pymc3_pb_smoothness

August 2, 2019

1 Adding a background to the simple peakbag

I'm going to add a proper treatment of the mode frequencies. The remaining caveats are:

- I will not impose a complex prior on linewidth
- I will not impose a complex prior on mode heights
- I am not accounting for any asphericities due to near-surface magnetic fields

The expected effect of this will be, in order:

- Increased uncertainty on linewidths
- Increased runtime as the mode heights are less constrained
- Possible linewidth broadening or shifting of mode frequencies if there are significant frequency shifts

In [18]:
```
import numpy as np
import matplotlib.pyplot as plt

import lightkurve as lk
from astropy.units import cds
from astropy import units as u
import seaborn as sns
import mystyle as ms

import corner
import pystan
import pandas as pd
import pickle
import glob
from astropy.io import ascii
import os

import pymc3 as pm
import arviz
```

In [2]:
```
target = 3632418
mal = pd.read_csv('../../data/malatium.csv', index_col=0)
```

```
idx = np.where(mal.KIC == target)[0][0]
star = mal.loc[idx]
kic = star.KIC
numax_ = star.numax
dnu_ = star.dnu
```

In [3]:
```
sfile = glob.glob('../../data/*{}*.pow'.format(kic))
data = ascii.read(sfile[0]).to_pandas()
ff, pp = data['col1'].values, data['col2'].values
```

In [4]:
```
# Read in the fit data
cad = pd.read_csv('../../data/cadmium.csv', index_col=0)
cad = cad.loc[cad.KIC == target]
```

In [5]:
```
# Read in the mode locs
cop = pd.read_csv('../../data/copper.csv',index_col=0)
cop = cop[cop.l != 3]
modelocs = cop[cop.KIC == str(kic)].Freq.values[18:27]
elocs = cop[cop.KIC == str(kic)].e_Freq.values[18:27]
modeids = cop[cop.KIC == str(kic)].l.values[18:27]
overtones = cop[cop.KIC == str(kic)].n.values[18:27]

lo = modelocs.min() - .25*dnu_
hi = modelocs.max() + .25*dnu_

sel = (ff > lo) & (ff < hi)
f = ff[sel]
p = pp[sel]
```

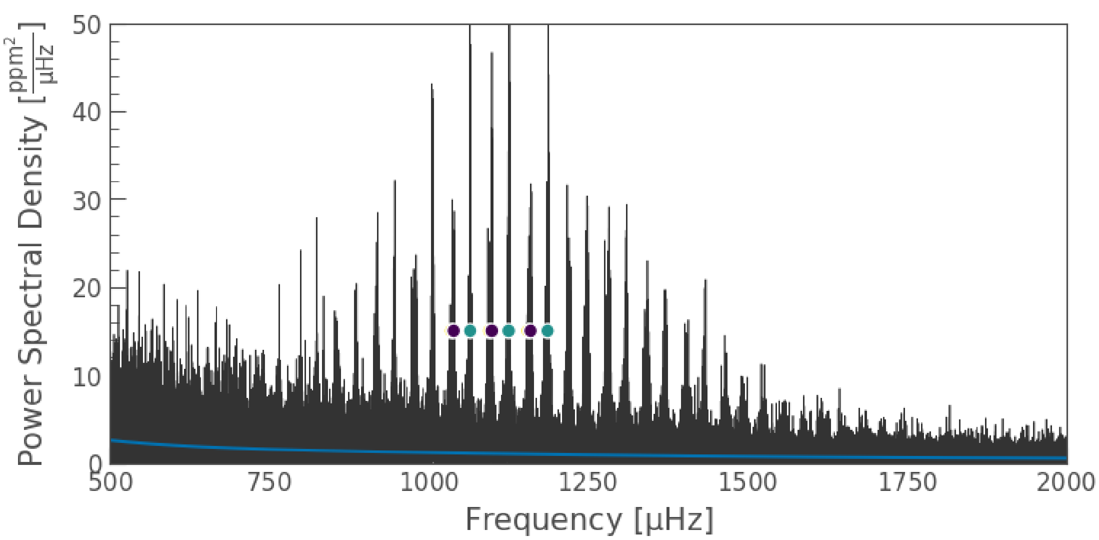
In [6]:
```
def harvey(f, a, b, c):
    harvey = 0.9*a**2/b/(1.0 + (f/b)**c);
    return harvey

def get_apodization(freqs, nyquist):
    x = (np.pi * freqs) / (2 * nyquist)
    return (np.sin(x)/x)**2

def get_background(f, a, b, c, d, j, k, white, scale, nyq):
    background = np.zeros(len(f))
    background += get_apodization(f, nyq) * scale\
                    * (harvey(f, a, b, 4.) + harvey(f, c, d, 4.) + harvey(f, j, k, 2.)
                    + white
    return background
```

In [7]:
```
try:
    backdir = glob.glob('/home/oliver/PhD/mnt/RDS/malatium/backfit/'
                        +str(kic)+'/*_fit.pkl')[0]
    with open(backdir, 'rb') as file:
        backfit = pickle.load(file)
```

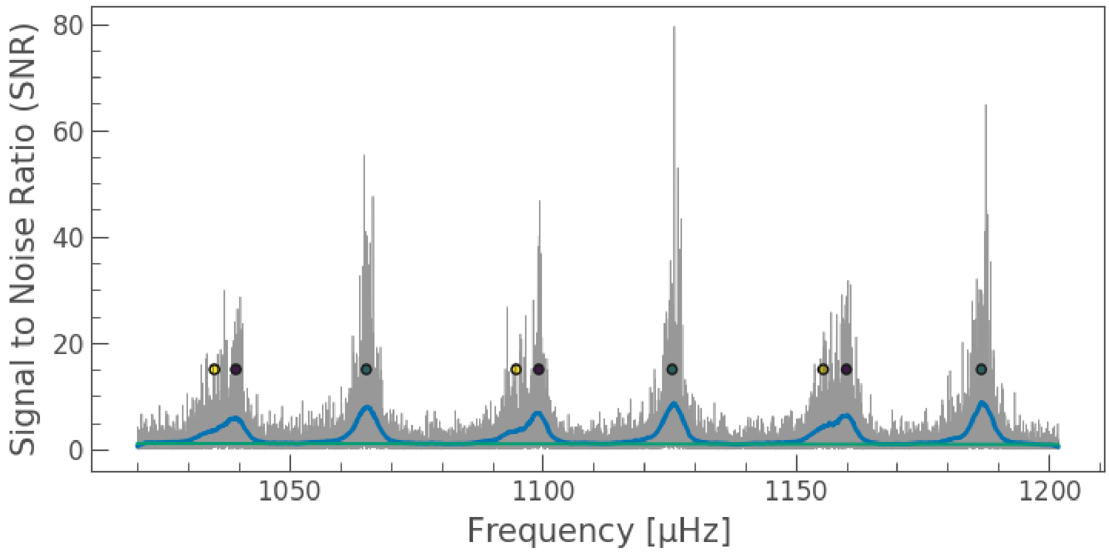
```
        labels=['loga','logb','logc','logd','logj','logk','white','scale','nyq']
        res = np.array([np.median(backfit[label]) for label in labels])
        res[0:6] = 10**res[0:6]

        phi_ = np.array([np.median(backfit[label]) for label in labels])
        phi_sigma = pd.DataFrame(backfit)[labels].cov()
        phi_cholesky = np.linalg.cholesky(phi_sigma)

        model = get_background(ff, *res)
        m = get_background(f, *res)
    except IndexError:
        pg = lk.periodogram.SNRPeriodogram(f*u.microhertz, pf*(cds.ppm**2/u.microhertz))
        p = pg.flatten().power.value * 2
```

In [8]:
```
pg = lk.Periodogram(ff*u.microhertz, pp*(cds.ppm**2 / u.microhertz))
ax = pg.plot()
ax.plot(ff, model)
plt.scatter(modelocs, [15]*len(modelocs),c=modeids, s=50, edgecolor='w',zorder=100)
ax.set_xlim(500,2000)
ax.set_ylim(0, 50)
```

Out[8]: (0, 50)



In [9]:
```
pg = lk.periodogram.SNRPeriodogram(f*u.microhertz, p*(cds.ppm**2/u.microhertz))
ax = pg.plot(alpha=.5)
pg.smooth(filter_width=3.).plot(ax=ax, linewidth=2)
ax.plot(f, m)
plt.scatter(modelocs, [15]*len(modelocs),c=modeids, s=20, edgecolor='k')
plt.show()
```



1.1 Finding the mode frequencies through the asymptotic relation

We have good constraints on the mode frequencies from previous studies, but since we are using the same data, we don't want to use their posteriors as priors on our frequencies. Instead we want to find a way to include the prior knowledge from previous studies without making our study dependent on them.

