Lab 5: Random Numbers and Simulations

2024-05-03

Simulations

You discussed simulations at length in Lecture 04, but in general simulations are a useful tool for understanding estimators' behaviors

Today we are going to discuss how to simulate data and test the reliability of estimators

Random Variables in R

First, we should become familiar with how we can generate data. To do this, we will need to know how R handles random numbers.

In Base R, the following common random number distributions can be generated using the following functions:

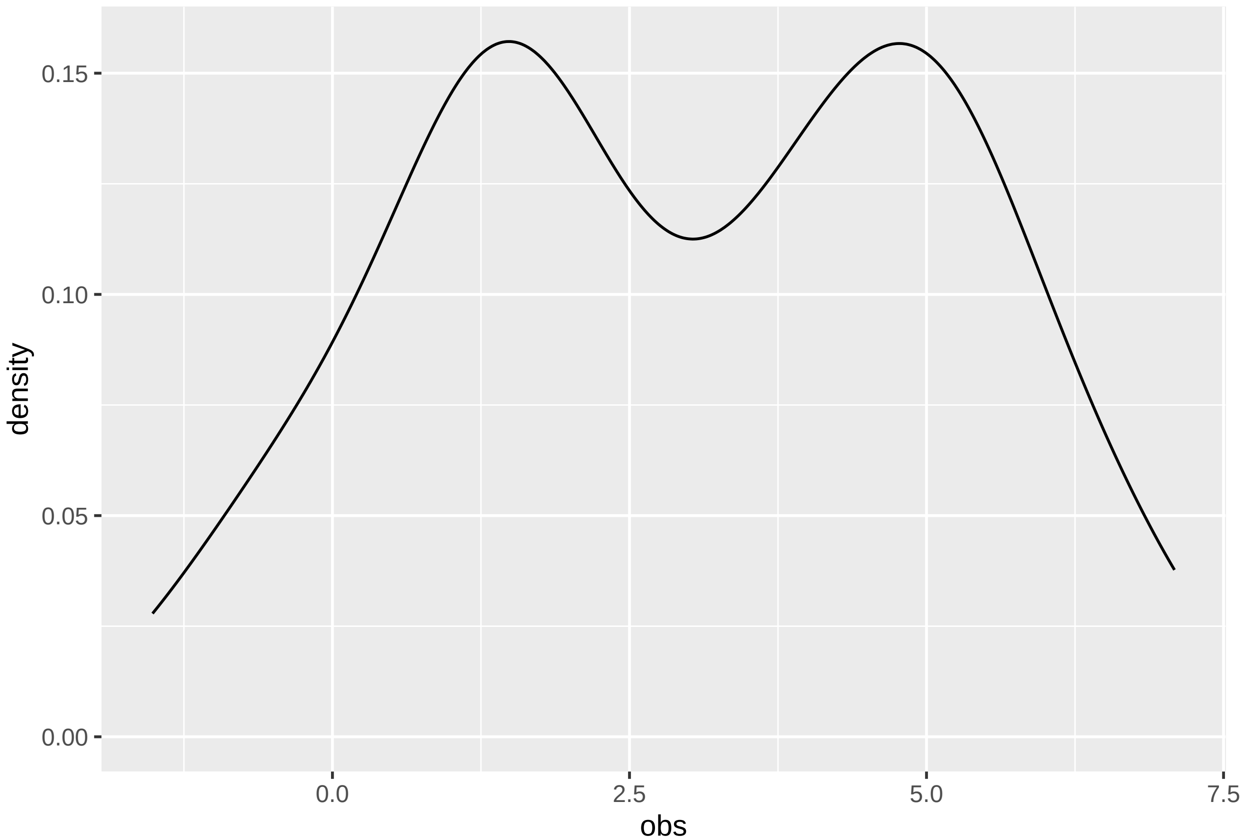
- Binomial: *rbinom(N, x, p)*
  - N trials, outcome is either x or 0, and it yields x with the probability p
- Normal: *rnorm(N, mean = μ, sd = σ)*
  - N trials, each drawn from $N(\mu, \sigma)$, i.e. the distribution is mean μ and has *standard deviation* σ
- Uniform: *runif(N, min, max)*
  - N trials, each drawn from $U(min, max)$

Let's try an example:

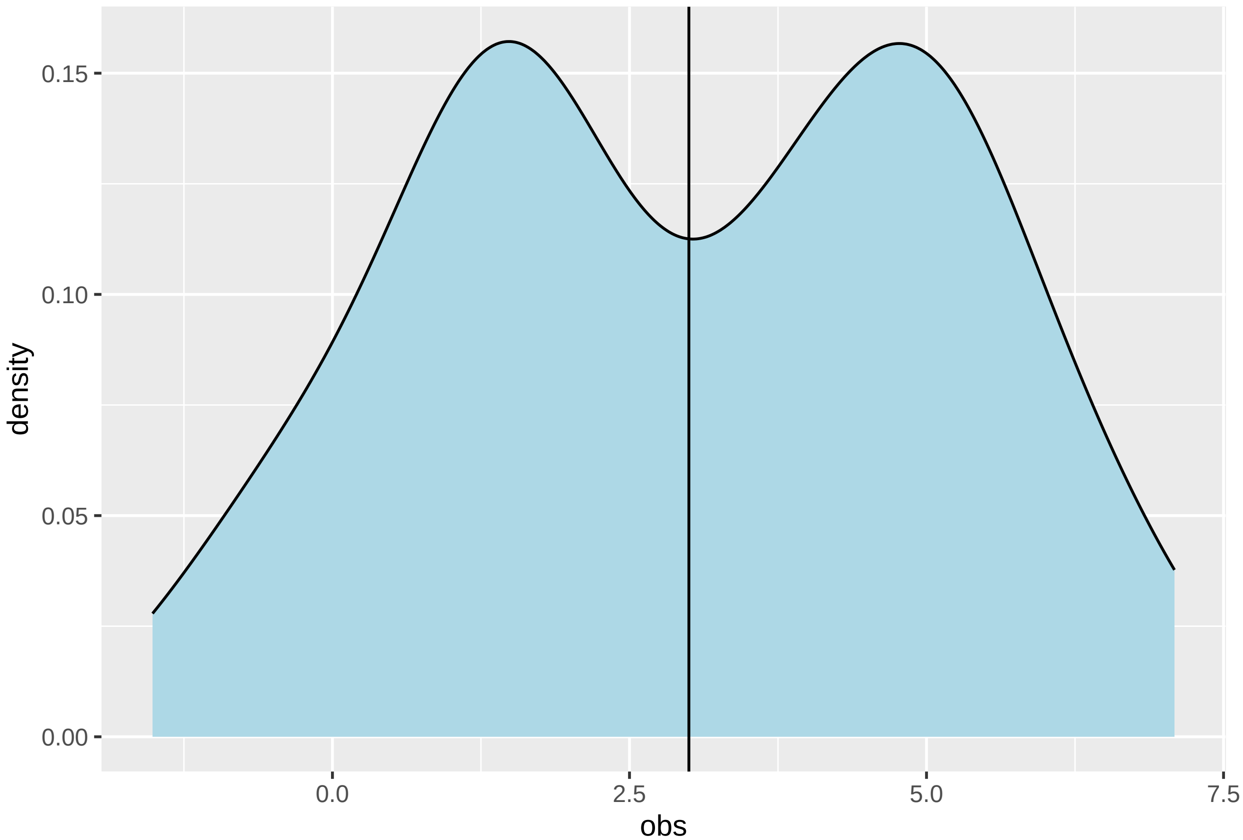
Exercise 1: Generate 100 samples of outcomes from the following distribution and graph these as a density plot:

$$N(3, 2) + U(-1, 1)$$

```
# Grab 2 packages
library(pacman)
p_load(tidyverse, ggplot2)

# Generate in a tibble
ex_1_data = tibble(
              obs = rnorm(100, mean = 3, sd = 2) + runif(100, min = -1, max = 1)
)


# Basic graph
ggplot(ex_1_data) +
  geom_density(aes(obs))
```




```r
# Let's make it look nicer!
ggplot(ex_1_data) +
  # fill the density plot
  geom_density(aes(obs), fill = "lightblue") +
  # vertical line where the mean is
  geom_vline(xintercept = 3)
```



Seeds?

You may have seen the term "seed" around when it comes to generating random numbers on a computer.

