World Congress on Global Optimization
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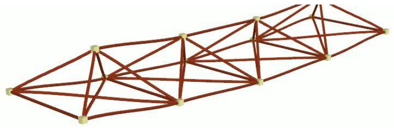

Global Optimization Algorithms for Strucutral Static Analysis

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Presentation Outline

- ✓ Background and Research Objective
- ✓ Arc-Length Method
- ✓ Equilibrium States and Objective Function
- ✓ Global Optimization Framework
- ✓ Case Studies
- ✓ Conclusions and Current Simulations

BACKGROUND AND RESEARCH OBJECTIVE

Spatial truss structures are employed to build some of the most impressive structures worldwide.
Steel Bridges are an example of this structural typology.

Harbour Bridge across the Sydney Harbor, Australia.
500 meters span.

Bayonne Bridge over Kill Van Kull river, New York City (US).
510 meters span

BACKGROUND AND RESEARCH OBJECTIVE

Large-span roof systems including spatial domes represent another important example of 3D truss structures

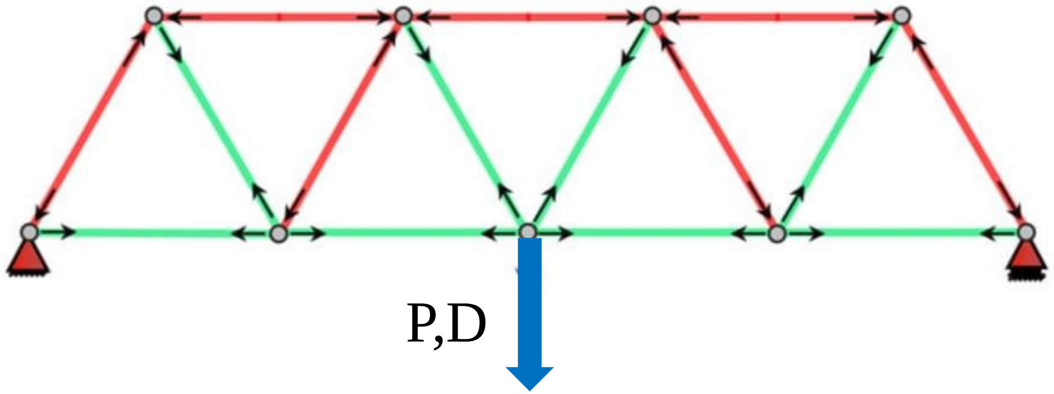
Sport Hub, Singapore

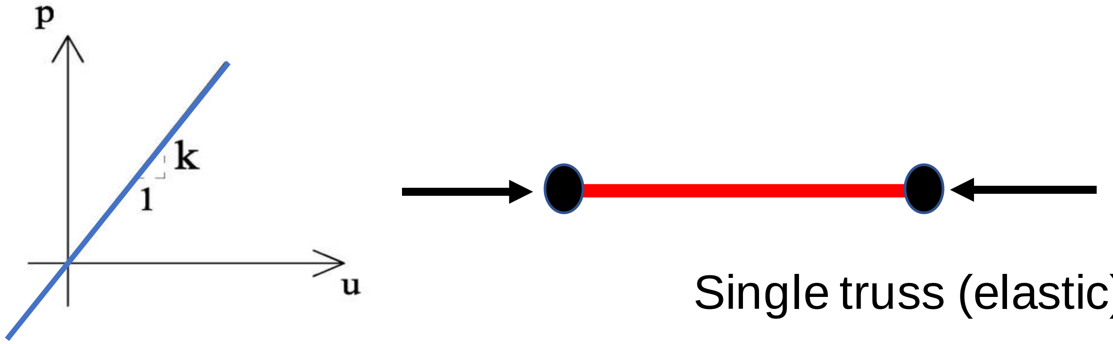
Taegu Stadium, South Korea

BACKGROUND AND RESEARCH OBJECTIVE

- ✓ Each element (truss) is subjected only to axial load: tension or compression
- ✓ These structures shows nonlinear responses due to **geometric nonlinearity**.

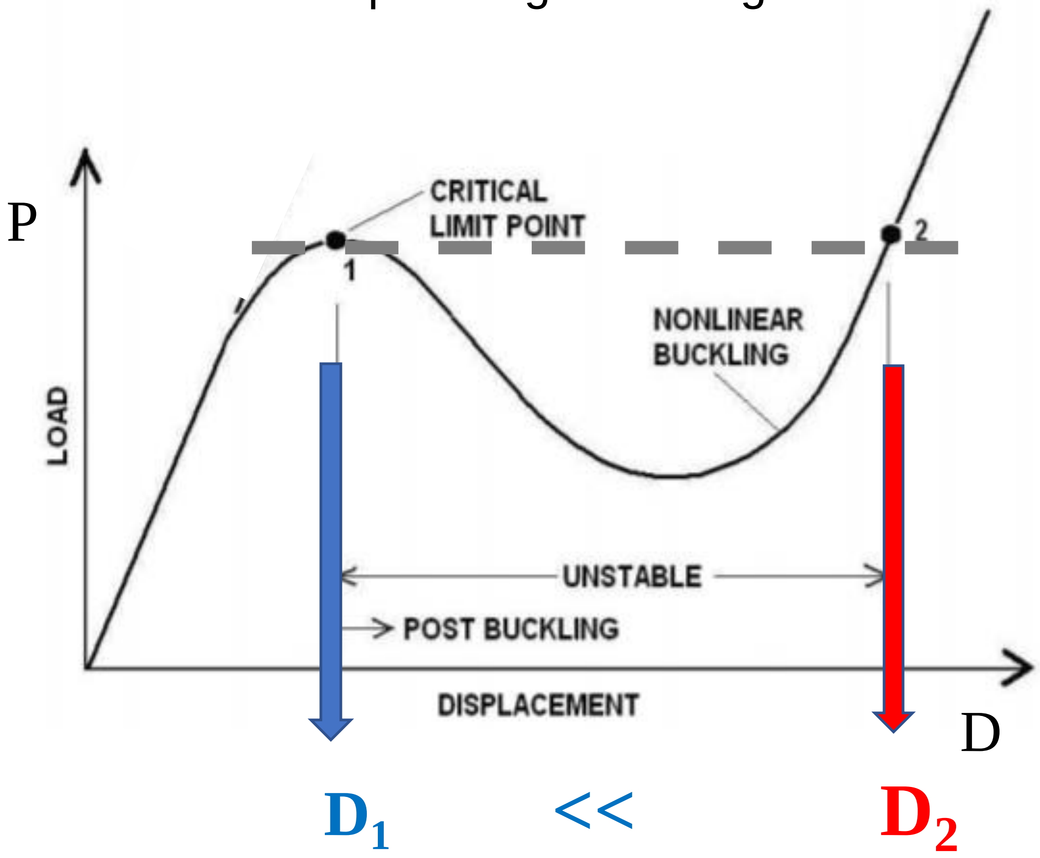
2D simply supported truss-beam





Single truss (elastic)

snap-through buckling



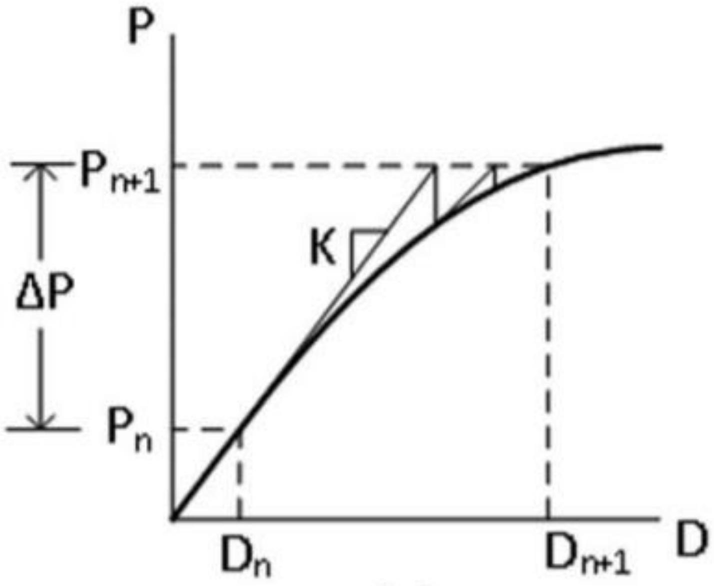
ARC LENGTH METHOD

The system response is governed by the Stiffness Matrix (**K**) by the relation:

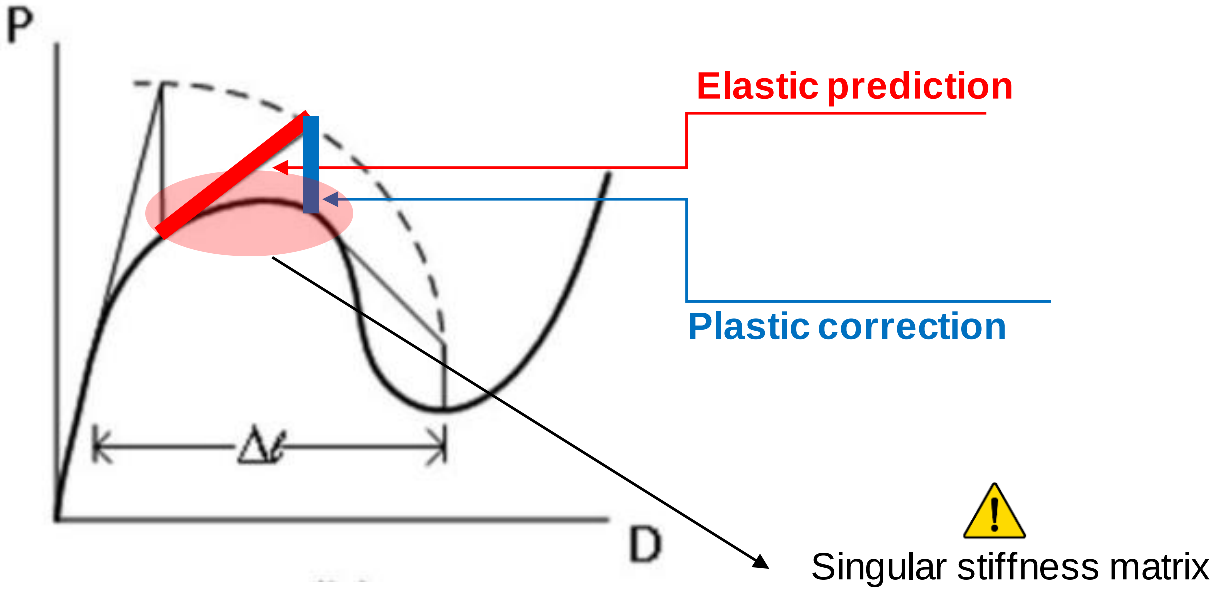
$$\Delta \mathbf{F} = [\mathbf{K_M} + \mathbf{K_G}]\Delta \mathbf{U}$$