The locations of the radial $l = 0$ modes can be predicted from the asymptotic relation. Radial modes of consecutive overtones are separated by the large frequency separation $\Delta\nu$, in principle. However in practice, these mode frequencies can be subject to some curvature, as well as noise from glitches in the sound speed profile in the stellar interior.

We're going to omit a detailed treatment of glitches for now, but we *will* include a curvature term, using the astymptotic relation presented in Vrard et al. 2015, which goes as

$\nu_{l=0} = (\bar{n} + \epsilon + (\frac{\alpha}{2}(n_{\rm max} - \bar{n})^2))\Delta\nu + \Delta$

where $\nu_{\rm l=0}$ is the frequency locations of all l=0 modes at overtones $\bar{n}$, ϵ is the phase offset, α determines the curvature, $\Delta\nu$ is the large separation, and $n_{\rm max}$ is the overtone closest to $\nu_{\rm max}$, round which the curvature is centered, and is given by

$n_{\rm max} = \frac{\nu_{\rm max}}{\Delta\nu} - \epsilon,$

and Δ is the noise on the frequency positions. Including this noise term allows us to formalize the 'smoothness condition' (Davies et al. 2016), where we specify that the difference in large frequency separation between subsequent radial modes should be close to zero, with some scatter. We therefore specify Δ as:

$\Delta = \mathcal{N}(0, \sigma_\Delta),$

where σ_Δ is a free parameter. I guess eventually we should really upgrade this to a Gaussian Process periodic Kernel, so that we take care of glitch patterns.

The positions of the dipole and octopole $l = 1, 2$ modes are then determined from the radial frequencies, as

$\nu_{l=1} = \nu_{l=0} + \delta\nu_{01} + \mathcal{N}(0, \sigma_{01})$

$\nu_{l=2} = \nu_{l=0} - \delta\nu_{02} + \mathcal{N}(0, \sigma_{02})$

where $\delta\nu_{01}$ and $\delta\nu_{02}$ are the small separations between the radial frequency and the dipole and octopole frequencies of the same radial degree. σ_{01} and σ_{02} are the uncertainties on these separations. All are free parameters, and as before we're adding on noise.

So we know have a complex hierarchical system where the mode frequencies are latent parameters, and we have a bunch of hyperparameters controlling them, giving them noise and curvature. These hyperparameters are where we include our *prior* information from the Kages and LEGACY papers, as first guesses and as means on the distributions from which they are drawn.

$\epsilon \sim \mathcal{N}(\epsilon_{\rm prior}, 1.)$
$\alpha \sim \mathcal{N}_{\rm log}(\alpha_{\rm prior}, 1.)$
$\Delta\nu \sim \mathcal{N}(\Delta\nu_{\rm prior}, \Delta\nu_{\rm prior} * 0.1)$
$\nu_{\rm max} \sim \mathcal{N}(\nu_{\rm max,prior}, \nu_{\rm max,prior} * 0.1)$
$\delta\nu_{01} \sim \mathcal{N}(\delta\nu_{01,\rm prior}, \Delta\nu_{\rm prior} * 0.1)$
$\delta\nu_{02} \sim \mathcal{N}(\delta\nu_{02,\rm prior}, 3.)$

ϵ, α and the small separations will be determined from a fit to the mode frequencies of each star. The remainder are taken as reported in the literature.

The noise terms σ_Δ, σ_{01} and σ_{02} will all be drawn from lognormal distributions, to ensure they don't go too close to zero.

Notice that I've flipped the sign in the equation for $\nu_{l=2}$. In a typical $l = 0, 2$ pair, the octopole mode is one overtone lower than the radial mode. When passing the overtone numbers into the model, we simply add $+1$ to those of the octopole modes to make this equation work and maintain our traditional measure of $\delta\nu_{02}$.

We use the overtone numbers n reported in LEGACY and Kages, and also do not fit for any modes of oscillation not reported in those papers.

2 Build the model

```
In [10]: class model():
             def __init__(self, f, n0, n1, n2, f0_, f1_, f2_):
                 self.f = f
                 self.n0 = n0
                 self.n1 = n1
                 self.n2 = n2
                 self.npts = len(f)
                 self.M = [len(f0_), len(f1_), len(f2_)]


             def harvey(self, a, b, c):
                 harvey = 0.9*a**2/b/(1.0 + (self.f/b)**c);
                 return harvey

             def get_apodization(self, nyquist):
                 x = (np.pi * self.f) / (2 * nyquist)
                 return (np.sin(x)/x)**2

             def get_background(self, loga, logb, logc, logd, logj, logk, white, scale, nyq):
                 background = np.zeros(len(self.f))
                 background += self.get_apodization(nyq) * scale  \
```

```
                    * (self.harvey(10**loga, 10**logb, 4.) \
                    +  self.harvey(10**logc, 10**logd, 4.) \
                    +  self.harvey(10**logj, 10**logk, 2.))\
                    +  white
    return background

def epsilon(self, i, l, m):
#We use the prescriptions from Gizon & Solanki 2003 and Handberg & Campante 2012
    if l == 0:
        return 1
    if l == 1:
        if m == 0:
            return np.cos(i)**2
        if np.abs(m) == 1:
            return 0.5 * np.sin(i)**2
    if l == 2:
        if m == 0:
            return 0.25 * (3 * np.cos(i)**2 - 1)**2
        if np.abs(m) ==1:
            return (3/8)*np.sin(2*i)**2
        if np.abs(m) == 2:
            return (3/8) * np.sin(i)**4
    if l == 3:
        if m == 0:
            return (1/64)*(5*np.cos(3*i) + 3*np.cos(i))**2
        if np.abs(m) == 1:
            return (3/64)*(5*np.cos(2*i) + 3)**2 * np.sin(i)**2
        if np.abs(m) == 2:
            return (15/8) * np.cos(i)**2 * np.sin(i)**4
        if np.abs(m) == 3:
            return (5/16)*np.sin(i)**6

def lor(self, freq, h, w):
    return h / (1.0 + 4.0/w**2*(self.f - freq)**2)

def mode(self, l, freqs, hs, ws, i, split=0):
    for idx in range(self.M[l]):
        for m in range(-l, l+1, 1):
            self.modes += self.lor(freqs[idx] + (m*split),
                                 hs[idx] * self.epsilon(i, l, m),
                                 ws[idx])

def model(self, p):
    f0, f1, f2, g0, g1, g2, h0, h1, h2, split, i, phi = p

    # Unpack background parameters
    loga = phi[0]
    logb = phi[1]
```

```
            logc = phi[2]
            logd = phi[3]
            logj = phi[4]
            logk = phi[5]
            white = phi[6]
            scale = phi[7]
            nyq = phi[8]

            # Calculate the modes
            self.modes = np.zeros(self.npts)
            self.mode(0, f0, h0, g0, i)
            self.mode(1, f1, h1, g1, i, split)
            self.mode(2, f2, h2, g2, i, split)
            self.modes *= self.get_apodization(nyq)

            #Calculate the background
            self.back = self.get_background(loga, logb, logc, logd, logj, logk,
                                           white, scale, nyq)

            #Create the model
            self.mod = self.modes + self.back
            return self.mod

        def asymptotic(self, n, numax, deltanu, alpha, epsilon):
            nmax = (numax / deltanu) - epsilon
            over = (n + epsilon + ((alpha/2)*(nmax - n)**2))
            return over * deltanu

        def f0(self, p):
            numax, deltanu, alpha, epsilon, d01, d02 = p

            return self.asymptotic(self.n0, numax, deltanu, alpha, epsilon)

        def f1(self, p):
            numax, deltanu, alpha, epsilon, d01, d02 = p

            f0 = self.asymptotic(self.n1, numax, deltanu, alpha, epsilon)
            return f0 + d01

        def f2(self, p):
            numax, deltanu, alpha, epsilon, d01, d02 = p

            f0 = self.asymptotic(self.n2+1, numax, deltanu, alpha, epsilon)
            return f0 - d02
```

