The Search for
“One Norm to Rule Them All”

BobFest
University of San Diego
June 8, 2024

Robert Kosut
Friend & Colleague

The Search for

"One $\|\cdot\|$ to Rule Them All"

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Start the *Search* with these:

Stability theorems for the relaxation of the strictly positive real condition in hyperstable adaptive schemes
BDO Anderson, RR Bitmead, CR Johnson, RL Kosut
The 23rd IEEE Conference on Decision and Control, 1286-1291, 1984

Positive real conditions for adaptive control are not really necessary
BDO Anderson, RR Bitmead, CR Johnson, RL Kosut
IFAC Proceedings Volumes 18 (5), 1003-1008, 1985

Fixed-point Theorems for Stability Analysis of Adaptive Systems
RL Kosut, RR Bitmead
Adaptive Systems in Control and Signal Processing 1986, 137-142

Stability of adaptive systems: Passivity and averaging analysis
BDO Anderson, RR Bitmead, CR Johnson Jr, PV Kokotovic, RL Kosut, I. M. Y. Mareels, L. Praly, B. D. Riedle
MIT press , 1986

How exciting can a signal really be?
IMY Mareels, RR Bitmead, M Gevers, CR Johnson Jr, RL Kosut,
Systems & control letters 8 (3), 197-204, 1987

Transient analysis of adaptive control
RL Kosut, IMY Mareels, BDO Anderson, RR Bitmead, CR Johnson Jr
IFAC Proceedings Volumes 20 (5), 127-132, 1987

1984

Proceedings of 23rd Conference
on Decision and Control
Las Vegas, NV, December 1984

FA2 - 10:45

STABILITY THEOREMS FOR THE RELAXATION OF THE STRICTLY POSITIVE REAL CONDITION IN HYPERSTABLE ADAPTIVE SCHEMES

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ABSTRACT

The hyperstability theorems of Popov have played an important role in establishing the convergence of adaptive schemes, notably adaptive output error identification and adaptive control. The error system of these schemes has the form of a feedback loop with a time-invariant forward path and a passive time-varying feedback path. The strict positive realness of the forward path suffices to establish asymptotic stability of the feedback loop and therefore establishes convergence of the adaptive scheme. In this paper we study conditions which preserve the asymptotic stability but permit relaxation of the strict positive real condition at high frequencies, subject to restrictions on algorithm gain parameters and frequency content of the input signals. These theorems are important for the design of robust adaptive methods.

cedures. The results of this paper concentrate on the issues in adaptive identification since this permits the more straightforward derivation of theoretical results and tools. The direct extension to adaptive control is contained in a companion paper [25]. These results may also be applied to more novel forms of adaptive control [26] which actively attempt to satisfy the stability conditions of this paper through the introduction of filtering. Related control ideas have been expressed in [24] and [27].

PROBLEM STATEMENT

The specific system which we consider is dep
which represents the error equations as derived in mo
tive schemes. With obvious abuse of notation we s
exponential asymptotic stability (EAS) of

---

step discontinuities.) We then have immediately the following characterization of the asymptotic stability of (5) from application of the small gain theorem, [13].

THEOREM 1

Suppose that: $Z_1(s)$ is SPR, u(t) is persistently exciting *and* satisfies (13) and (14). Further, denote the L_p gain of $Z_3(s)$ for some particular $p \in [1,\infty]$ by g_3. Then the solution $x(\cdot)$ of the differential equation (5) will be in L_p if

$$\epsilon \frac{m}{\lambda} m_1 m_2 g_3 < 1 \qquad (15)$$

where λ and m are defined in (12) and represent the convergence rate of the "ideal" differential equation (10).

We have, after application of the small gain theorem,

THEOREM 2

Suppose that the conditions of Theorem 1 hold, constant a is chosen such that $0 < a < \lambda$ and $Z_3(s-a)$ has no poles in $Re(s) \geq 0$. Then x(t), the solution of (5) satisfies

$$e^{at} x(t) \in L_2 \text{ if } \epsilon m_1 m_2 \tilde{g}_3(a) m(\lambda-a)^{-1} < 1 \qquad (23)$$

and

$$e^{at} x(t) \in L_\infty \text{ if } \epsilon m_1 m_2 g_3(a) m(\lambda-a)^{-1} < 1 \qquad (24)$$

Further, condition (23) implies that $e^{at}x(t) \in L_\infty$ and $e^{at}x(t) \to 0$ as $t \to \infty$

1985

POSITIVE REAL CONDITIONS FOR ADAPTIVE CONTROL ARE NOT REALLY NECESSARY

B. D. O. Anderson*, R. R. Bitmead*, C. R. Johnson** and R. L. Kosut***

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Abstract. Model reference adaptive control with constrained complexity controllers is studied and stability results obtained. The use and advantages of regression vector filtering are also explained in connection with the stability argument.