F = External forces
U= independent displacements
$\mathbf{K_M}$ = Material stiffness matrix
$\mathbf{K_G}$ = Geometric stiffness matrix

Standard approaches used to predict the structural response are based upon **Arc-length** approach and **Newton-Raphson** iterative algorithms[1,2,3]



Newton-Raphson procedure



Arc-length procedure

References:
Crisfield, M. A. (1981). “A fast incremental/iterative solution procedure that handles ‘snap-through.’” Comput. Struct., 13(1), 55–62.
Riks, E. (1979). “An incremental approach to the solution of snapping and buckling problems.” Int. J. Solids Struct., 15(7), 529–551.
Riks, E. (1972). “The application of Newton’s method to the problem of elastic stability.” J. Appl. Mech., 39(4), 1060–106500`

EQUILIBRIUM STATES

Displacements of nodes represent the Degrees of Freedom (**DoFs**) of the structure

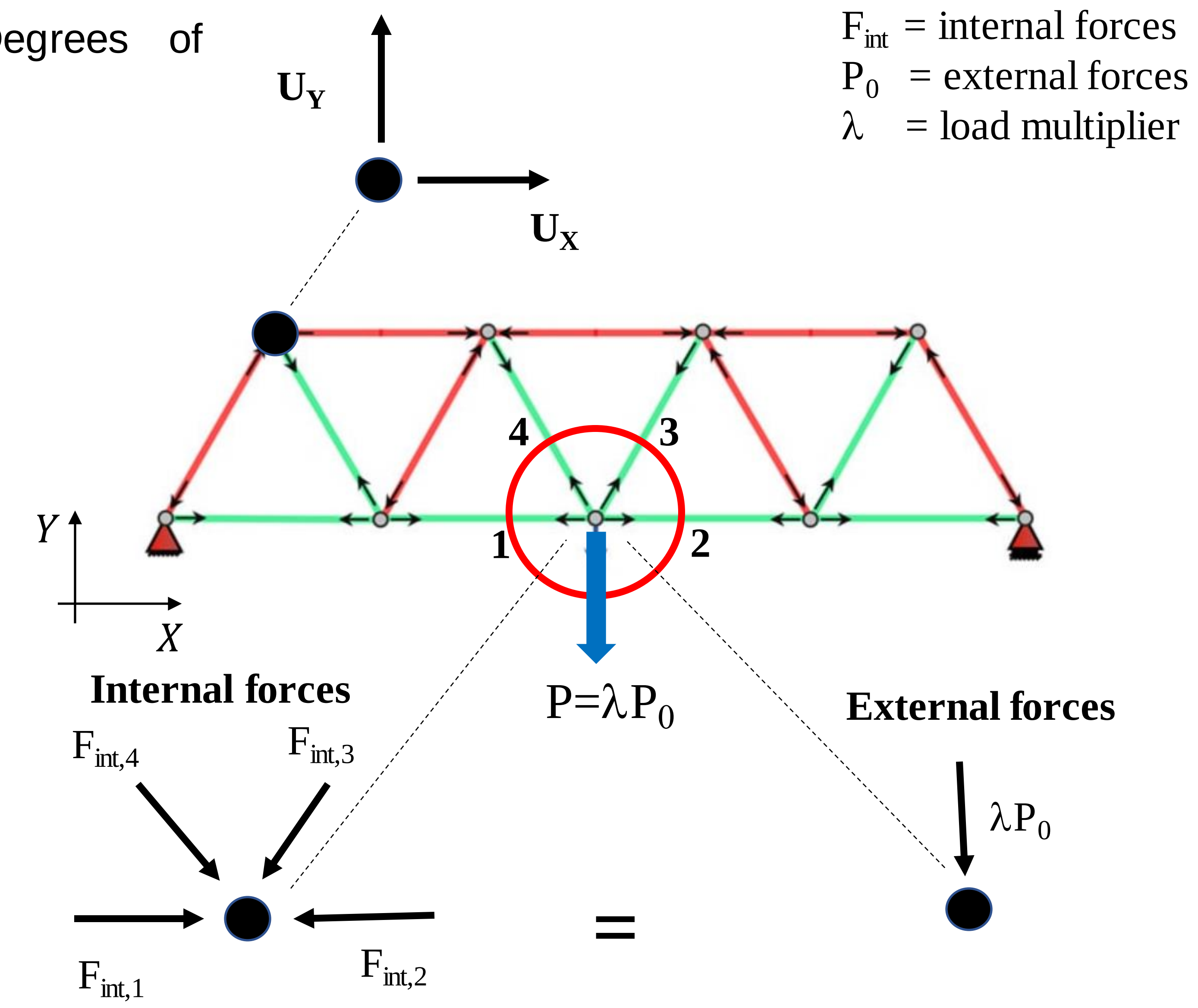
Considering a generic equilibrium state, each node is in equilibrium under **external** and **internal** forces.

The following expression hold:

$$\begin{cases} R_X = F_{int,X} - \lambda P_{0,X} = 0 \\ R_Y = F_{int,Y} - \lambda P_{0,Y} = 0 \end{cases}$$

Where:

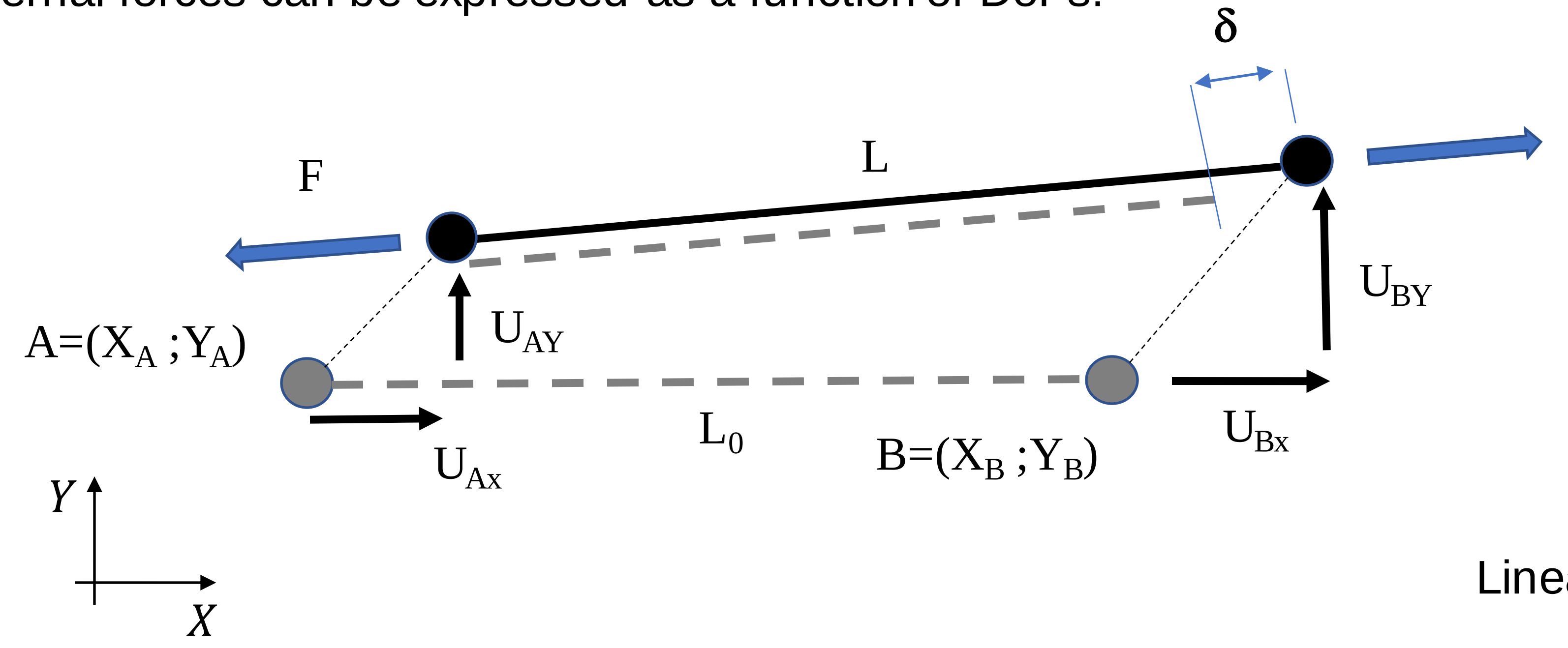
$R_x, R_Y =$ unbalance forces



EQUILIBRIUM STATES

Internal forces can be expressed as a function of DoFs:

δ = truss elongation
F = truss reactive force
L = truss current length
L_0 = truss initial length



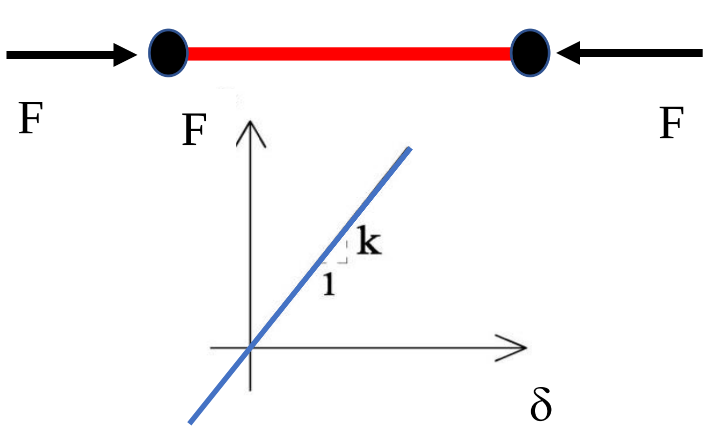
$$L_0 = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}$$

$$L = \sqrt{(X_B + U_{BX} - X_A - U_{AX})^2 + (Y_B + U_{BX} - Y_A - U_{AX})^2}$$

$$\delta = L - L_0$$

$$F(\delta) = k\delta$$ **only for linear-elastic materials !**

Linear elastic mechanical behaviour



THE OBJECTIVE FUNCTION

n = number of nodes
nDoFs = number of degrees of freedom
λ = load multiplier

- **Variables to optimise**
  - $\mathbf{U}\,\{1 \times \mathrm{nDoFs}\} = [U_{\mathrm{X}1}, U_{\mathrm{Y}1}, \ldots, U_{\mathrm{Xn}}, U_{Yn}]$
  - load multiplier factor (λ)

- **Objective function (Equilibrium)**

$$f(\mathbf{U}, \lambda) = \sqrt{\sum_{i=1}^{n}\left[F_{int,x,i} - \lambda P_{0x,i}\right]^2 + \sum_{i=1}^{n}\left[F_{int,Y,i} - \lambda P_{0Y,i}\right]^2}$$

Where:

$\mathbf{F_{int}}(\mathbf{i}) = [F_{int,X,i} \quad F_{int,Y,i}]$ = resultant of the internal forces applied to the **i**-th node

$\mathbf{P_{0,i}} = [P_{0X,i} \quad P_{0Y,i}]$ = external force applied to the **i**-th node

Global Optimization Framework

Initialise a structure (i.e., define forces, nodes, elements, etc.)