A "seed" initializes a pseudorandom number generator. This allows your results to be replicated elsewhere. Try it out:

```r
# Set seed
set.seed(90325) # button mash

# Generate some numbers
rnorm(10, 0, 3)
```

```
##  [1] -1.0079275  2.2570383  1.9844698  1.1620589 -0.8240209 -1.0463759
##  [7] -2.7948056  2.6680005 -3.4627577  0.7473596
```

```r
# Do it again:
set.seed(90325)
rnorm(10, 0, 3)
```

```
##  [1] -1.0079275  2.2570383  1.9844698  1.1620589 -0.8240209 -1.0463759
##  [7] -2.7948056  2.6680005 -3.4627577  0.7473596
```

Another fun exercise would be to find out what the probability of getting these exact numbers twice without re-setting the seed… But we won't do that now.

Tibble as a Function

We already saw 1 way to implement the *tibble* function to generate random data. Now let's do that within a function. This function will generate a dataframe of:

- N observations
- $ID_i = i$ numbers the observations
- $X_i = N(10, 2)$
- $Z_i = U(-3, 3) - 0.2 * X_i$
- $\varepsilon_i = N(0, 1)$
- $Y_i = \alpha + \beta X_i + \delta Z_i + \varepsilon_i$

For now, keep N, α, β and δ as variables that we can change.

```r
# Data generating function
data_gen = function(N, alpha, beta, delta) {

  # create the dataset
  data = tibble(
    ID = 1:N,
    X = rnorm(N, mean = 10, sd = 2),
    Z = runif(N, min = -3, max = 3) - 0.2*X,
    e = rnorm(N, mean = 0, sd = 1),

    # create Y as a function of other variables
    Y = alpha + beta*X + delta*Z + e
  )

  return(data)
}
```

Try with the following parameters:

$$N = 1000, \quad \alpha = 4, \quad \beta = 1/2, \quad \delta = 2$$

```r
test_data = data_gen(N = 1000, alpha = 4, beta = 0.5, delta = 2)

head(test_data)
```

```
## # A tibble: 6 x 5
##      ID     X     Z       e      Y
##   <int> <dbl> <dbl>   <dbl>  <dbl>
## 1     1  8.87 -3.58  0.827   2.10
## 2     2  8.68 -3.50 -0.563   0.780
## 3     3  8.70 -2.73  1.59    4.47
## 4     4  6.06  1.55 -0.0454 10.1
## 5     5  7.18 -4.16 -1.05   -1.78
## 6     6  8.70 -3.95 -0.296   0.153
```

Simulate Regressions

Create a function that simulates the data above (with the same parameters) 100 times, each time performing the following regression:

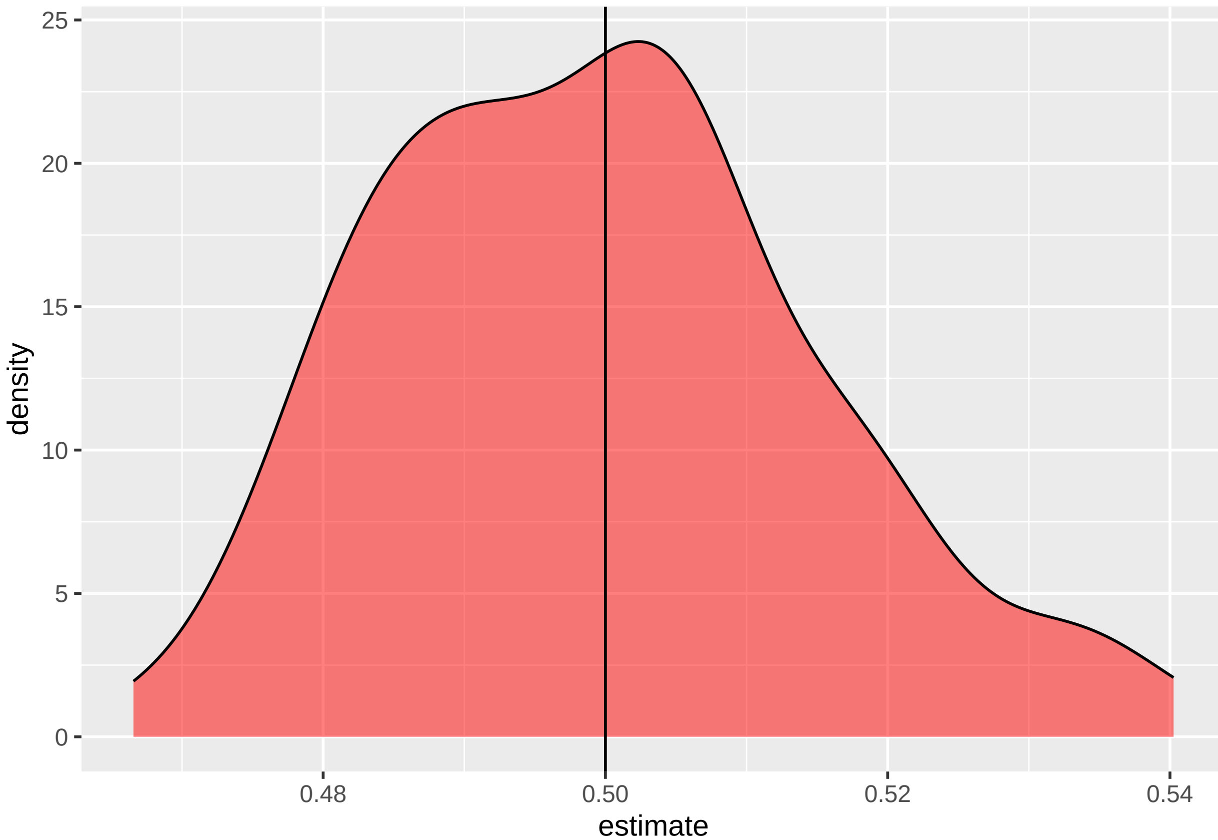
$$Y_i = a + bX_i + cZ_i$$

Collect the estimates for b.

```r
# Grab some packages
p_load(fixest, broom)


# regression simulation, function of the number of iterations
reg_sim = function(iter){

  # get data
  data_i = data_gen(N = 1000, alpha = 4, beta = 0.5, delta = 2)

  # regression
  reg_i = feols(data_i, Y ~ X + Z)

  # Clean a bit
  bind_rows(tidy(reg_i)) %>%
      # only want the estimate of b
        filter(term == "X") %>%
      # grab the estimate
        select(2)
}


# Simulate for 100 periods
iter = 100

results_1 = bind_rows(map(1:iter, reg_sim))
```

Now plot the density of the estimates from the results of the simulation

```r
# Plot
ggplot(results_1) +
  # fill the density plot
  geom_density(aes(estimate), fill = "red", alpha = 0.5) + # alpha makes it see-through!

  # recall that the true Beta = 0.5
  geom_vline(xintercept = 0.5)
```



Omitted Variable Bias

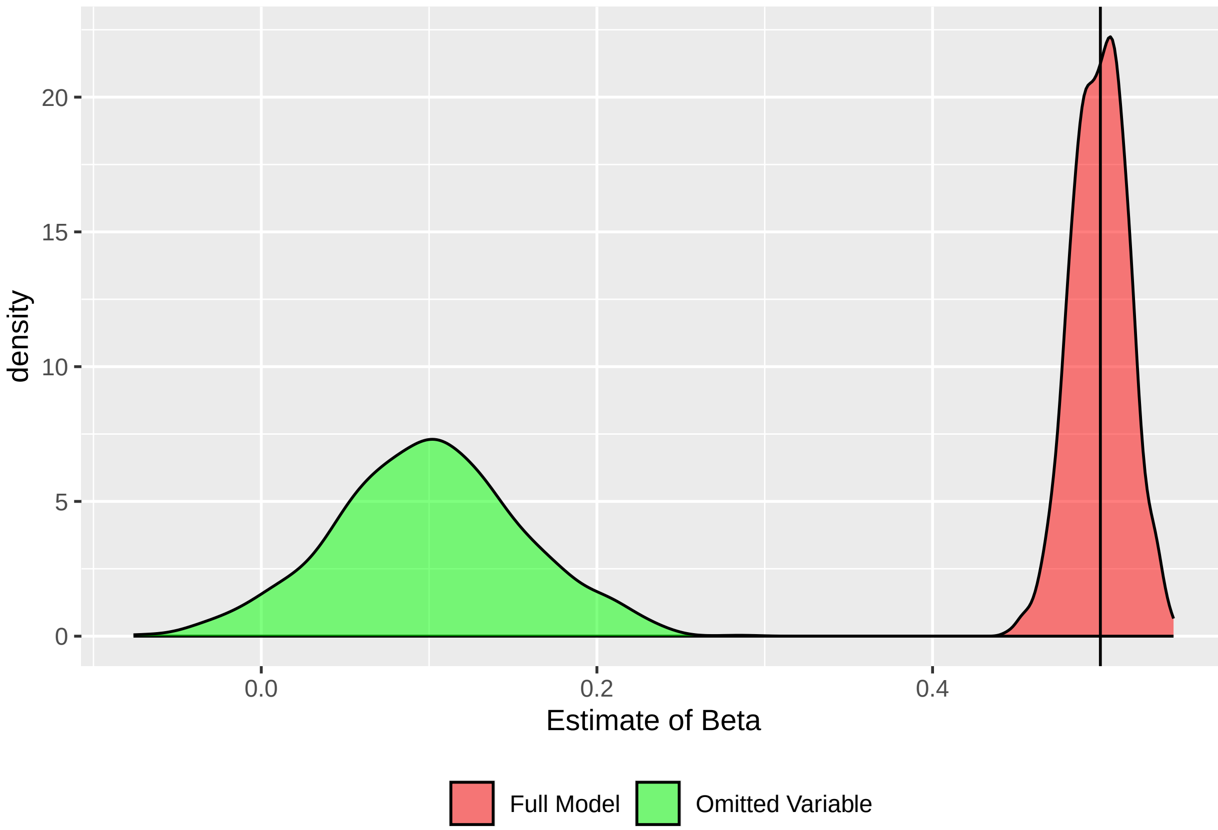
Do the same simulation from the previous part but this time only estimating the regression:

$$Y_i = a + bX_i$$

```
# Omitted Variable Bias Regression
reg_sim_2 = function(iter){

  # get data (same)
  data_i = data_gen(N = 1000, alpha = 4, beta = 0.5, delta = 2)

  # new regression
  reg_i = feols(data_i, Y ~ X)

  # Clean a bit
  bind_rows(tidy(reg_i)) %>%
      # only want the estimate of b
        filter(term == "X") %>%
      # grab the estimate
        select(2)
}
```

Simulate the full regression model and the OVB simulation 1000 times each and graph the estimates for b on the same density plot

```r
# 1000 periods
iter = 1000


# Full regression
results_full = bind_rows(map(1:iter, reg_sim))

# OVB regression
results_ovb = bind_rows(map(1:iter, reg_sim_2))


# Graph
# Plot
ggplot() +
  # Full reg estimates
  geom_density(aes(results_full$estimate, fill = "Full Model"), alpha = 0.5) +

  # OVB estimates
  geom_density(aes(results_ovb$estimate, fill = "Omitted Variable"), alpha = 0.5) +

  # true Beta = 0.5
  geom_vline(xintercept = 0.5) +

  # Label
  labs(x = "Estimate of Beta") +

  # Fill colors
  scale_fill_manual(name = "", values = c("red", "green")) +

  # Legend position
  theme(legend.position = "bottom")
```



You can estimate the mean of the biased estimator:

$$E(b^{OVB}) = \beta + \delta \frac{cov(X, Z)}{var(X)} = 0.5 + 2\frac{-0.8}{4} = 0.1$$

T-Stats OVB Simulation

Run the simulation again, except this time grab the t-stat on the coefficient on X

First recreate the simulation function, as *1 function*

```
# Full Model Regression Simulation
reg_t_sim = function(iter){

  # get data
  data_i = data_gen(N = 1000, alpha = 4, beta = 0.5, delta = 2)

  # regressions
  reg_full = feols(data_i, Y ~ X + Z) # full regression
  reg_ovb = feols(data_i, Y ~ X)  # OVB regression

  # Clean a bit
  bind_rows(tidy(reg_full),tidy(reg_ovb)) %>%
        filter(term == "X") %>%
      # t-stat is the 4th term
        select(4) %>%
      # name whether estimate came from full or ovb regression
        mutate(OVB = c("No", "Yes"))
```