Keywords. Adaptive control; adaptive systems; stability

3. FORMULATION OF AN ERROR MODEL

Let us begin by making a simple observation.

Lemma 3.1. In the adaptive algorithm stated at the end of the preceding section, and with

$$\theta^* = [w_0^* \; w_1^* \; \dots \; w_\alpha^* \; n_0^* \; \dots \; n_\beta^*] \qquad (3.1)$$

$$\tilde{\theta}(k) = \theta^* - \hat{\theta}(k) \qquad (3.2)$$

there holds

$$u(k) - u^*(k) = \frac{1}{w_0^*} X^T(k)\tilde{\theta}(k) \qquad (3.3)$$

Proof is omitted due to space limitations.

Theorem 5.1. Let θ_1^* define the controller, assumed stabilizing, which causes the average value of the solution of (3.17), with $O(\gamma^2)$ terms neglected, to be zero. Let θ_2^* define the (fixed) controller, assumed stabilizing which minimizes the index

$$J = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} [\frac{G(q^{-1})}{F(q^{-1})} C(q^{-1})[z(k)-y(k)]]^2 \qquad (5.1)$$

Then θ_1^* causes

$$\overline{[Z(k-d)[K(q^{-1})C(q^{-1})-1]Z^T(k-d)}\theta_1^* = 0 \qquad (5.2)$$

and θ_2^* causes

$$\overline{[K(q^{-1})C(q^{-1})Z(k-d)[K(q^{-1})C(q^{-1})-1]Z^T(k-d)}\theta_2^* = 0 \qquad (5.3)$$

with $K(\cdot)$ depending in (5.2) and (5.3) on θ_1^*, θ_2^*. Note that in case $K(e^{j\omega})C(e^{j\omega})-1$ is small in the region where $Z(\cdot)$ has significant frequency content, (5.2) and (5.3) are evidently approximately equivalent. In Johnson, Anderson and Bitmead (1984) equality of θ_1^* and θ_2^* was conjectured.

Adaptive Systems in Control and Signal Processing 1986

IFAC Workshop Series

1987, Pages 137-142

FIXED-POINT THEOREMS FOR STABILITY ANALYSIS OF ADAPTIVE SYSTEMS

R.L. Kosut, R.R. Bitmead *

https://doi.org/10.1016/B978-0-08-034085-2.50027-4

ABSTRACT

The use of fixed point theorems is considered for the stability analysis of adaptive systems. The particular fixed point theorems considered are the Contraction Mapping Principle of Banach and the Schauder Fixed Point Theorem. It is shown how the contraction property can be achieved by exponential stability of the homogeneous part of the linearized adaptive system. The region of linearization validity is estimated by considering fixed as well as adaptive tuned systems. Fixed-point theorems are shown also to be useful for a transient analysis of the adaptive system.

1987

into the so called contraction mapping principle by Banach and Cacciopoli. If $\mathscr{F}$ is a subset of a Banach space $\mathscr{X}$ and T is a transformation taking $\mathscr{F}$ into a Banach space $\mathscr{B}$ (written as T: $\mathscr{F} \to \mathscr{B}$), then T is a *contraction* on $\mathscr{F}$ if there is a λ, $0 \leq \lambda < 1$, such that

$$|Tx - Ty| \leq \lambda |x - y|, \qquad x, y \in \mathscr{F}.$$

The constant λ is called the *contraction constant* for T on $\mathscr{F}$.

THEOREM 3.1. (*Contraction mapping principle of Banach-Cacciopoli*). If $\mathscr{F}$ is a closed subset of a Banach space $\mathscr{X}$ and T: $\mathscr{F} \to \mathscr{F}$ is a contraction on $\mathscr{F}$, then T has a unique fixed point $\bar{x}$ in $\mathscr{F}$. Also, if x_0 in $\mathscr{F}$ is arbitrary, then the sequence $\{x_{n+1} = Tx_n,\ n = 0, 1, 2, \ldots\}$ converges to $\bar{x}$ as $n \to \infty$ and $|\bar{x} - x_n| \leq \lambda^n |x_1 - x_0|/(1 - \lambda)$, where $\lambda < 1$ is the contraction constant for T on $\mathscr{F}$.