Set variables boundary (i.e., x-Min and x-Max vectors)

Select an optimizer and set their hyper-parameters

(e.g., Ant Colony Optimization (ACO), Differential Evolution (DE), Particle Swarm Optimization (PSO), DIRECT, MCS, etc.)

Initialise solution vector(s)

Repeat until max number of function evaluations (or desired accuracy) reached

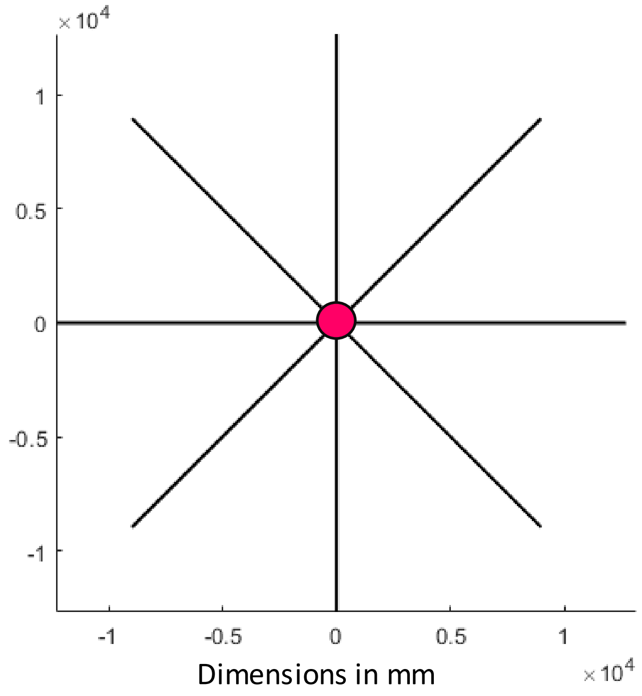
Update solution vector(s)

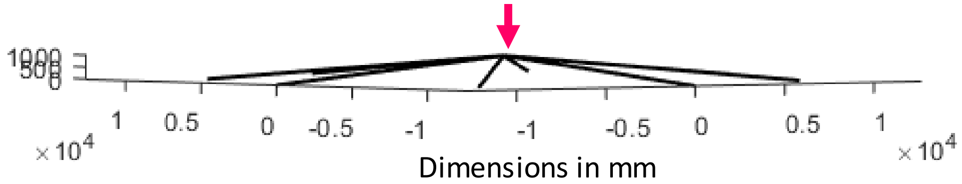
Evaluate (Equilibrium) updated solution vector(s)

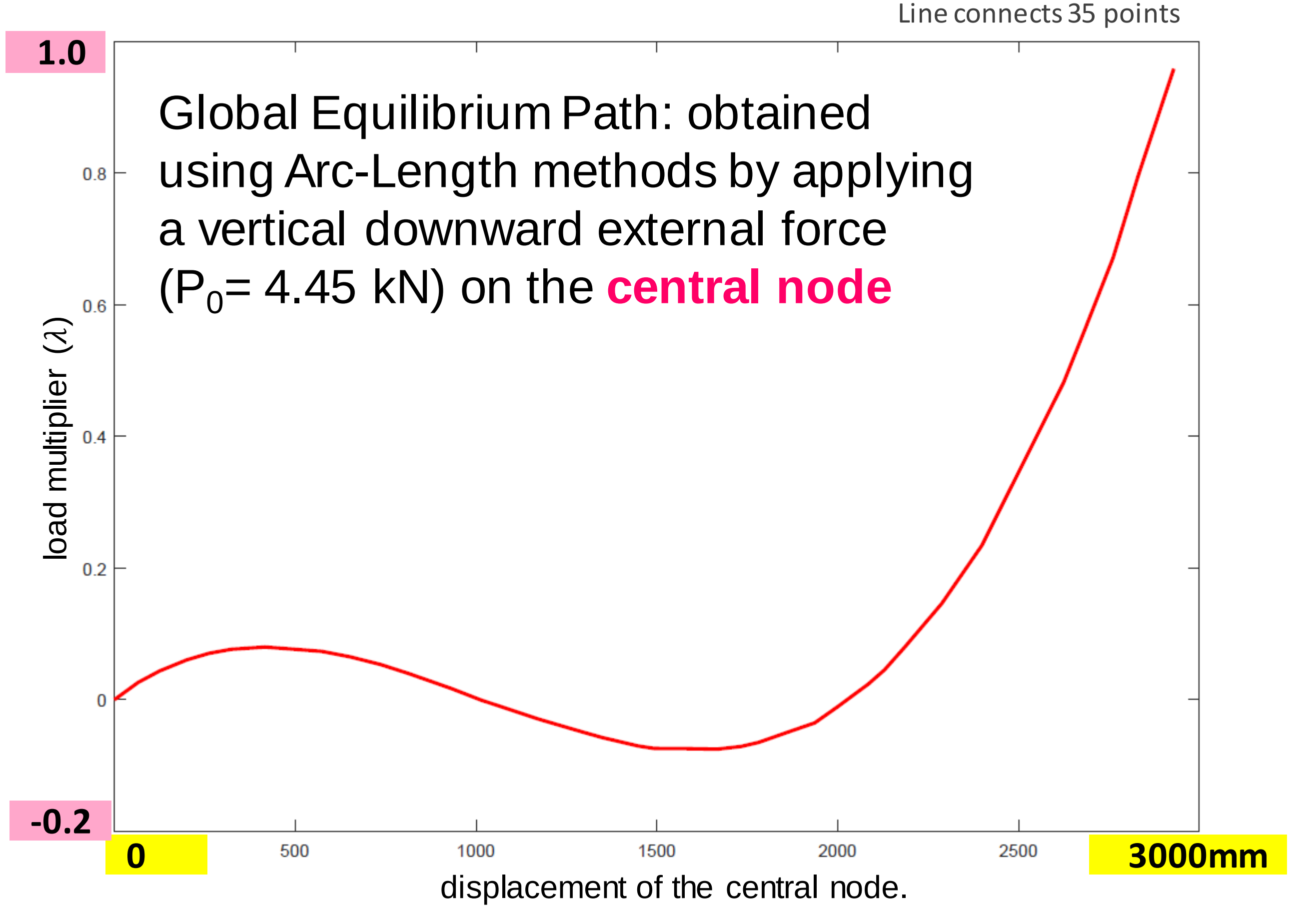
End

Reference:
Socha, K., & Dorigo, M. (2008). Ant colony optimization for continuous domains. *European journal of operational research*, *185*(3), 1155-1173.
Storn, R., & Price, K. (1997). Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of global optimization*, *11*(4), 341-359.
Kennedy, J., & Eberhart, R. (1995, November). Particle swarm optimization. In *Proceedings of ICNN'95-international conference on neural networks* (Vol. 4, pp. 1942-1948). IEEE.