In [11]:
```
f0_ = modelocs[modeids==0]
f1_ = modelocs[modeids==1]
f2_ = modelocs[modeids==2]
```

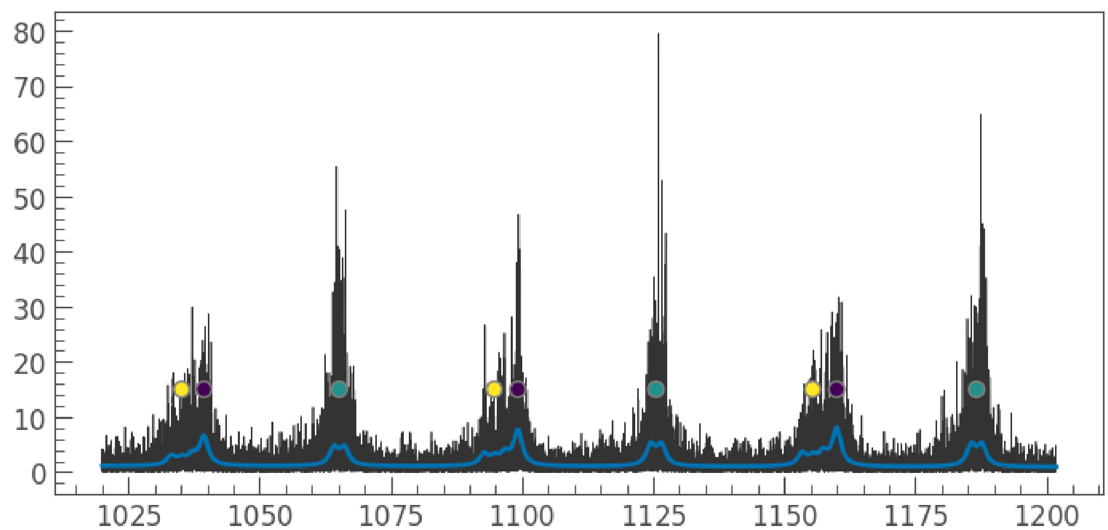
```
f0_e = elocs[modeids==0]
f1_e = elocs[modeids==1]
f2_e = elocs[modeids==2]
n0 = overtones[modeids==0]
n1 = overtones[modeids==1]
n2 = overtones[modeids==2]

alpha_ = cad.alpha.values[0]
epsilon_ = cad.epsilon.values[0]
d01_ = cad.d01.values[0]
d02_ = cad.d02.values[0]
numax_ = star.numax
numax_e = star.enumax
deltanu_ = star.dnu
deltanu_e = star.ednu
```

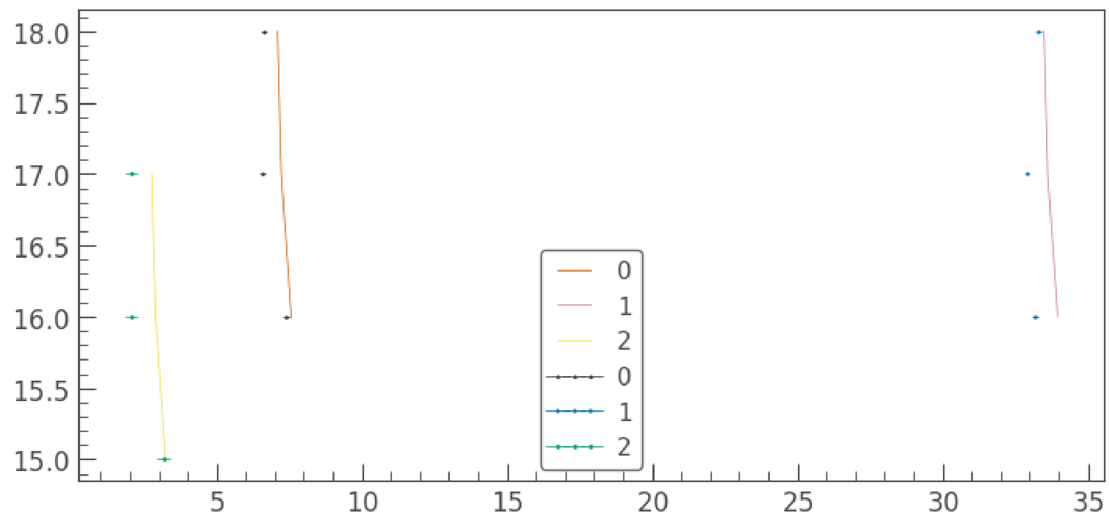
Do some first guesses for height

```
In [12]: def gaussian(locs, l, numax, Hmax0):
             fwhm = 0.25 * numax
             std = fwhm/2.355

             Vl = [1.0, 1.22, 0.71, 0.14]

             return Hmax0 * Vl[l] * np.exp(-0.5 * (locs - numax)**2 / std**2)
```

```
In [13]: init_m =[f0_,                               # l0 modes
                 f1_,                               # l1 modes
                 f2_,                               # l2 modes
                 np.ones(len(f0_)) * 2.0,           # l0 widths
                 np.ones(len(f1_)) * 2.0,           # l1 widths
                 np.ones(len(f2_)) * 2.0,           # l2 widths
                 np.sqrt(gaussian(f0_, 0, numax_, 15.) * 2.0 * np.pi / 2.0) ,# l0 heights
                 np.sqrt(gaussian(f1_, 1, numax_, 15.) * 2.0 * np.pi / 2.0) ,# l1 heights
                 np.sqrt(gaussian(f2_, 2, numax_, 15.) * 2.0 * np.pi / 2.0) ,# l2 heights
                 1.0 * np.sin(np.pi/2),             # projected splitting
                 np.pi/2.,                          # inclination angle
                 np.copy(phi_)                     # background parameters (in log)
                  ]

         init_f =[numax_,                            # numax
                 dnu_,                               # deltanu
                 alpha_,                            # curvature term
                 epsilon_,                          # phase term
                 d01_ ,                             # small separation l=0,1
                 d02_                              # small separation l=0,2
                  ]
```

```
mod = model(f, n0, n1, n2, f0_, f1_, f2_)
with plt.style.context(lk.MPLSTYLE):
    fig, ax = plt.subplots()
    ax.plot(f, p)
    ax.plot(f, mod.model(init_m), lw=2)
    ax.scatter(modelocs, [15]*len(modelocs),c=modeids, s=50, edgecolor='grey', zorder=
    plt.show()

    fig, ax = plt.subplots()
    ax.errorbar(f0_%dnu_, n0, xerr=f0_e, fmt='^',label='0')
    ax.errorbar(f1_%dnu_, n1, xerr=f1_e, fmt='>',label='1')
    ax.errorbar(f2_%dnu_, n2, xerr=f2_e, fmt='o',label='2')

    ax.plot(mod.f0(init_f)%dnu_, n0, label='0')
    ax.plot(mod.f1(init_f)%dnu_, n1, label='1')
    ax.plot(mod.f2(init_f)%dnu_, n2, label='2')

    ax.legend()
    plt.show()
```





3 Build the priors in PyMC3

In [58]:
```
pm_model = pm.Model()
BoundedNormal = pm.Bound(pm.Normal, lower= 0.0)

with pm_model:
    # Frequency Hyperparameters
    alpha   = pm.HalfNormal('alpha',   sigma=1.,            testval=alpha_)
    epsilon = BoundedNormal('epsilon', mu=epsilon_, sigma=1.,            testval=epsil
    d01     = BoundedNormal('d01',     mu=d01_,     sigma=0.1*deltanu_, testval=d01_)
    d02     = BoundedNormal('d02',     mu=d02_,     sigma=3.,            testval=d02_)
    numax   = pm.Normal('numax',   mu=numax_,   sigma=numax_e,   testval=numax_)
    deltanu = pm.Normal('deltanu', mu=deltanu_, sigma=deltanu_e, testval=deltanu_)

    # Noise hyperparameters and latent parameters
    sigma0 = pm.HalfNormal('sigma0', sigma=2., testval=1.)
    sigma01 = pm.HalfNormal('sigma01', sigma=2., testval=1.)
    sigma02 = pm.HalfNormal('sigma02', sigma=2., testval=1.)

    Delta0 = pm.Normal('Delta0', mu=0., sigma=1., shape=len(f0_))
    Delta1 = pm.Normal('Delta1', mu=0., sigma=1., shape=len(f1_))
    Delta2 = pm.Normal('Delta2', mu=0., sigma=1., shape=len(f2_))

    #Frequencies
    f0 = pm.Deterministic('f0', mod.f0([numax, deltanu, alpha, epsilon, d01, d02]) + D
    f1 = pm.Deterministic('f1', mod.f1([numax, deltanu, alpha, epsilon, d01, d02]) + D
    f2 = pm.Deterministic('f2', mod.f2([numax, deltanu, alpha, epsilon, d01, d02]) + D
```