ORDINARY DIFFERENTIAL EQUATIONS

JACK K. HALE

1987

TRANSIENT ANALYSIS OF ADAPTIVE CONTROL

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Abstract *The transient properties of adaptive control systems are examined. A simple example system is presented which exhibits both slow and fast transients. Methods for analyzing the transient behavior are discussed, including averaging, Floquet analysis, and fixed point theory.*

A more general approach to the analysis of (4.1) can be formulated by appealing to the Contraction Mapping Principal(CMP) of Banach and Cacciopoli. We need the following definitions and statement of the CMP from Hale(1969).

If $\mathcal{M}$ is a subset of a Banach space $\mathcal{B}$ with norm $\|\cdot\|$, and Γ is an operator mapping $\mathcal{M} \to \mathcal{B}$, then Γ is a *contraction on* $\mathcal{M}$ if there is a constant $\sigma \in [0,1)$ such that

$$\|\Gamma x - \Gamma y\| \leq \sigma \|x - y\|, \forall x, y \in \mathcal{M} \tag{4.6}$$

The constant σ is referred to as the *contraction constant for* Γ *on* $\mathcal{M}$. A *fixed point* of Γ in $\mathcal{M}$ is a point (function) $x \in \mathcal{M}$ such that $x = \Gamma x$. We now state the following result.

Contraction Mapping Principal: *If $\mathcal{M}$ is a closed subset of a Banach space $\mathcal{B}$, and $\Gamma : \mathcal{M} \to \mathcal{M}$ is a contraction on $\mathcal{M}$, then Γ has a unique fixed point in $\mathcal{M}$.*

In order to apply the CMP to the adaptive system (4.1) we need to identify the operator Γ and the space $\mathcal{M}$. For example, let Γ be the operator mapping $\tilde\theta \mapsto \tilde\theta$ defined implicitly by

$$\boxed{\begin{aligned} \phi &= \phi_* - G(\tilde\theta^T \phi) && (4.7a) \\ \dot{\tilde\theta} &= \epsilon\phi[e_* - H(\phi^T \tilde\theta)] && (4.7b) \end{aligned}}$$

It is clear that fixed points of Γ are solutions to the adaptive system, i.e., $\tilde\theta = \Gamma\tilde\theta$ is equivalent to (4.1) with ϕ implicitly defined. A convenient choice of the space $\mathcal{M}$ is

$$\mathcal{M} = \{\tilde\theta \in C[0,T] : \|\tilde\theta(t)\| \leq \delta \exp(-\lambda t)\} \tag{4.8}$$

1987

Systems & Control Letters 8 (1987) 197–204
North-Holland

How exciting can a signal really be?

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Received 12 May 1986
Revised 13 August 1986

Abstract: The rate of parameter convergence in a number of adaptive estimation schemes is related to the smallest eigenvalue of the average information matrix determined by the regression vector. Using a very simple example, we illustrate that the input signals that maximize this minimum eigenvalue may be quite different from the input signals that optimize more classical input design criteria, e.g. D-optimal criterion.

Keywords: Exponential convergence, Persistence of excitation, Experiment design.

Selection Criteria. Over the class of input functions $(u(t), t \in \mathbb{R}^+)$, which have a power spectrum (as defined in (2.4)–(2.5)) and which satisfy the constraint

$$0 < \int_{-\infty}^{+\infty} S(\omega)\, d\omega \leqslant 1, \tag{2.9}$$

maximize, either

(C1) $\det(R)$, or
(C2) $\lambda_{\min}(R)$, or
(C3) $\lambda_{\min}(R)/\lambda_{\max}(R)$.

4. Conclusions

We have argued that, in the case of slow adaptation, the smallest eigenvalue and the condition number of the average information matrix determined by the regression vector should be considered as input design criteria in order to maximize the rate of exponential convergence. We have then performed this optimal input design in the simplest possible case, which allows a complete description of all optimal solutions. One should be very careful in extrapolating the conclusions of this simple example to more general situations.