Case 1:
Eight Member Symmetric Shallow Truss



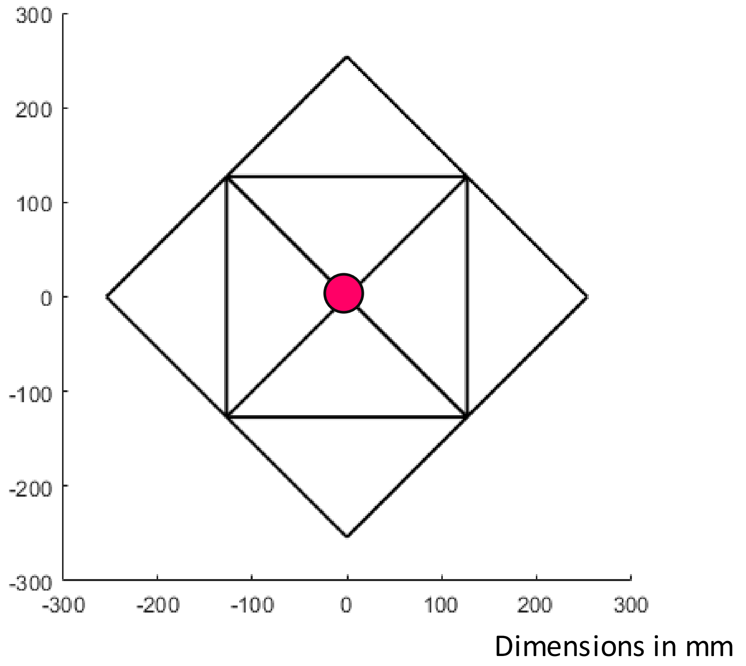


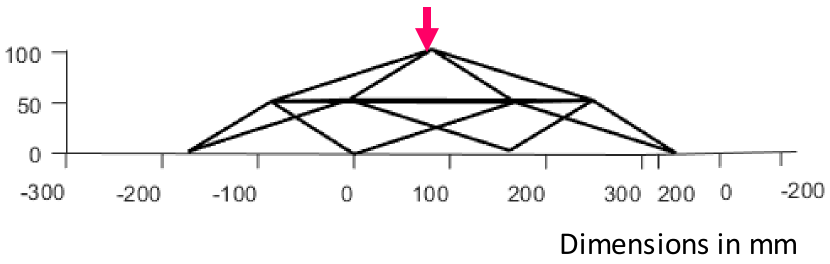


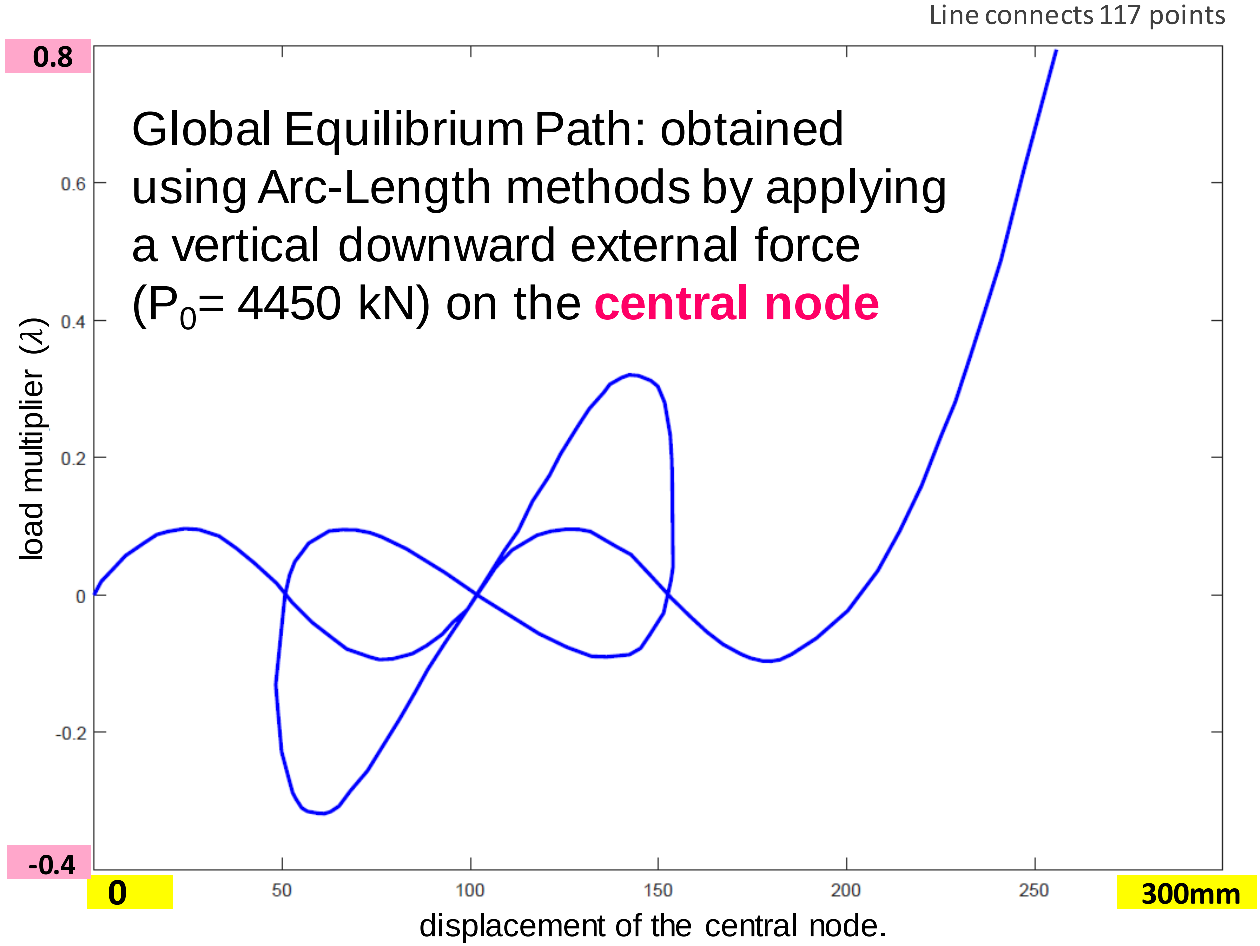
References:

Hrinda, G. (2010, April). Snap-through instability patterns in truss structures. In 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 18th AIAA/ASME/AHS Adaptive Structures Conference 12th (p. 2611).

Case 2: Sixteen Member Symmetric Shallow Truss



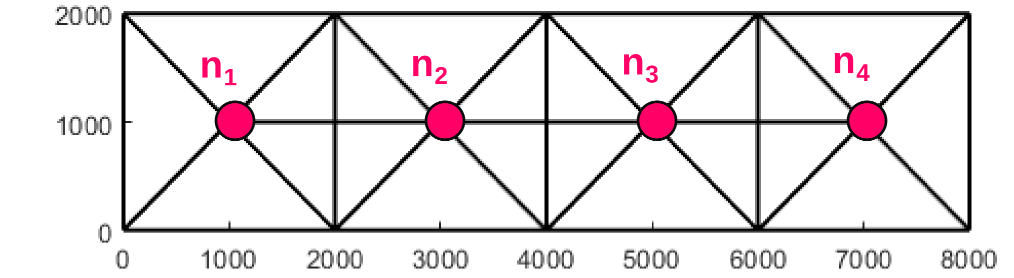




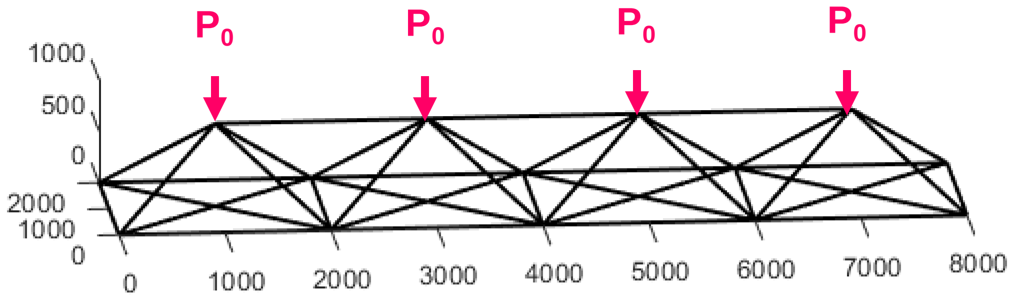
References:

Hrinda, G. (2010, April). Snap-through instability patterns in truss structures. In 51st AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 18th AIAA/ASME/AHS Adaptive Structures Conference 12th (p. 2611).