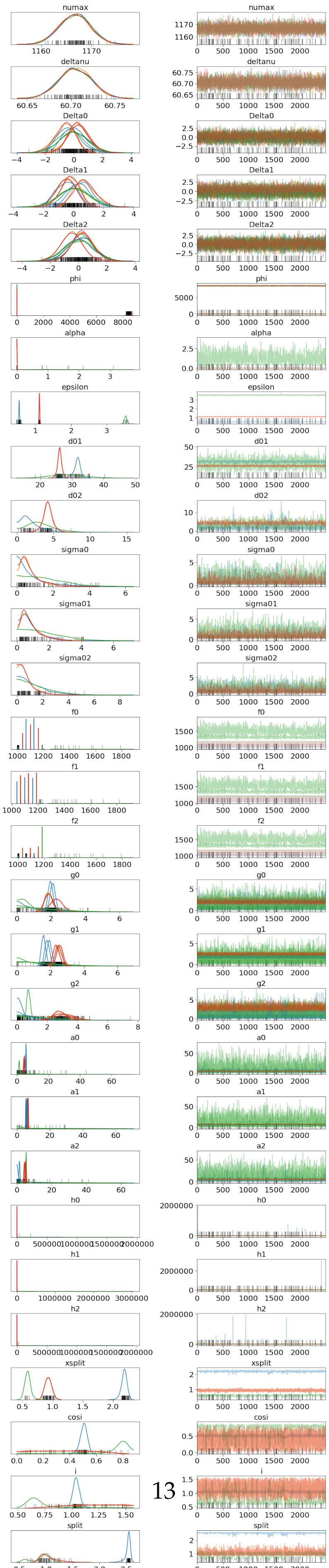
```
    #Testing
#     like0 = pm.Normal('like0', mu=f0, sigma=f0_e, observed=f0_)
#     like1 = pm.Normal('like1', mu=f1, sigma=f1_e, observed=f1_)
#     like2 = pm.Normal('like2', mu=f2, sigma=f2_e, observed=f2_)

#     # Mode linewidths
    g0 = pm.HalfNormal('g0', sigma=2.0, testval=init_m[3], shape=len(init_m[3]))
    g1 = pm.HalfNormal('g1', sigma=2.0, testval=init_m[4], shape=len(init_m[4]))
    g2 = pm.HalfNormal('g2', sigma=2.0, testval=init_m[5], shape=len(init_m[5]))

    # Mode amplitudes
    a0 = pm.HalfNormal('a0', sigma=20., testval=init_m[6], shape=len(init_m[6]))
    a1 = pm.HalfNormal('a1', sigma=20., testval=init_m[7], shape=len(init_m[7]))
    a2 = pm.HalfNormal('a2', sigma=20., testval=init_m[8], shape=len(init_m[8]))

    # Mode heights (determined by amplitude and linewidth)
    h0 = pm.Deterministic('h0', 2*a0**2/np.pi/g0)
    h1 = pm.Deterministic('h1', 2*a1**2/np.pi/g1)
    h2 = pm.Deterministic('h2', 2*a2**2/np.pi/g2)

    # Rotation and inclination parameterizations
    xsplit = pm.HalfNormal('xsplit', sigma=2.0, testval=init_m[10])
    cosi = pm.Uniform('cosi', 0., 1.)

    # Detangled inclination and splitting
    i = pm.Deterministic('i', np.arccos(cosi))
    split = pm.Deterministic('split', xsplit/pm.math.sin(i))

    # Background treatment
    phi = pm.MvNormal('phi', mu=phi_, chol=phi_cholesky, testval=phi_, shape=len(phi_

    #Model
    fit = mod.model([f0, f1, f2, g0, g1, g2, h0, h1, h2, split, i, phi])

    like = pm.Gamma('like', alpha=1, beta=1.0/fit, observed=p)
```

In [59]:
```
init = 5000
with pm_model:
    trace = pm.sample(chains=4, tune=int(init/2), draws=int(init/2))
```

```
INFO:pymc3:Auto-assigning NUTS sampler...
INFO:pymc3:Initializing NUTS using jitter+adapt_diag...
INFO:pymc3:Multiprocess sampling (4 chains in 4 jobs)
INFO:pymc3:NUTS: [phi, cosi, xsplit, a2, a1, a0, g2, g1, g0, Delta2, Delta1, Delta0, sigma02, s
Sampling 4 chains: 100%|| 20000/20000 [9:53:14<00:00,  4.18s/draws]
ERROR:pymc3:There were 48 divergences after tuning. Increase `target_accept` or reparameterize
ERROR:pymc3:There were 34 divergences after tuning. Increase `target_accept` or reparameterize
ERROR:pymc3:There were 7 divergences after tuning. Increase `target_accept` or reparameterize.
```

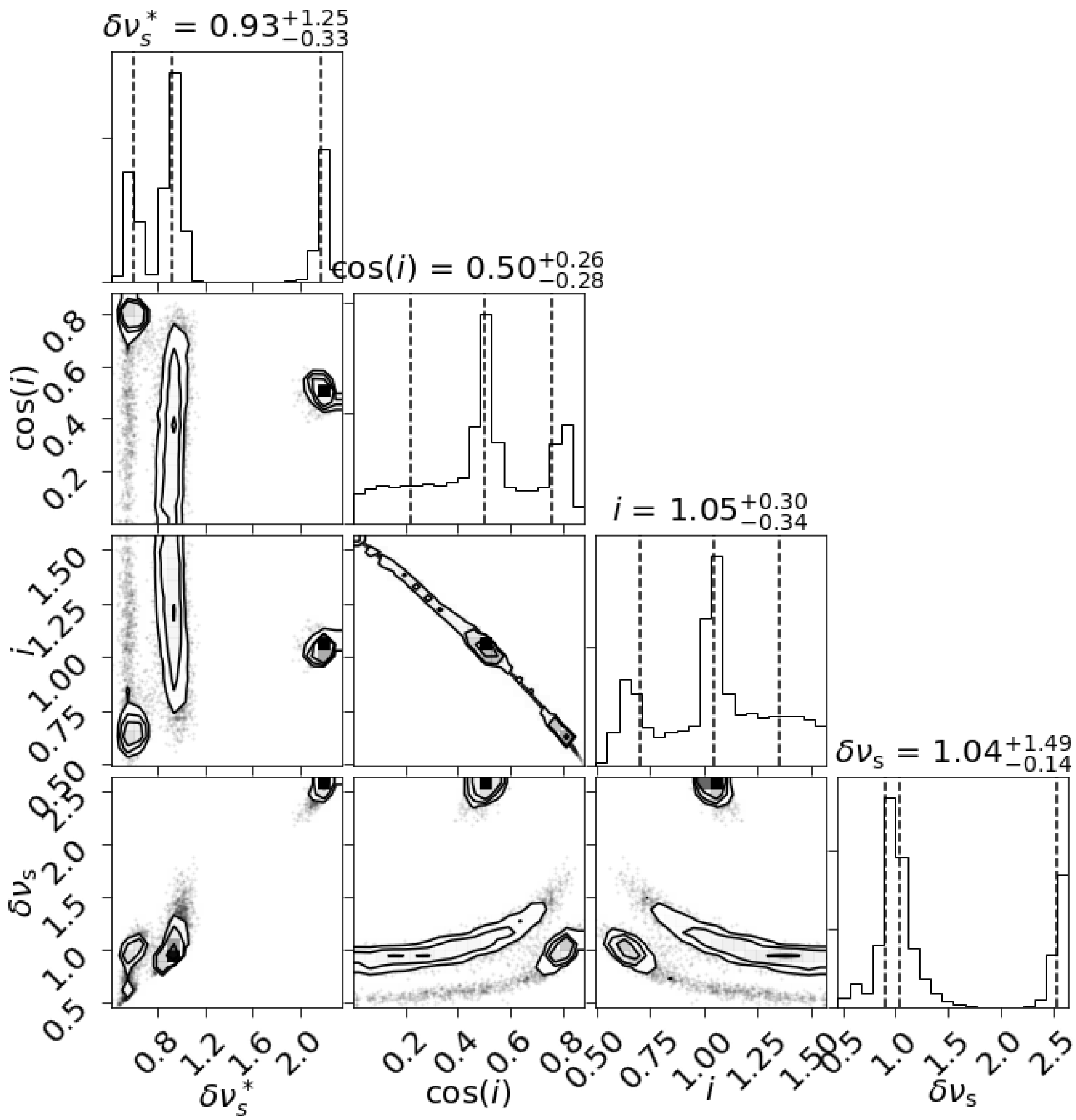
```
ERROR:pymc3:There were 18 divergences after tuning. Increase `target_accept` or reparameterize
ERROR:pymc3:The gelman-rubin statistic is larger than 1.4 for some parameters. The sampler did
ERROR:pymc3:The estimated number of effective samples is smaller than 200 for some parameters.
```