More attention has to be paid to the effects of modelling errors, noise spectra, etc. which tend to complicate matters even further. Including observations of this sort leads one to concentrate the energy of the input signal in low frequency regimes [1,9] which as our results indicate, causes an attendant decrease in the level persistency of excitation which in its turn may accentuate the effect of unmodelled dynamics [1]. Given this dilemma, and its practical importance in designing well performing (robust) adaptive control/estimation algorithms, we believe that this problem deserves much more attention but its resolution will probably depend strongly on the particular circumstances.

14th IFAC Symposium on System Identification, Newcastle, Australia, 2006

AN APPROACH TO JOINT IDENTIFICATION AND MODEL ORDER SELECTION

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Abstract:
An explicit constraint representing a standard whiteness test is added to the framework of least-squares ARX identification in order to provide a means for model order selection without relying on a second data set. This is accomplished by a convex approximation of the non-convex standard whiteness test, thereby forming a convex optimization problem.

Keywords: system identification for control design, model uncertainty estimation

2006

3. CHARACTERIZING WHITE NOISE

Suppose we have a finite sample of prediction errors from the n-th order ($p = 2n$) ARX model (5), specifically, $\{ \varepsilon_t \mid t = 1, \ldots, \ell \}$. A standard *whiteness test* is based on the *Portmanteau Statistic*,

$$Q(\varepsilon) = \ell \, \|\rho\|_2^2 = \ell \sum_{\tau=1}^{m} \rho_\tau^2$$

$$\rho_\tau = \frac{\sum_{t=\tau+1}^{\ell} \varepsilon_t \varepsilon_{t-\tau}}{\sum_{t=1}^{\ell} \varepsilon_t^2}, \quad \tau = 1, \ldots, m \tag{9}$$

where m, $m \leq \ell$ is the width of the *lag window* which is used to smooth ρ_τ, the normalized correlation function; typically $m \ll \ell$. If the time series $\{\varepsilon_t\}$ is white, then as the data length ℓ increases, the distribution of the statistic $Q(\varepsilon)$ asymptotically approaches

2006

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The ν- gap Metric

- The Vinnicombe ν-gap metric measures the distance between two systems or controllers.

- Definition 1 Two controllers, $C_0(s)$ and $C_1(s)$, satisfy the Winding Number Condition if

$$\det(I + C_1^* C_0)(j\omega) \neq 0, \ \forall\omega \ \text{and}$$
$$\text{wno}\det(I + C_1^* C_0) + \eta(C_0) - \eta(C_1) = 0$$

where wno(.) indicates the winding number of the Nyquist diagram of the scalar transfer function evaluated on a contour enclosing the RHP and indented along the imaginary axis to the right around any pure imaginary axis to the right around any pure imaginary poles, and $\eta(C)$ is the number of open RHP poles of det(C).

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The ν– gap metric

- **Definition 2**

$$\delta_\nu(C_0, C_1) = \begin{cases} \left\| (I + C_1 C_1^*)^{-1/2}(C_1 - C_0)(I + C_0^* C_0)^{-1/2} \right\|_\infty & \text{if the WNC holds,} \\ 1 & \text{else.} \end{cases}$$

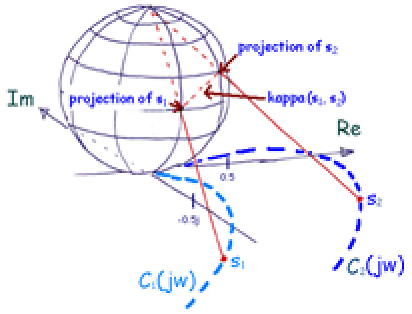

Fig. 2 Projection onto the Riemann sphere

Partial summary of "$|| \cdot ||$"

Small Gain Theorem

Strictly Positive Real & The Positive Real Lemma

Method of Averaging

Contraction Mapping Theorems (Banach, Brower, …)

Total Stability Theorem

Portmanteau Statistic

Gap Metric

Partial summary of "$\|\cdot\|$"

Small Gain Theorem

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Robust Quantum Control: Analysis & Synthesis via Averaging

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[1]*SC Solutions, Sunnyvale CA, 94085*
[2]*Department of Chemistry, Princeton University, Princeton, NJ, 08544*

(Dated: August 31, 2022)

