ACM40290: NUMERICAL ALGORITHMS

Assignment 1: Calculating Constants

Archimedes's Constant π: The ratio of circumference to diameter of a circle is a constant and is denoted by π. The calculation of the transcendental number π has a history that stretches back over 2000 years and is still a computational task of interest today.

Euler's Number e: The exponential function e^x has the property $\frac{de^x}{dx} = e^x$. Using this property we can derive the power series expansion

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

from which we get

$$e^1 = 1 + \frac{1}{1!} + \frac{1^2}{2!} + \frac{1^3}{3!} + \cdots$$

Alternatively we can apply the Binomial theorem to the definition

$$e = \lim_{n\to\infty} (1 + \frac{1}{n})^n.$$

Euler's Constant γ: The Harmonic series is defined as

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

and it diverges to $+\infty$ as $n \to \infty$. Euler assumed the sequence $H_n - \ln n$ converged and defined the limit

$$\gamma = \lim_{n\to\infty} (1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln n) = 0.5772156649015328606$$

The constant γ is something of a mystery because it is not known whether it is rational, irrational or transcendental.

EXERCISES

(1) Write and test four small MATLAB functions to:
   (a) Calculate π using $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$.
   (b) Calculate π using $\frac{\pi}{2} = \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \cdots}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 9 \cdots}$.
   (c) Calculate Euler's number using $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$.
   (d) Calculate Euler's constant using $\gamma = (1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln n)$.

   The MATLAB functions should be written with the headers `function value = Pi1(n)`, `function value = Pi2(n)`, `function value = EulerN(n)`, and `function value = EulerC(n)`, where n is the number of terms in the series.

(2) (a) Test each function for $n = 10^1, 10^3, 10^6, 10^9$.
   (b) Compare your results with MATLAB's pi and exp(1), and with the values

$$\pi = 3.14159265358979323846264338327950$$
$$e = 2.71828182845904523536028747135270$$
$$\gamma = 0.57721566490153286060651209008240$$

   which are exact to 32 digits.

(3) (a) Are the two methods for computing π equal?
   (b) Is one method better than the other, and if so, why?
   (c) How accurate can π be computed if you push n to a very large number? Can you explain why a limit is reached in this machine-precision calculation?

All answers should be submitted via Blackboard and should include MATLAB code implementing the functions in question (1) along with a brief writeup with answers to questions (2) and (3).

Due: 5pm Friday, September 22nd 2017