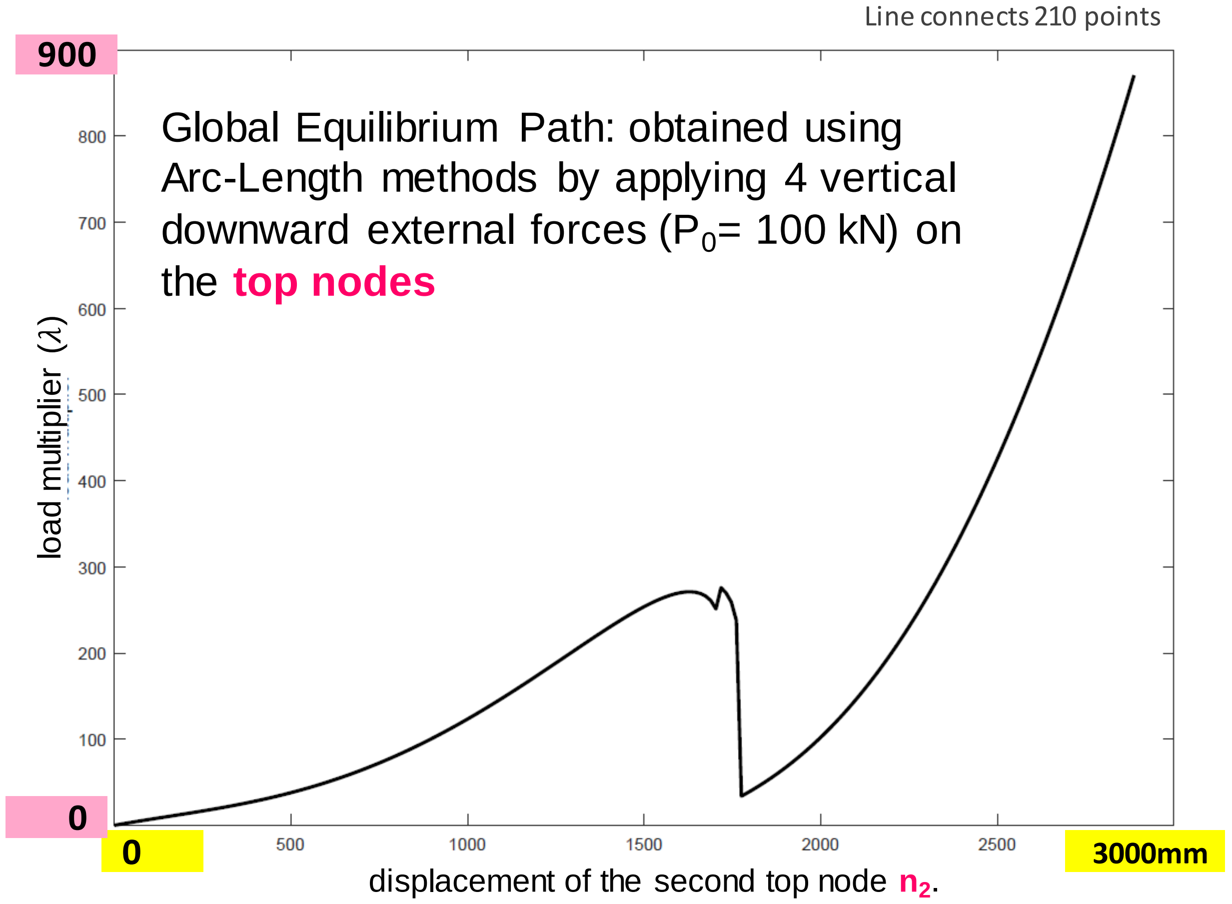
Case 3: Truss Bridge



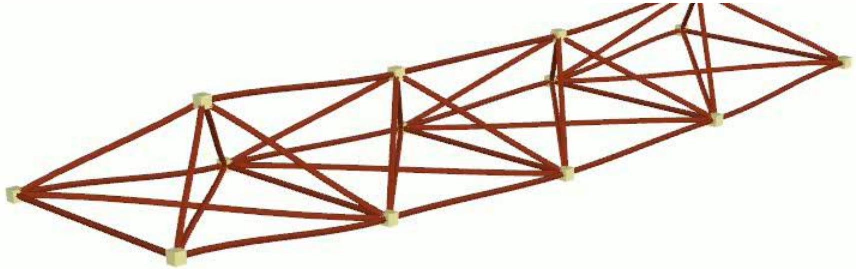
Dimensions in mm



Dimensions in mm

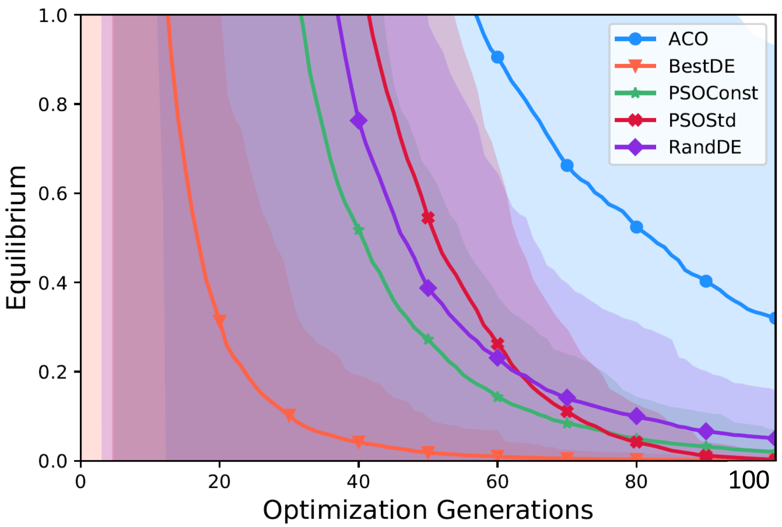


Deformed Configurations of Bridge Truss due to increasing forces

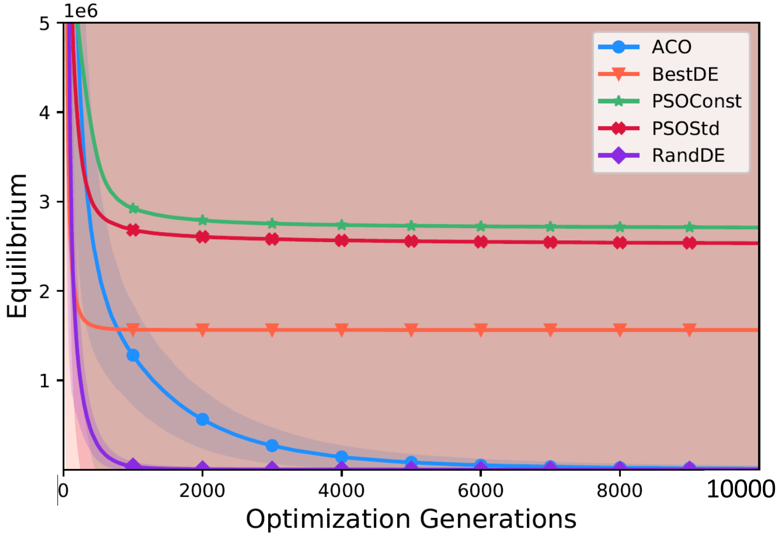


Mean Convergence of Three Cases

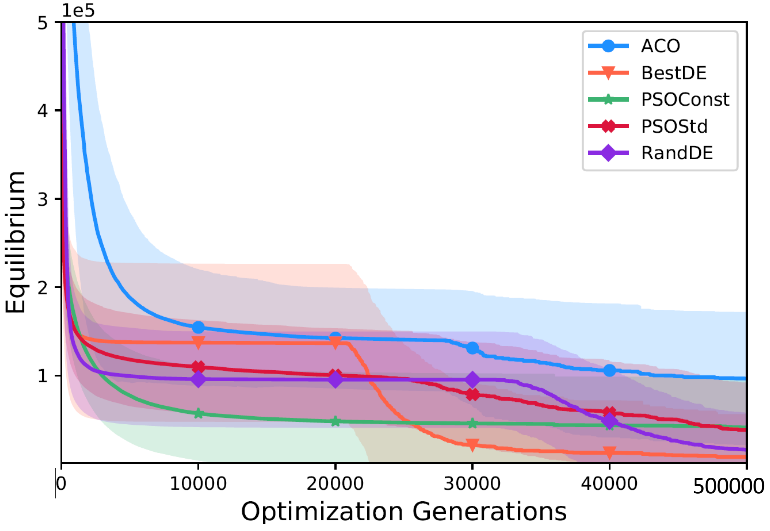
Eight Members



Sixteen Members



Bridge Truss



Eight Members Experiment.
Displacement Domain [0, 3000]
Load multiplier Domain [0, 1]

Variables: 4
Iterations: 10K
Trials: 1K
Optimal Solutions: 1K by each algorithm

Sixteen Members Experiment.
Displacement Domain [0, 250]
Load multiplier Domain [0, 1]

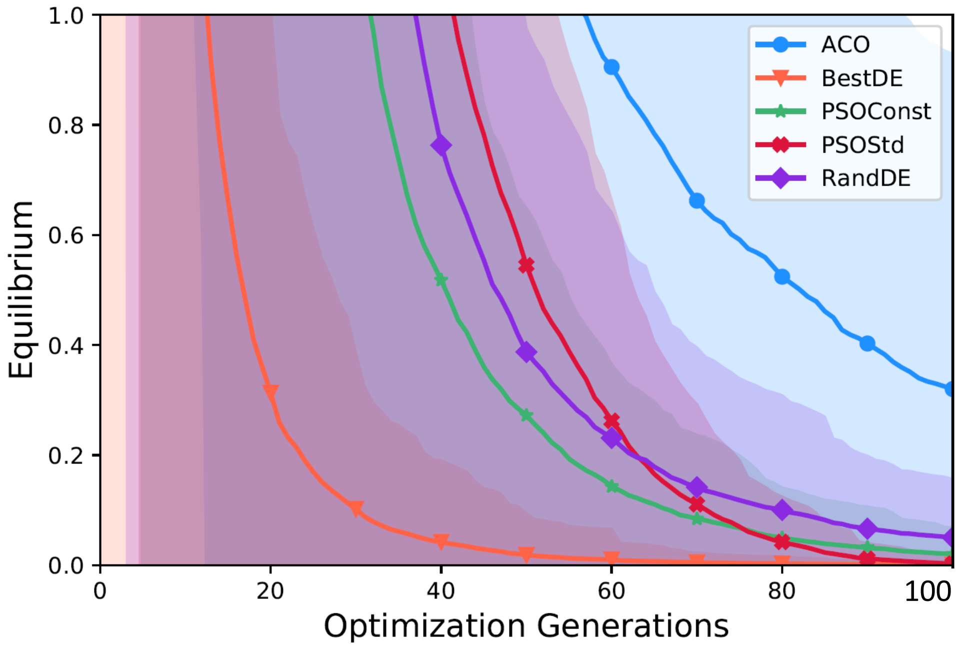
Variables: 16
Iterations: 100K
Trials: 500
Optimal solutions: different for each algorithm

Bridge Experiment.
Displacement Domain [0, 3000]
Load multiplier Domain [0, 900]

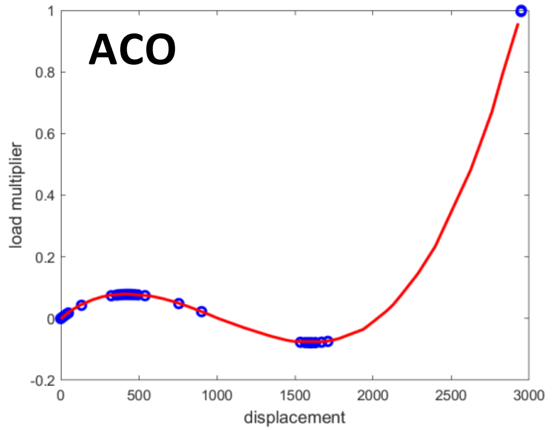
Variables: 31
Iterations: 500K
Trails: 500
Optimal solutions: different for each algorithm

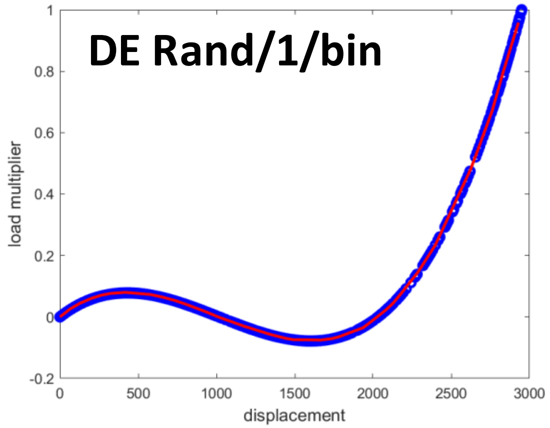
Solutions: Eight Members (Case 1) (Optimal Solutions)

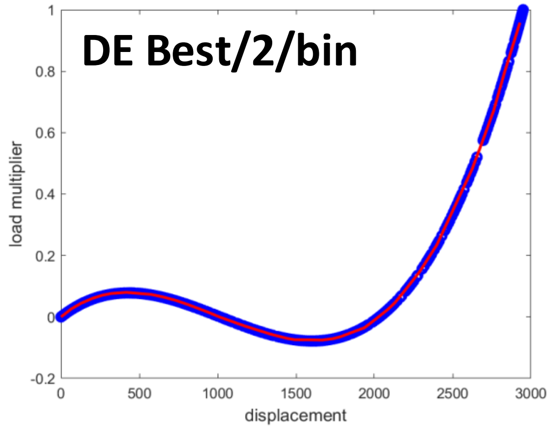
1K Solutions - all converged to global optimum