In [60]:
```
pm.traceplot(trace)
plt.show()
```


numax
deltanu
Delta0
Delta1
Delta2
phi
alpha
epsilon
d01
d02
sigma0
sigma01
sigma02
f0
f1
f2
g0
g1
g2
a0
a1
a2
h0
h1
h2
xsplit
cosi
i
split

4 Now lets plot some diagnostics...

```
In [67]: labels=['xsplit','cosi','i','split']
         chain = np.array([trace[label] for label in labels])
         verbose = [r'$\delta\nu_s^*$',r'$\cos(i)$',r'$i$',r'$\delta\nu_{\rm s}$']
         corner.corner(chain.T, labels=verbose, quantiles=[0.16, 0.5, 0.84]
                       ,show_titles=True)
         plt.show()
```



In [71]:
```
labels=['numax','deltanu','alpha','epsilon','d01','d02']
chain = np.array([trace[label] for label in labels])
verbose = [r'$\nu_{\rm max}$', r'$\Delta\nu$', r'$\alpha$',r'$\epsilon$',r'$\delta\nu_
corner.corner(chain.T, labels=verbose, quantiles=[0.16, 0.5, 0.84]
              ,show_titles=True)
plt.show()
```

```
WARNING:root:Too few points to create valid contours
WARNING:root:Too few points to create valid contours
