, with coordinates shown as (x_1, x_2) in the figure. Each data point's label is indicated by a blue circle or red square.



a. *[1.5 points]* First, we apply random forests to this dataset. We use $n = 2$ data points for each tree and train $m = 5$ trees. Find an expression for the probability that $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is used to train the fifth tree. Remember to show your work. You may assume that data points are sampled with replacement. If it is not possible to derive this probability without knowledge of the first four trees, explain why not.

b. *[1.5 points]* Now suppose that you try AdaBoost on this dataset, and you decide to use decision stumps (one-layer decision trees) as your base classifiers. Find the first decision stump that is created by AdaBoost. You can provide your answer by drawing the line in the figure below, or by providing the equation of the decision boundary.



c. *[1 points]* Continuing from the decision stump found in part (b), you decide to run AdaBoost for four more iterations, so that your final ensemble has five decision stumps. Find the probability that the data point $\begin{bmatrix}2\\1\end{bmatrix}$ has higher weight than the data point $\begin{bmatrix}-2\\1\end{bmatrix}$ in training the fifth decision stump in AdaBoost. Remember to show your work. If it is not possible to derive this probability without computing the second, third, and fourth trees, explain why not.

d. *[1 points]* How many data points in this dataset will be misclassified by the resulting ensemble of five trees from part (c)? Explain your answer or show your work.

e. *[1 points]* Considering again the ensemble learned in part (c), will the mis-classification rate on a test dataset be larger or smaller than the mis-classification rate on the training dataset? Briefly explain your answer. If it is not possible to know the answer without information on the testing dataset, explain why not.