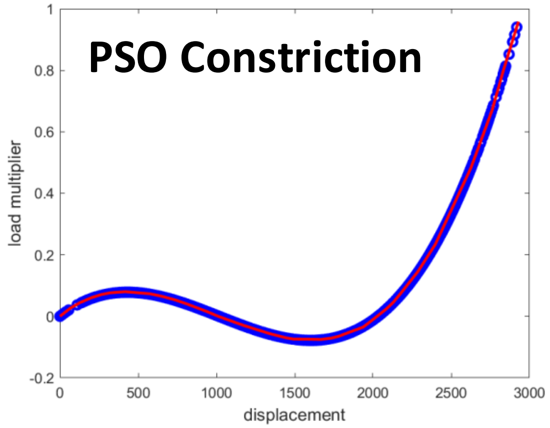


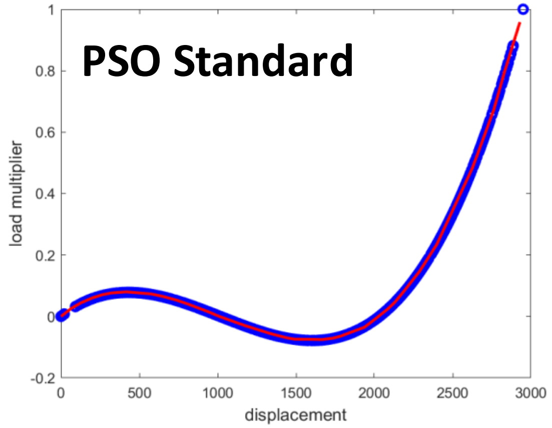
Eight Members



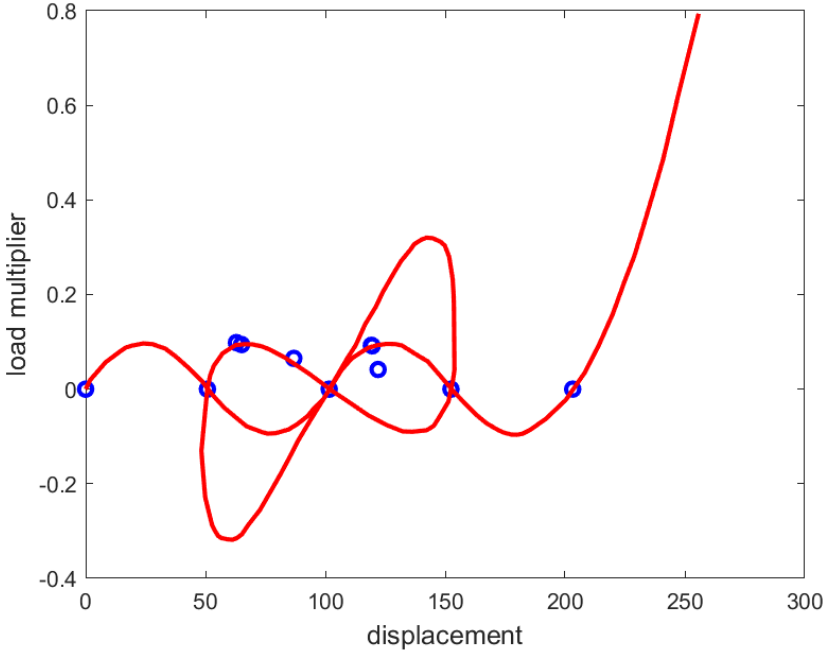






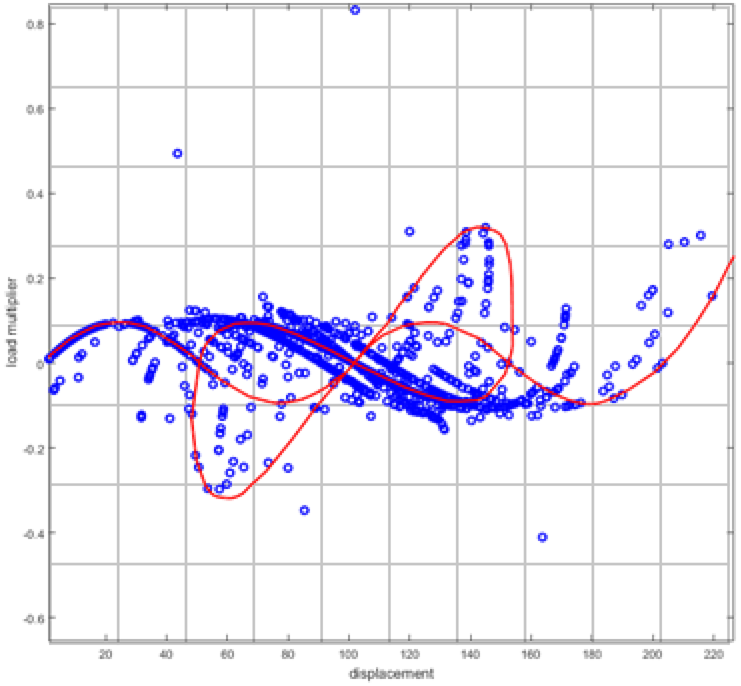


Solutions : Sixteen Members (Case 2) Domain Analysis



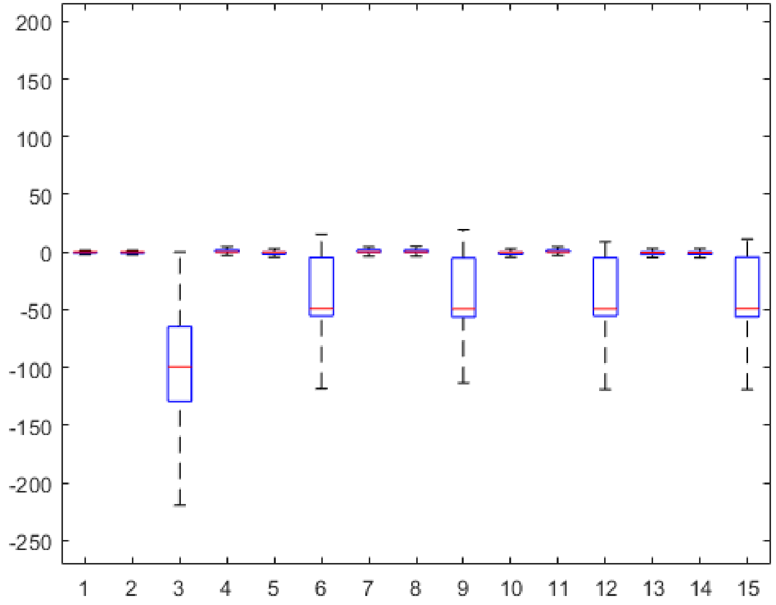
Full Domain Trial (DE Rand/1/Bin)

Unable to solve full curve but finds a few optimal solutions



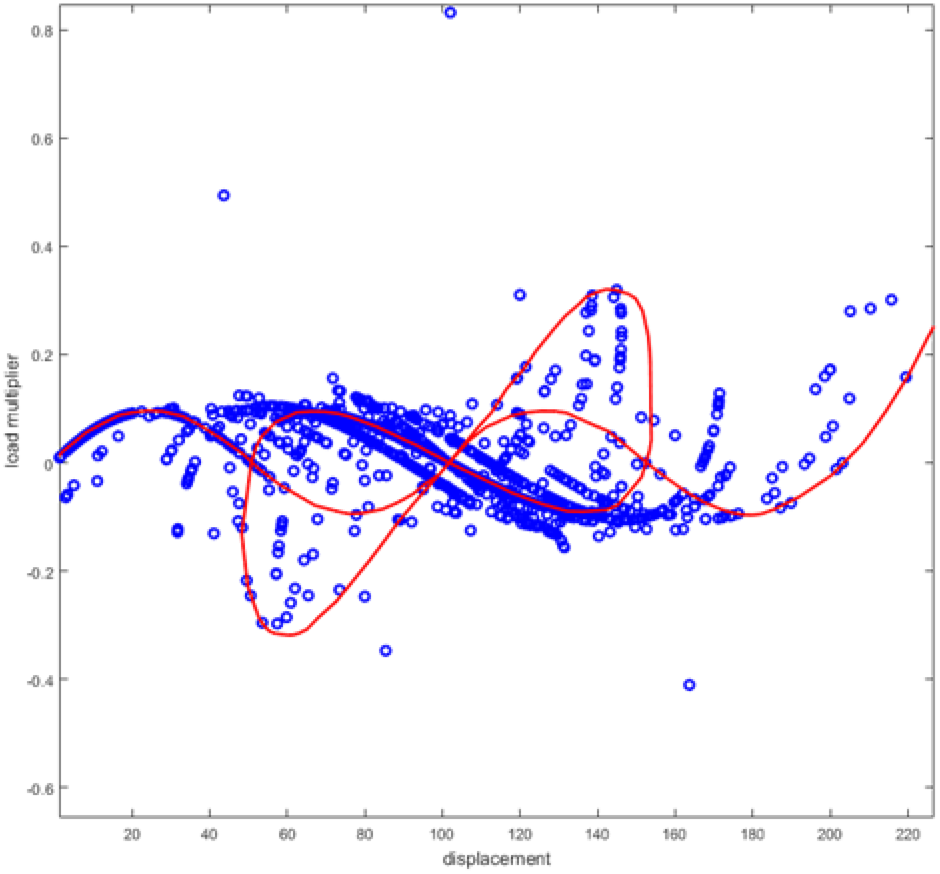
Discrete Domain Trial (DE Rand/1/Bin)