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WARNING:root:Too few points to create valid contours
```
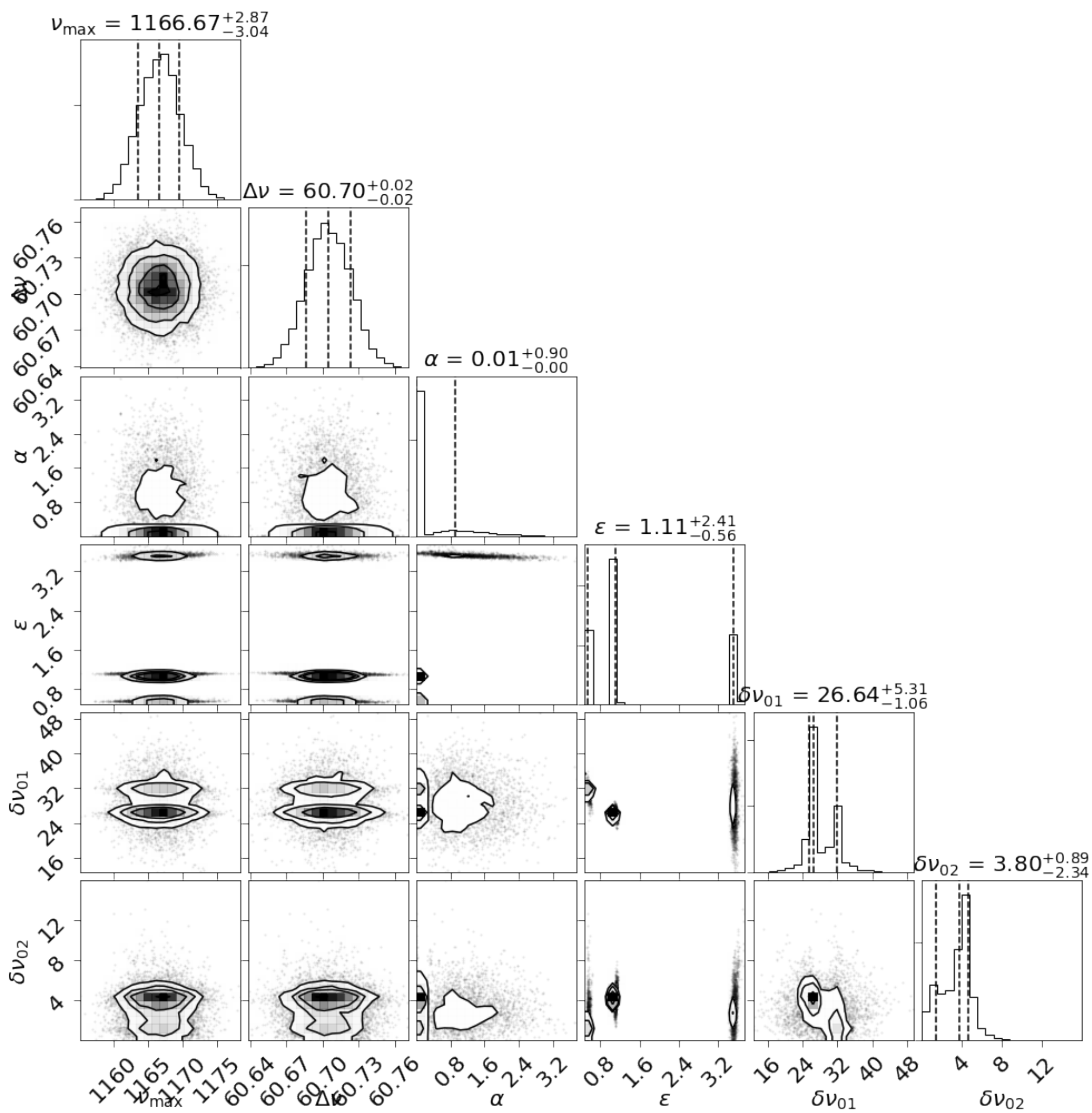
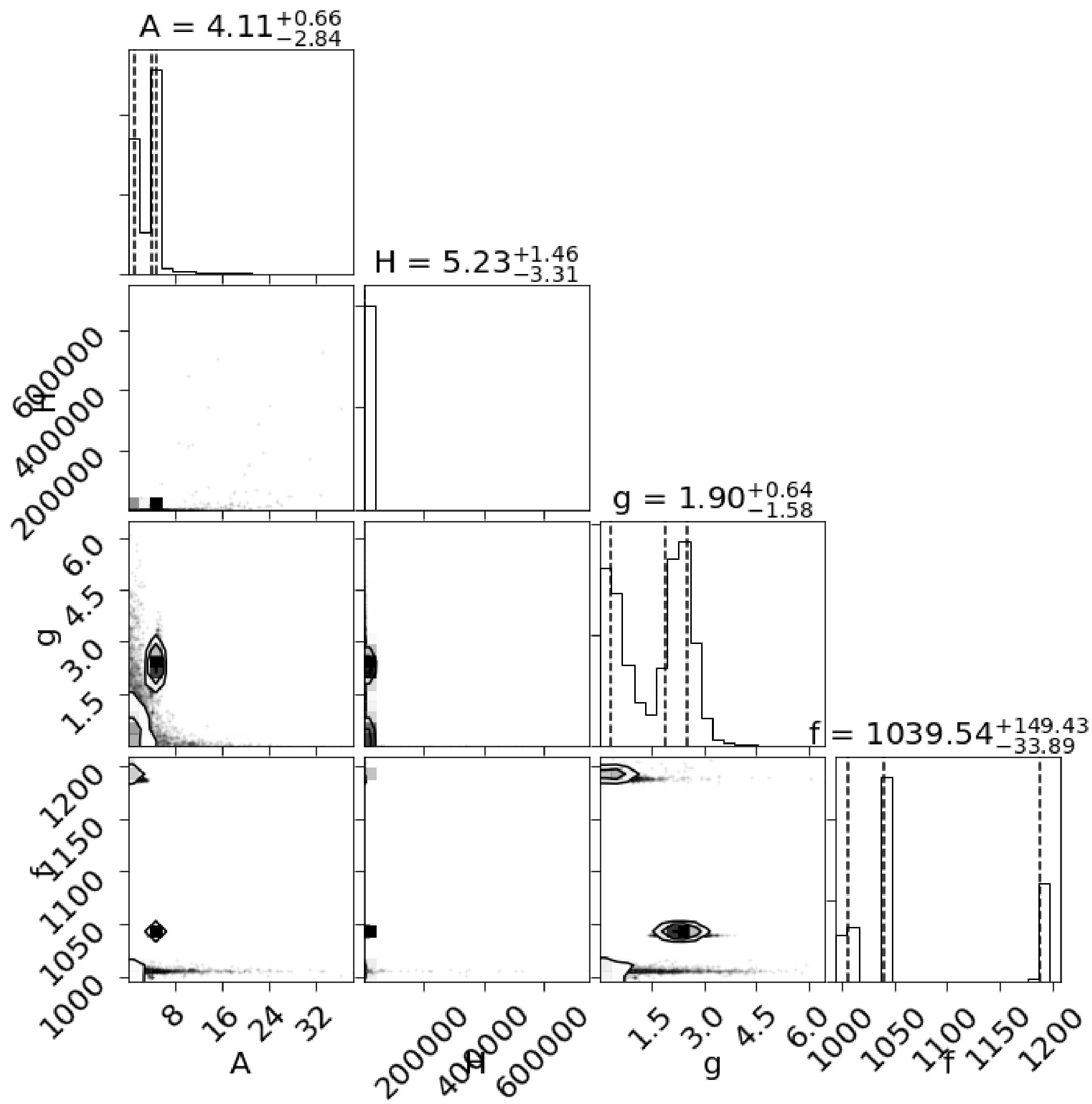



```
In [62]: for idx in range(len(trace['h0'].T)):
             chain = np.array([trace['a0'].T[idx], trace['h0'].T[idx], trace['g0'].T[idx], trac
             corner.corner(chain.T, labels=['A','H','g','f'],
                           quantiles=[0.16, 0.5, 0.84],show_titles=True)
             plt.show()
```

```
WARNING:root:Too few points to create valid contours
WARNING:root:Too few points to create valid contours
WARNING:root:Too few points to create valid contours
```




```
WARNING:root:Too few points to create valid contours
WARNING:root:Too few points to create valid contours
```

```
WARNING:root:Too few points to create valid contours
WARNING:root:Too few points to create valid contours
WARNING:root:Too few points to create valid contours
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```

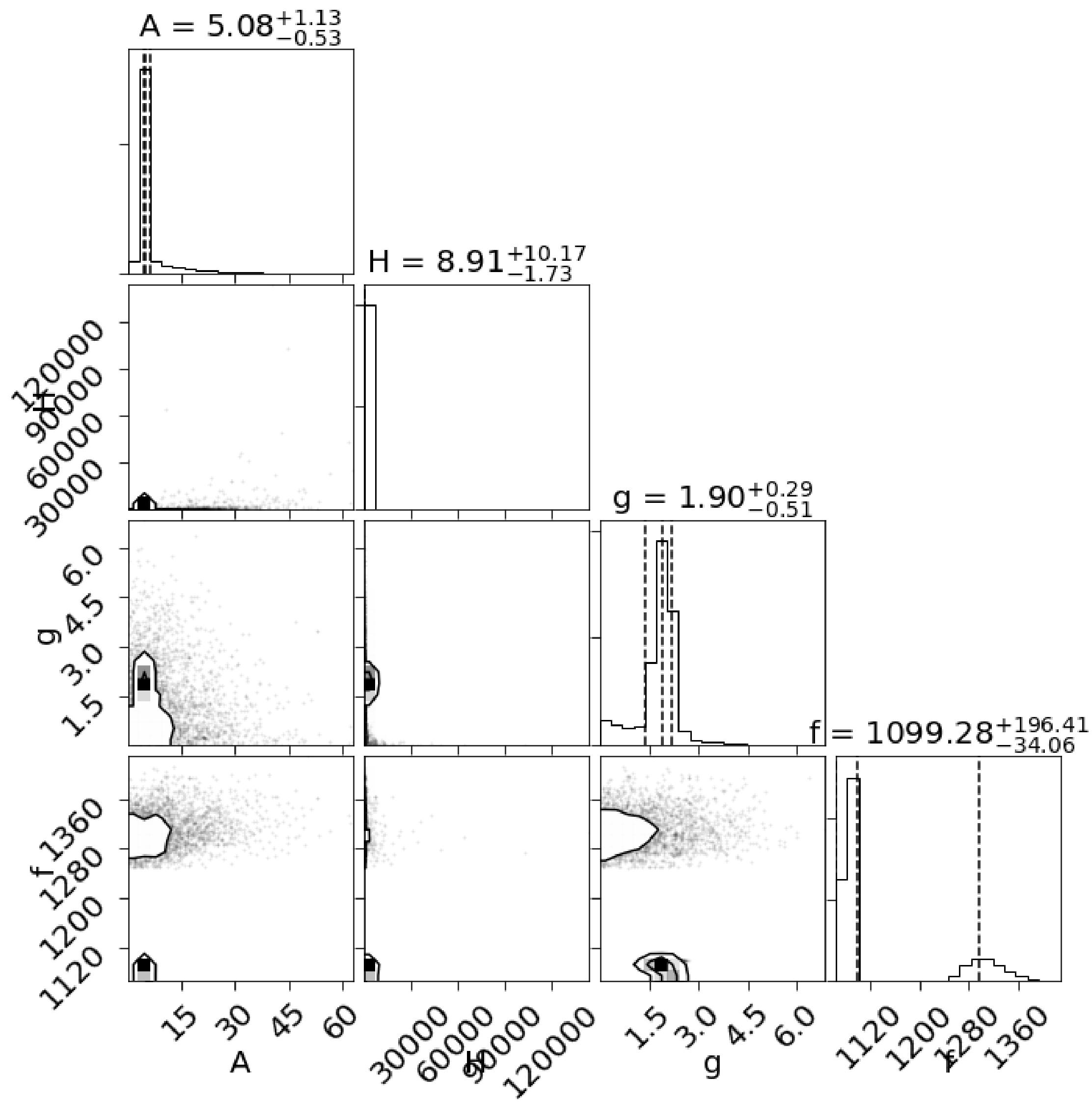