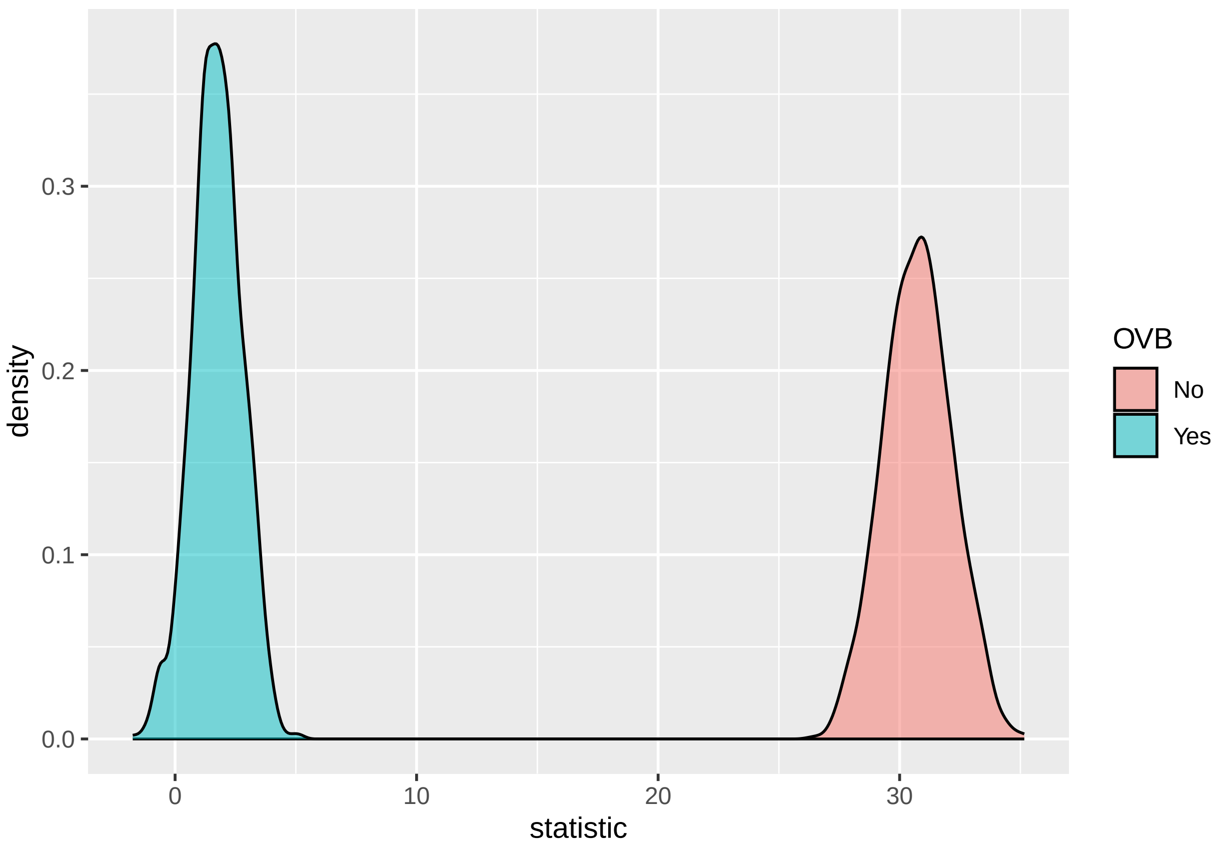
```
}
```

Second: iterate this 1000 times

```
# 1000 periods
iter = 1000

# Run Simulation
results_t_sim = bind_rows(map(1:iter, reg_t_sim))
```

Third: Graph!

```
ggplot(results_t_sim) +
  geom_density(aes(statistic, fill = OVB),
               stat = "density", position = "identity", alpha = 0.5)
```



Fourth: (optional) make it look nice :)

- Let's make the full model density red, the OVB density green
- Shade in the area where we fail to reject the null hypothesis $H_0 : \beta = 0$

```
ggplot(results_t_sim) +

  # Same as before
  geom_density(aes(statistic, fill = OVB),
               stat = "density", position = "identity", alpha = 0.5) +

  # Change colors
  scale_fill_manual(name = "Omitted Variable?", values = c("red", "green")) +
```

```r
# Shade region (basically a rectangle)
annotate("rect", xmin = -1.96, xmax = 1.96, ymin = 0, ymax = 0.4,
         alpha = .5) +

# Name "fail to reject" region.
annotate("text", x=-2, y=0.4, hjust=0, vjust=-.25, size=3, label = "Fail to Reject") +
  # hjust and vjust adjust the text's relative position to the coordinates

# Labels
labs(x = "t-statistic") +

theme(legend.position = "bottom")
```