Finds large number of optimal solutions on all ground truth curve

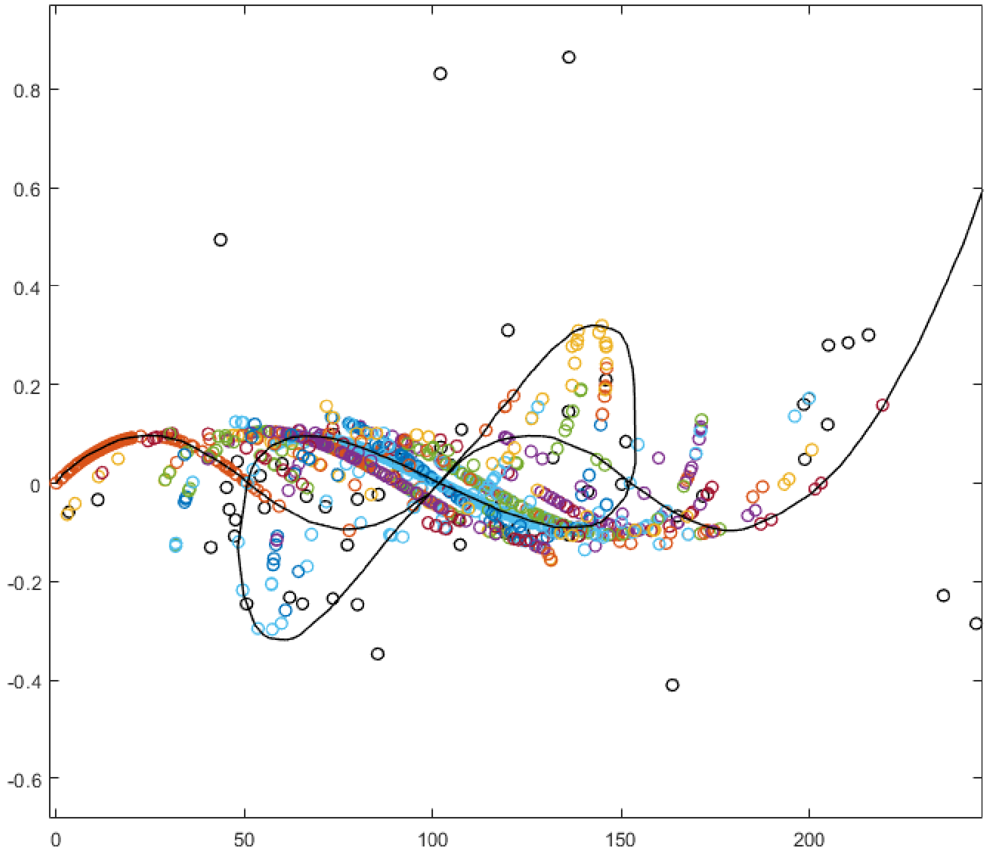


Variables (DoF) Domain Information

Solutions : Sixteen Members (Case 2)
Post Processing



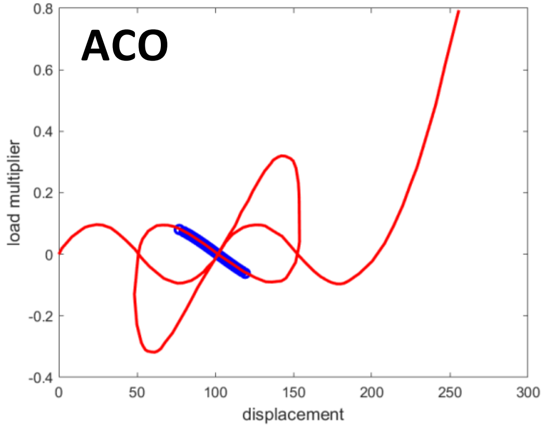
Sixteen Members

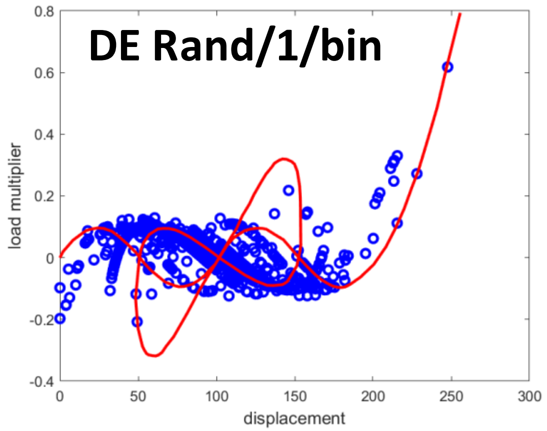


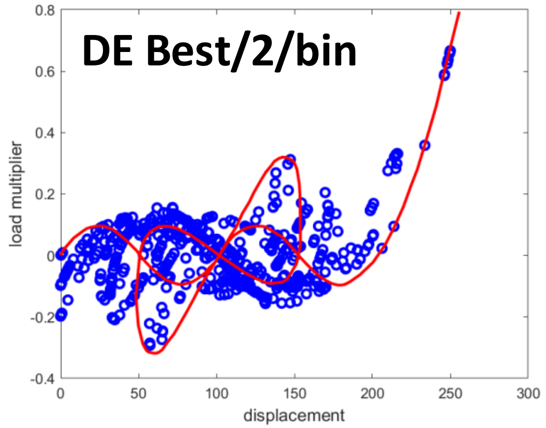
Sixteen Members
(Post Processing for Specificity)

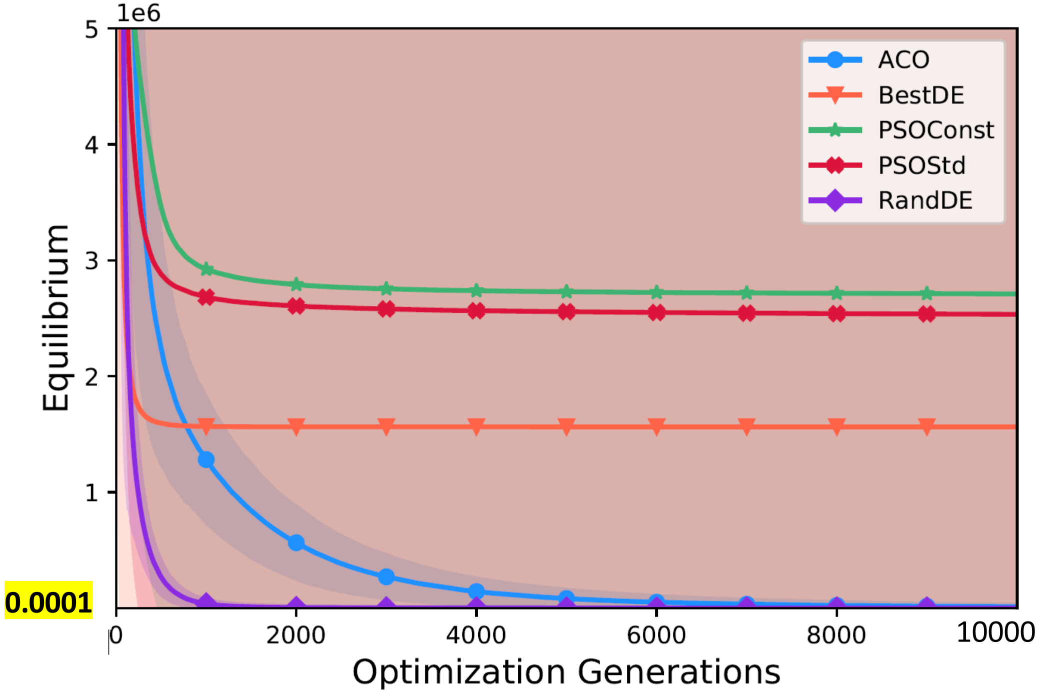
Solutions : Sixteen Members (Case 2)

1K Solutions - all DE Rand/1/bin solutions converged to global optimum, but other algorithms have a few solution values <= 10^2