```
WARNING:root:Too few points to create valid contours
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```
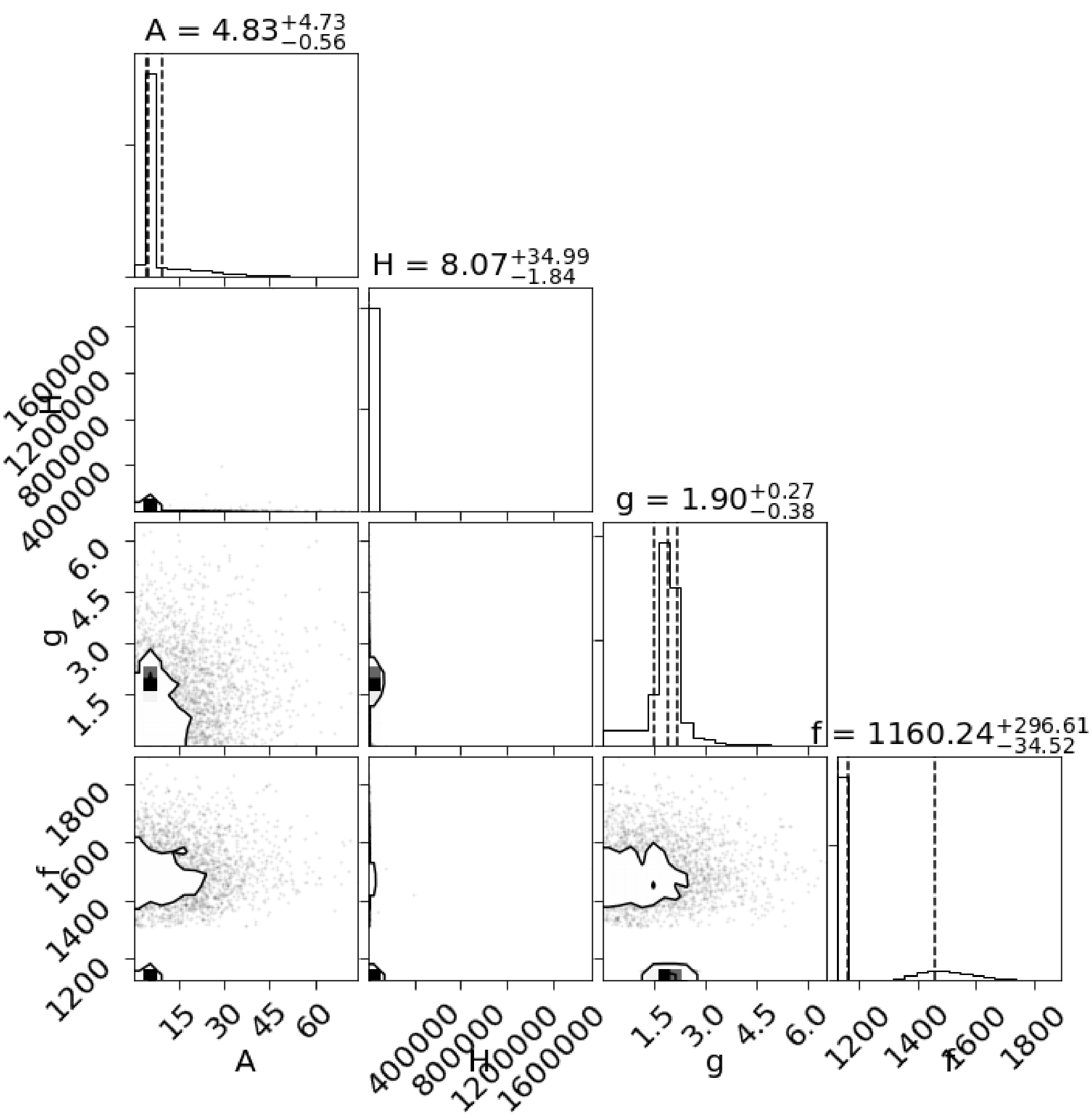
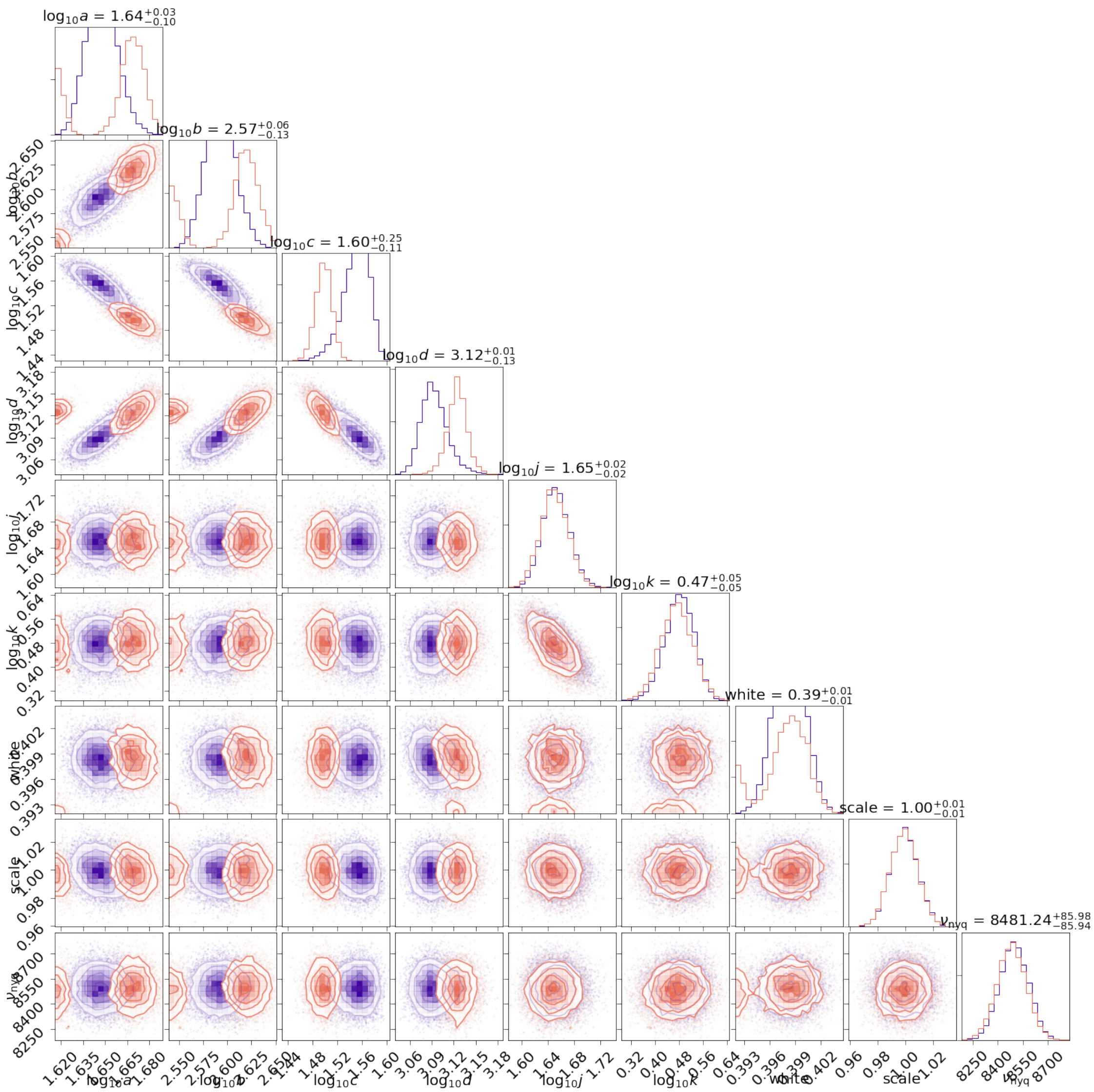




```
In [63]: cmap = sns.color_palette('plasma', n_colors = 10)
```

```
In [64]: labels=['loga','logb','logc','logd','logj','logk',
                 'white','scale','nyq']
         verbose=[r'$\log_{10}a$',r'$\log_{10}b$',
                 r'$\log_{10}c$',r'$\log_{10}d$',
                 r'$\log_{10}j$',r'$\log_{10}k$',
                 'white','scale',r'$\nu_{\rm nyq}$']

         backchain = np.array([backfit[label] for label in labels]).T
         phichain = np.array([trace['phi'][:,idx] for idx in range(len(phi_))]).T
         limit = [(backfit[label].min(), backfit[label].max()) for label in labels]
```

```
fig = corner.corner(backchain, color=cmap[0],range=limit)
corner.corner(phichain, fig=fig, show_titles=True, labels=verbose, color=cmap[6],range
plt.show()
```