An approach is presented for robustness analysis and quantum (unitary) control synthesis based on the classic method of averaging. The result is a multicriterion optimization competing the nominal (uncertainty-free) fidelity with a well known robustness measure: the size of an interaction (error) Hamiltonian, essentially the first term in the Magnus expansion of an interaction unitary. Combining this with the fact that the topology of the control landscape at high fidelity is determined by the null space of the nominal fidelity Hessian, we arrive at a new two-stage algorithm. Once the nominal fidelity is sufficiently high, we approximate both the nominal fidelity and robustness measure as quadratics in the control increments. An optimal solution is obtained by solving a convex optimization for the control increments at each iteration to keep the nominal fidelity high and reduce the robustness measure. Additionally, by separating fidelity from the robustness measure, more flexibility is available for uncertainty modeling.

arXiv [quant-phys]: 2022

Bringing Quantum Systems under Control

Julian Berberich[1], Robert L. Kosut[2], Thomas Schulte-Herbrüggen[3]

Tutorial submission:
CDC 2024, Milan

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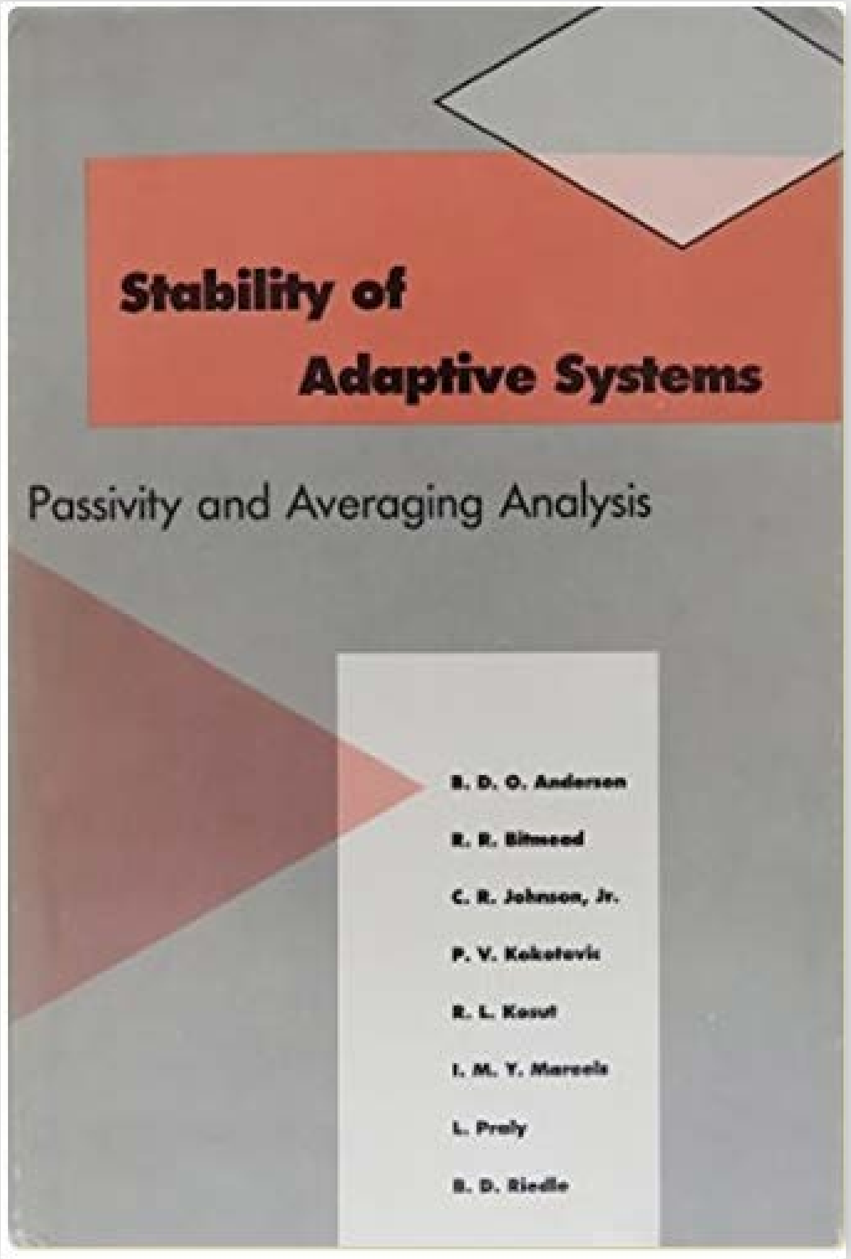
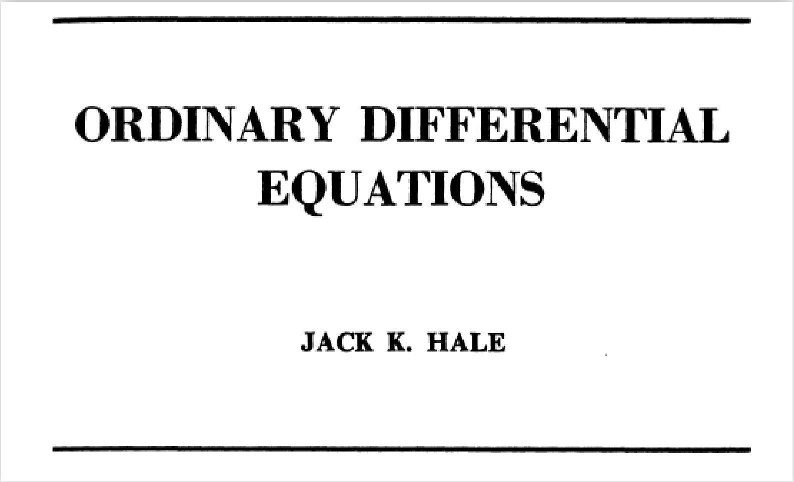
[3]Technical University of Munich, School of Natural Sciences, 85748 Garching-Munich, Germany & Munich Quantum Valley (MQV), Schellingstr. 4, 80799 Munich, Germany tosh@tum.de