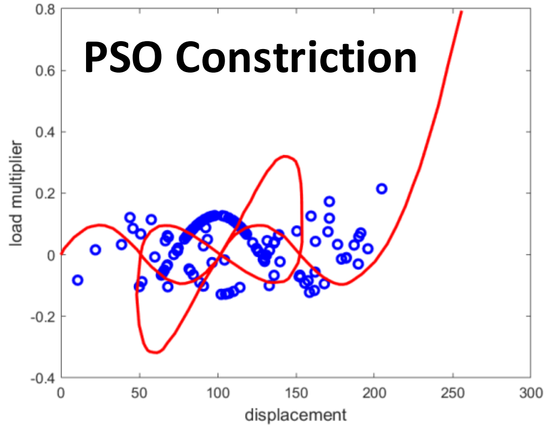


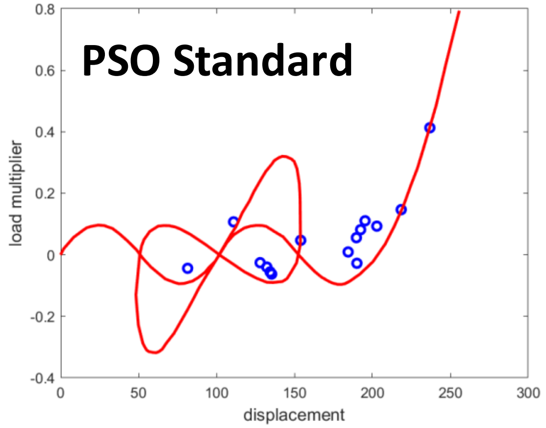






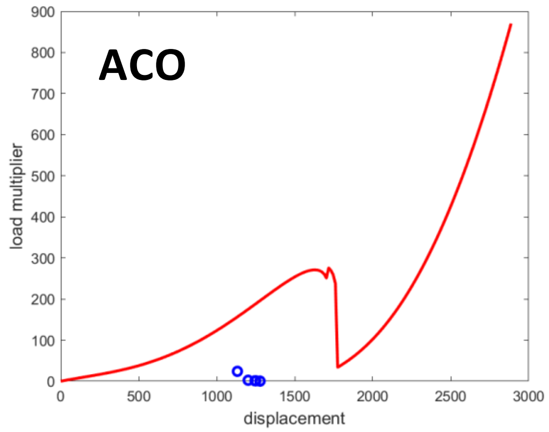
Sixteen Members

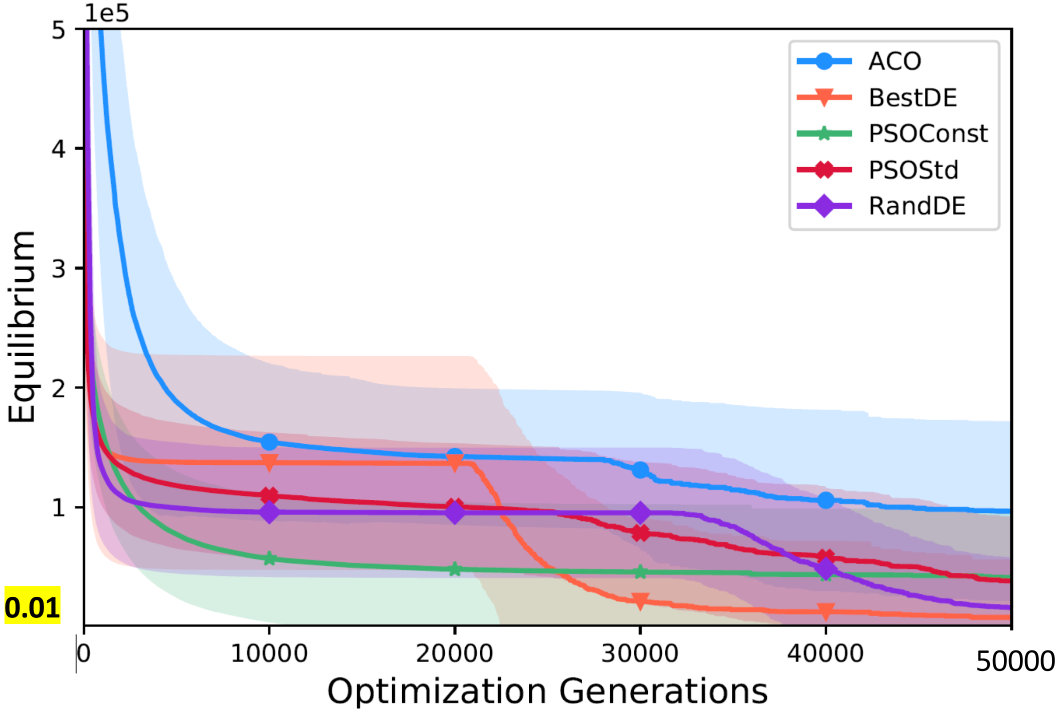


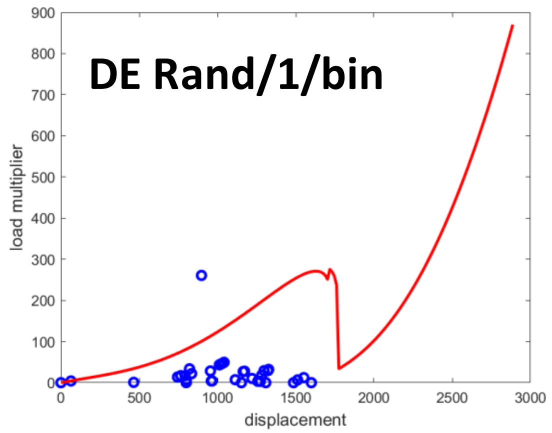


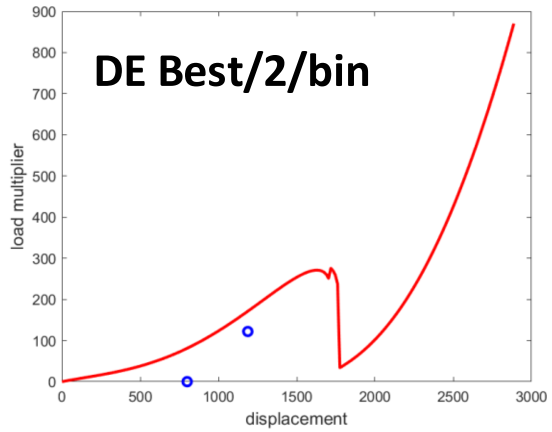
We are running simulations to solve all the path

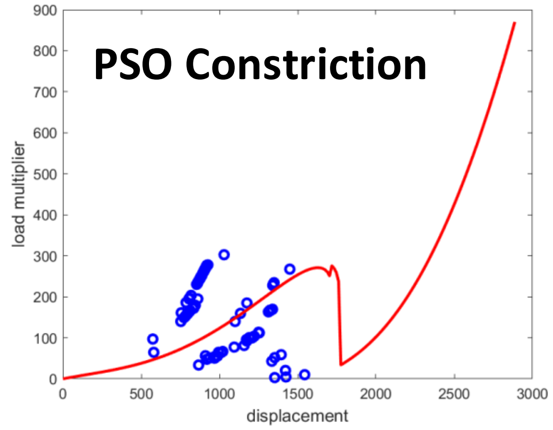
Solutions: Bridge (Case 3)

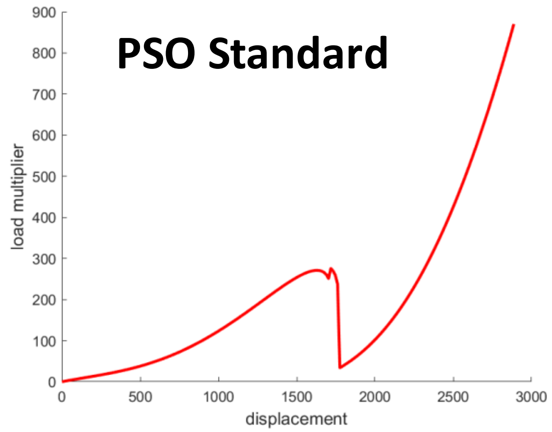








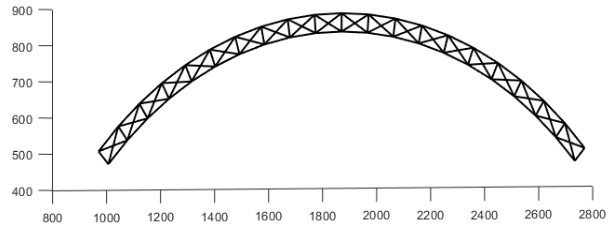


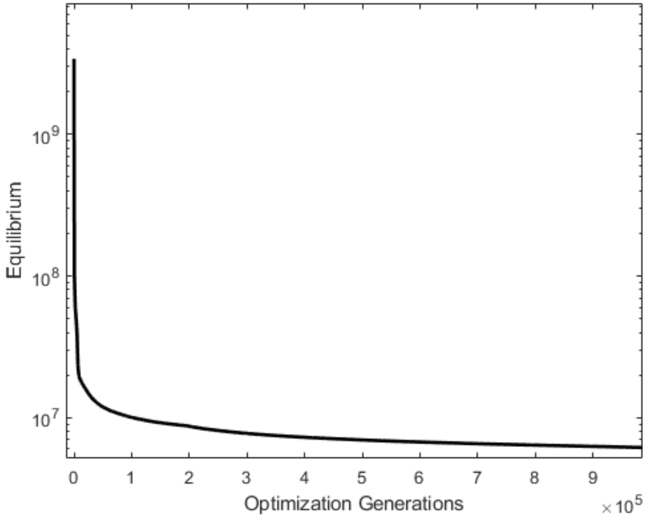


We are running simulations to solve all the path

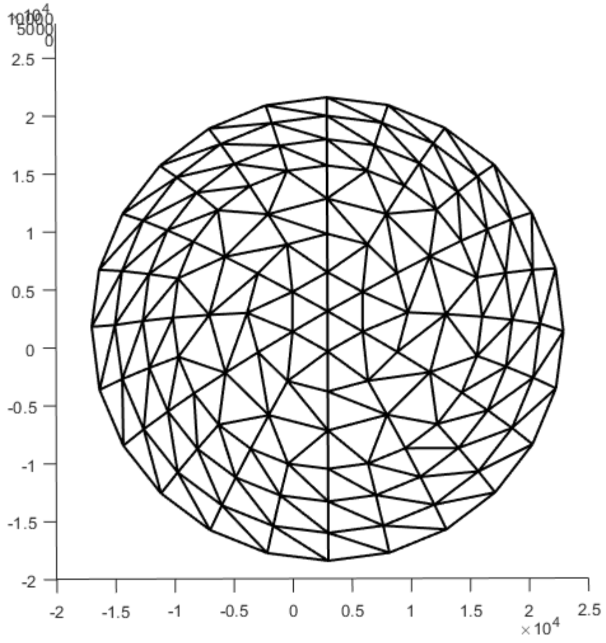
Ongoing tasks
2D Arch and 3D Dome

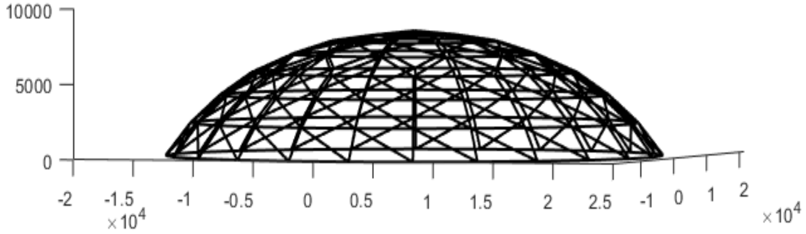
Trial solutions of DE Best/2/bin

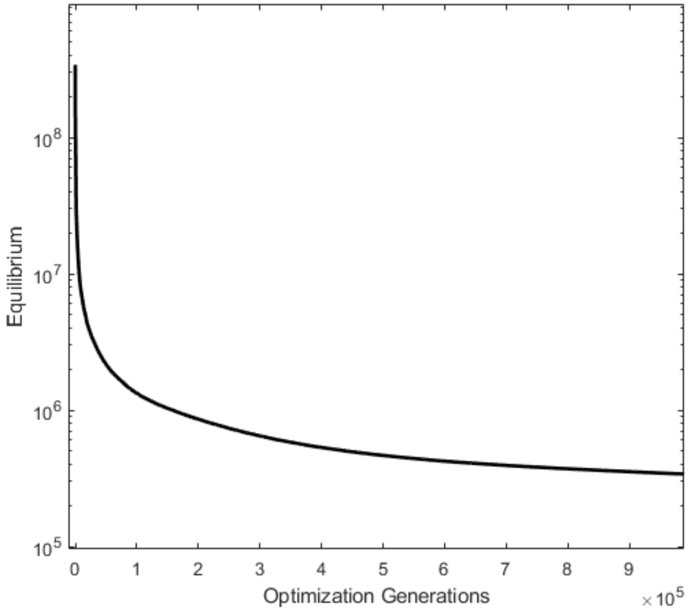




Convergence of 10^10 -> 10^6
In 1 million iterations (50 million function evolutions)







Convergence 10^9 -> 10^5
In 1 million iterations (50 million function evolutions)

Conclusions and Current Simulations

- We formulated spatial truss structures stability analysis problem using global optimization algorithms.
- We applied our method to three benchmarks and two real-world problems considering elastic materials and geometric non-linearities.
- We observe that the convergence slows heavily with increasing number of DoFs.
- We will include mechanical non-linearities and random uncertainties in our future problems.
- We are running simulation to fill as many point as possible in the equilibrium path of the Sixteen Members and Bridge Problem.