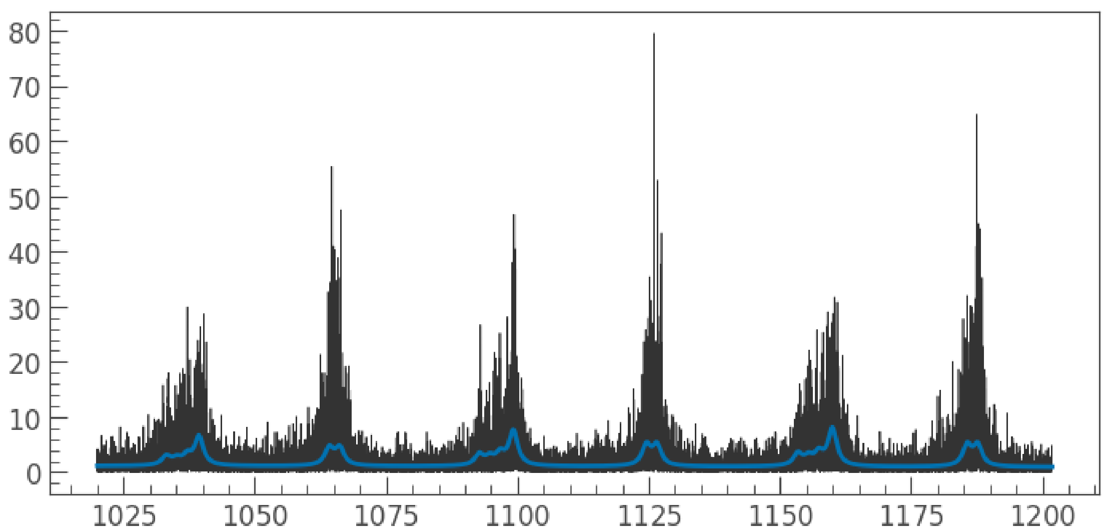


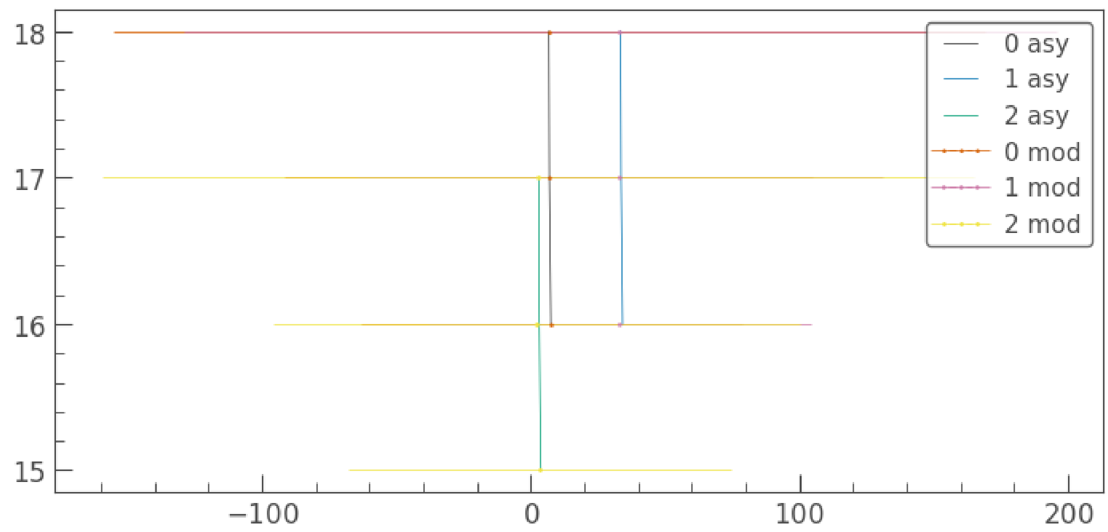
Looks like all the background parameters have been tightened up or remained within the priors. Always good to check!

4.0.1 Now let's plot some output evaluation:

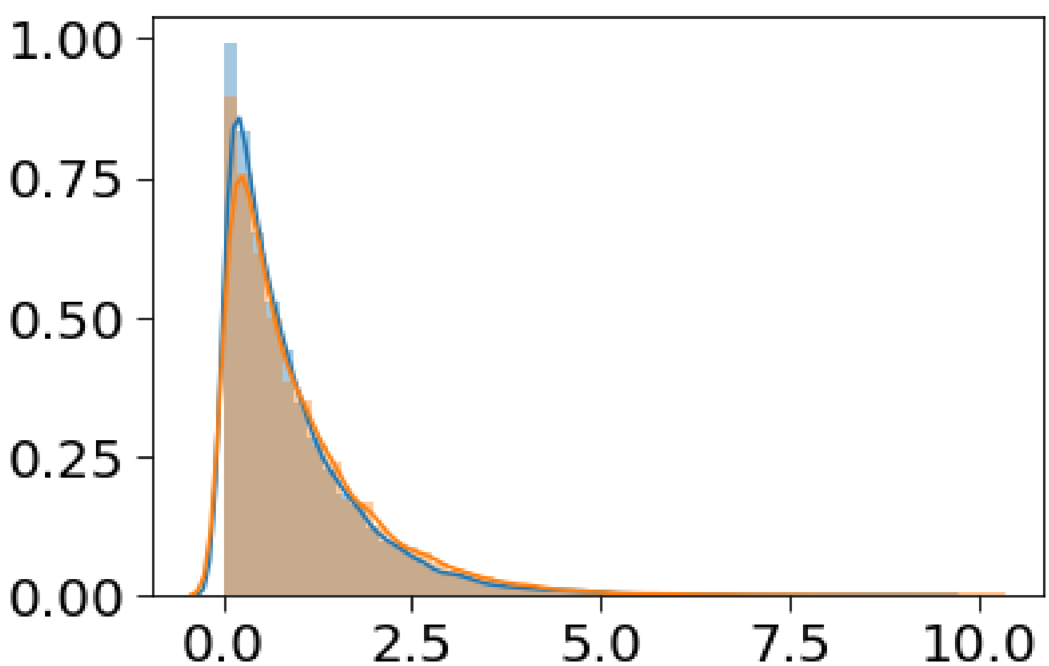
```
In [65]: with plt.style.context(lk.MPLSTYLE):
             fig, ax = plt.subplots()
             ax.plot(f, p)
             labels=['f0','f1','f2','g0','g1','g2','h0','h1','h2','split','i', 'phi']
```

```
res = np.array([np.median(trace[label],axis=0) for label in labels])
ax.plot(f, mod.model(init_m), lw=2)
plt.show()

fig, ax = plt.subplots()

res = [np.median(trace[label]) for label in ['numax','deltanu','alpha','epsilon',
resls = [np.median(trace[label],axis=0) for label in['f0','f1','f2']]
stdls = [np.std(trace[label],axis=0) for label in['f0','f1','f2']]

ax.plot(mod.f0(res)%res[1], n0, label='0 asy')
ax.plot(mod.f1(res)%res[1], n1, label='1 asy')
ax.plot(mod.f2(res)%res[1], n2, label='2 asy')

ax.errorbar(resls[0]%res[1], n0, xerr=stdls[0], fmt='^',label='0 mod')
ax.errorbar(resls[1]%res[1], n1, xerr=stdls[1], fmt='>',label='1 mod')
ax.errorbar(resls[2]%res[1], n2, xerr=stdls[2], fmt='o',label='2 mod')

ax.legend()
```






```
In [70]: labels=['f0','f1','f2','g0','g1','g2','h0','h1','h2','split','i', 'phi']
         res = np.array([np.median(trace[label],axis=0) for label in labels])
         sns.distplot(p/mod.model(res))
         sns.distplot(np.random.chisquare(2, size=10000)/2)

Out[70]: <matplotlib.axes._subplots.AxesSubplot at 0x7f27444c3dd8>
```



Model looks reasonable, The model fits the χ^2_2 noise, we're all good!

5 Leftovers

In [ ]:
```
res = [np.median(trace[label]) for label in ['numax','deltanu','alpha','epsilon','d01'

resls = [np.median(trace[label],axis=0) for label in['f0','f1','f2']]
stdls = [np.std(trace[label],axis=0) for label in['f0','f1','f2']]


with plt.style.context(ms.ms):
    fig, ax = plt.subplots()

    ax.plot(mod.f0(res)%res[1], n0, label='0 asy')
    ax.plot(mod.f1(res)%res[1], n1, label='1 asy')
    ax.plot(mod.f2(res)%res[1], n2, label='2 asy')

    ax.errorbar(resls[0]%res[1], n0, xerr=stdls[0], fmt='^',label='0 mod')
    ax.errorbar(resls[1]%res[1], n1, xerr=stdls[1], fmt='>',label='1 mod')
    ax.errorbar(resls[2]%res[1], n2, xerr=stdls[2], fmt='o',label='2 mod')

    ax.errorbar(f0_%res[1], n0, xerr=f0_e, fmt='^',label='0 lit')
    ax.errorbar(f1_%res[1], n1, xerr=f1_e, fmt='>',label='1 lit')
    ax.errorbar(f2_%res[1], n2, xerr=f2_e, fmt='o',label='2 lit')

    ax.legend()
```