To Oz, and Beyond!

1986



"To Hale with it!"


ORDINARY DIFFERENTIAL EQUATIONS

JACK K. HALE

Eight

1985-86

Brian Petar Rick Robert Brad **Bob** Laurent Iven

Chapter 1

ROBUST STABILITY FORMULATION

1.1 INTRODUCTION

Adaptive systems are a response to engineering design problems arising from uncertainty. In the representative areas of adaptive control, communications, and signal processing, uncertainty means an imprecise knowledge of the current system. The aim of adaptation is to provide on-line modification of system behavior in response to current measured performance. Thus, the formulation of adaptive systems involves concepts of performance measures, adjustment rules, and desired objectives, together with an appreciation of the role of uncertainty in describing the system.

In characterizing the effectiveness of adaptive methods one is led immediately to questions concerning the nature of the response of

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Chapter 2

PASSIVITY AND SMALL GAIN ANALYSIS

2.1 INTRODUCTION

Our legacy from the previous chapter and commitment to later ones is to study the behavior, and particularly the stability properties, of the linear system of ordinary differential equations represented by

$$\dot{\theta}(t) = -\epsilon\phi(t)H(s)\{\phi^T(t)\theta(t)\} \tag{1.1}$$

which describe the evolution of the adaptive error system. This system of equations is depicted in Figure 2.1. In discrete-time we equivalently analyze the ordinary difference equation‡

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Chapter 6

IMPLICATIONS AND INTERPRETATIONS

Chapter 1

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Chapter 2

PASSIVITY AND SMALL GAIN ANALYSIS

"With SGT by our side we are now prepared …"

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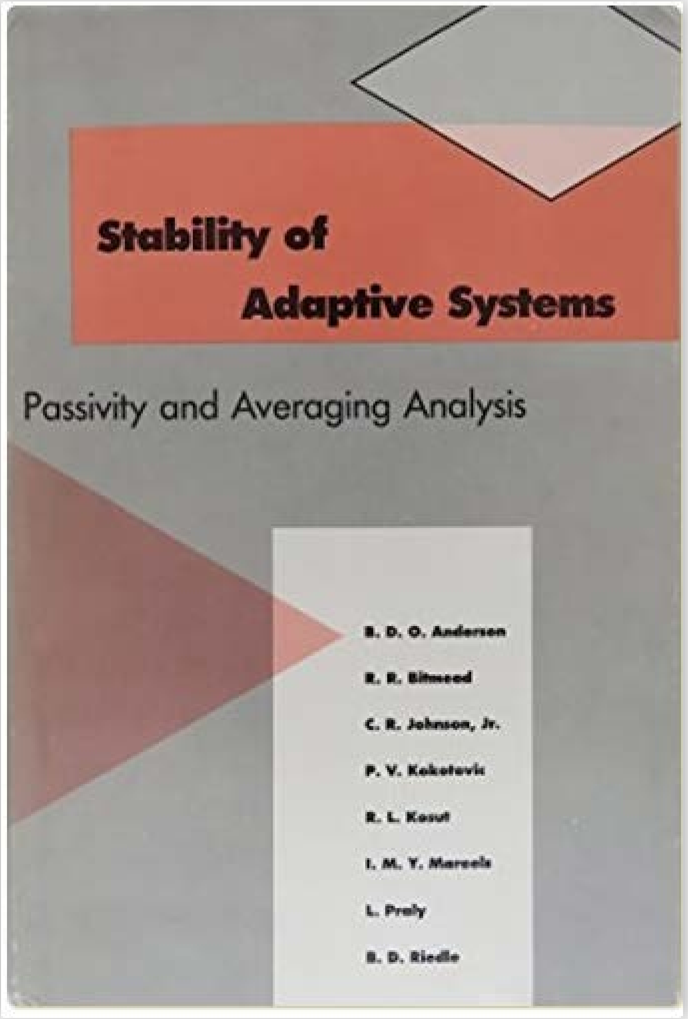
$$\dot{\theta}(t) = -\epsilon\phi(t)H(s)\{\phi^T(t)\theta(t)\} \tag{1.1}$$

This

Chapter 6

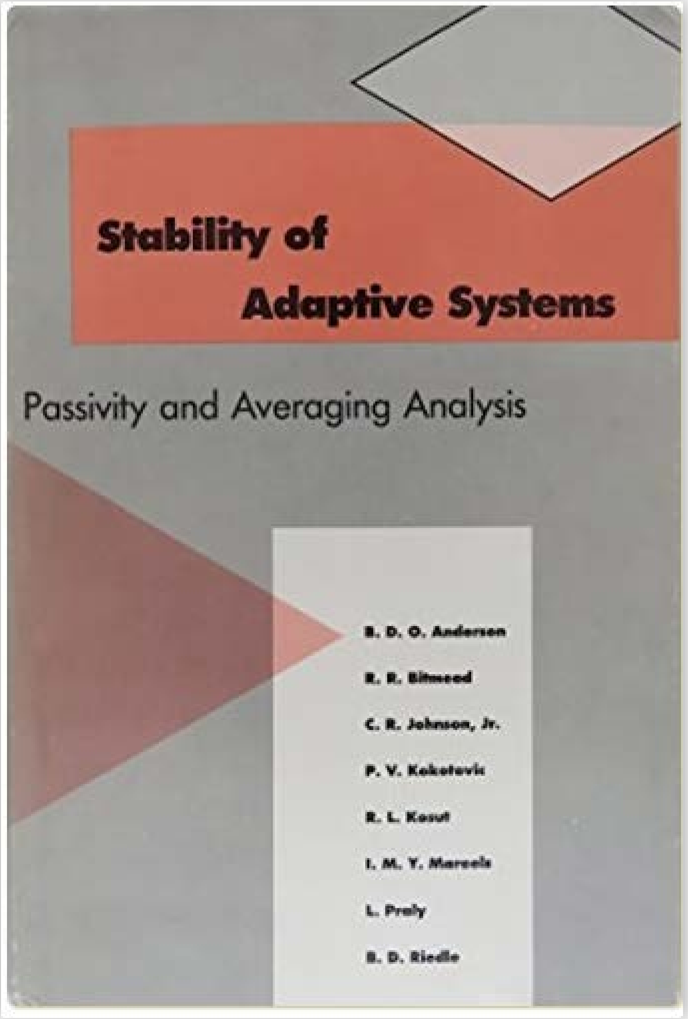
IMPLICATIONS AND INTERPRETATIONS



1986

“It is an error to imagine that evolution signifies a constant tendency to increased perfection. That process undoubtedly involves a constant remodeling of the organism in adaptation to new conditions; but it depends on the nature of those conditions whether the direction of the modifications effected shall be upward or downward.”

--- T. H. Huxley (1825-1895)



1986

"It is an error to imagine that evolution signifies a constant tendency to increased perfection. That process undoubtedly involves a constant remodeling of the organism in adaptation to new conditions; but it depends on the nature of those conditions whether the direction of the modifications effected shall be *upward* or *downward*."

--- T. H. Huxley (1825-1895)

$$''\textit{upward}'' = \text{minimize } \|\cdot\|$$

"Oh my, it's filled with $\|\cdot\|$!"