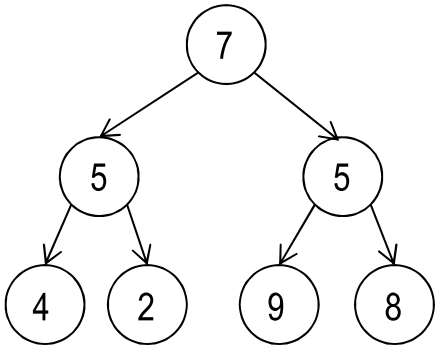
힙과 우선순위큐

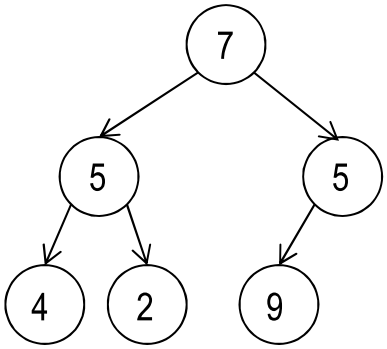
이진 힙(binary heap)

https://en.wikipedia.org/wiki/Binary_heap, CC-BY-SA

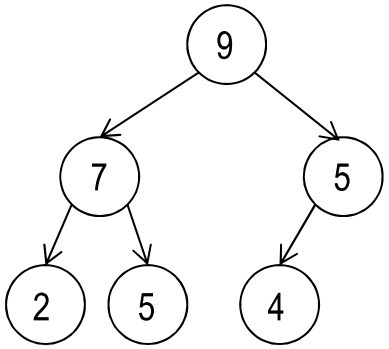
- 이진힙은 우선순위큐(priority queue)를 구현하는 자료구조이다.
- 최대이진힙(binary max heap)은 완전이진트리(complete binary tree)이면서,
  각 노드의 키 값이 자식 노드들의 키 값보다 크거나 같은 조건을 만족한다.



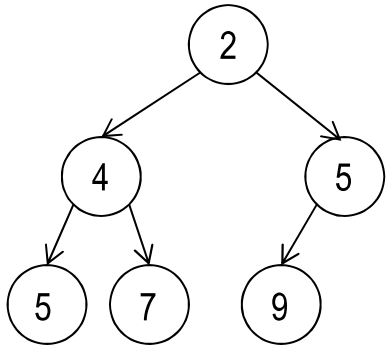
포화이진트리(perfect binary tree)



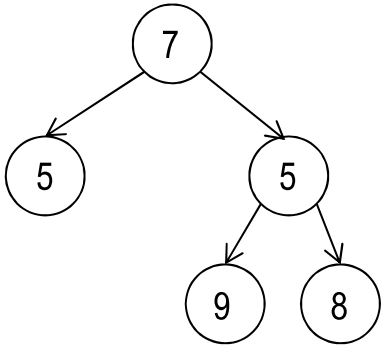
완전이진트리(complete binary tree)



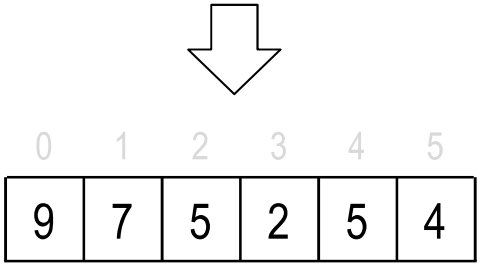
최대이진힙



최소이진힙



정이진트리(full binary tree):
모든 각 노드의 자식 노드 수 0개 혹은 2개

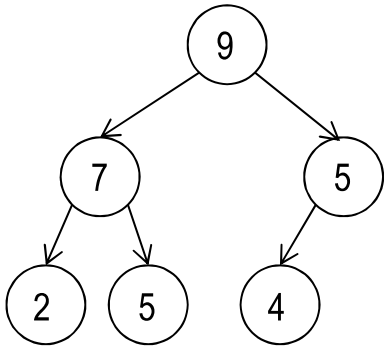


최대이진힙은 완전이진트리이므로 배열 내 효율적 저장 가능

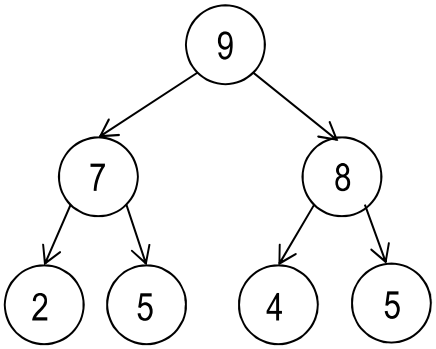
- 배열 i번째 위치 노드의 왼쪽자식노드 위치 → (2 * i) + 1
- 배열 i번째 위치 노드의 오른쪽자식노드 위치 → (2 * i) + 2
- 배열 i번째 위치 노드의 부모노드 위치 → (i - 1) / 2

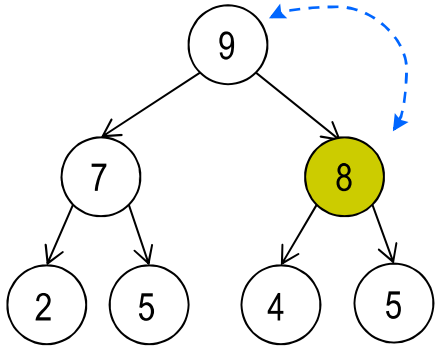
이진 힙(binary heap): 삽입

https://en.wikipedia.org/wiki/Binary_heap, CC-BY-SA

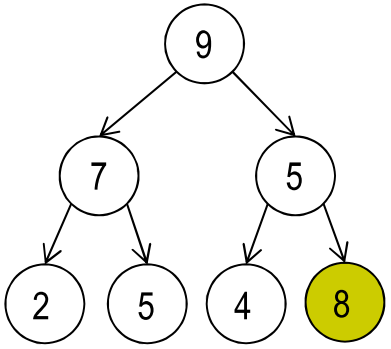


최대힙에 키 값 8 삽입

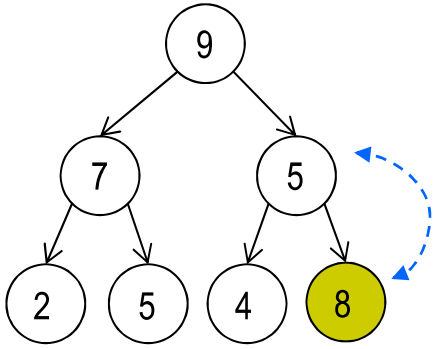




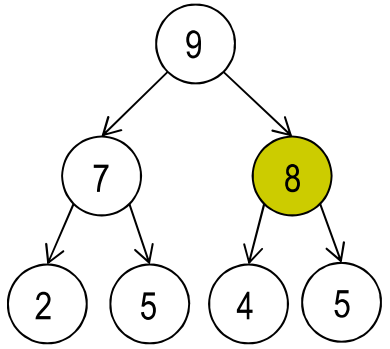
부모노드와 키 값 비교하여
최대힙조건 불만족 시 교환



삽입할 노드를 마지막 위치에 추가



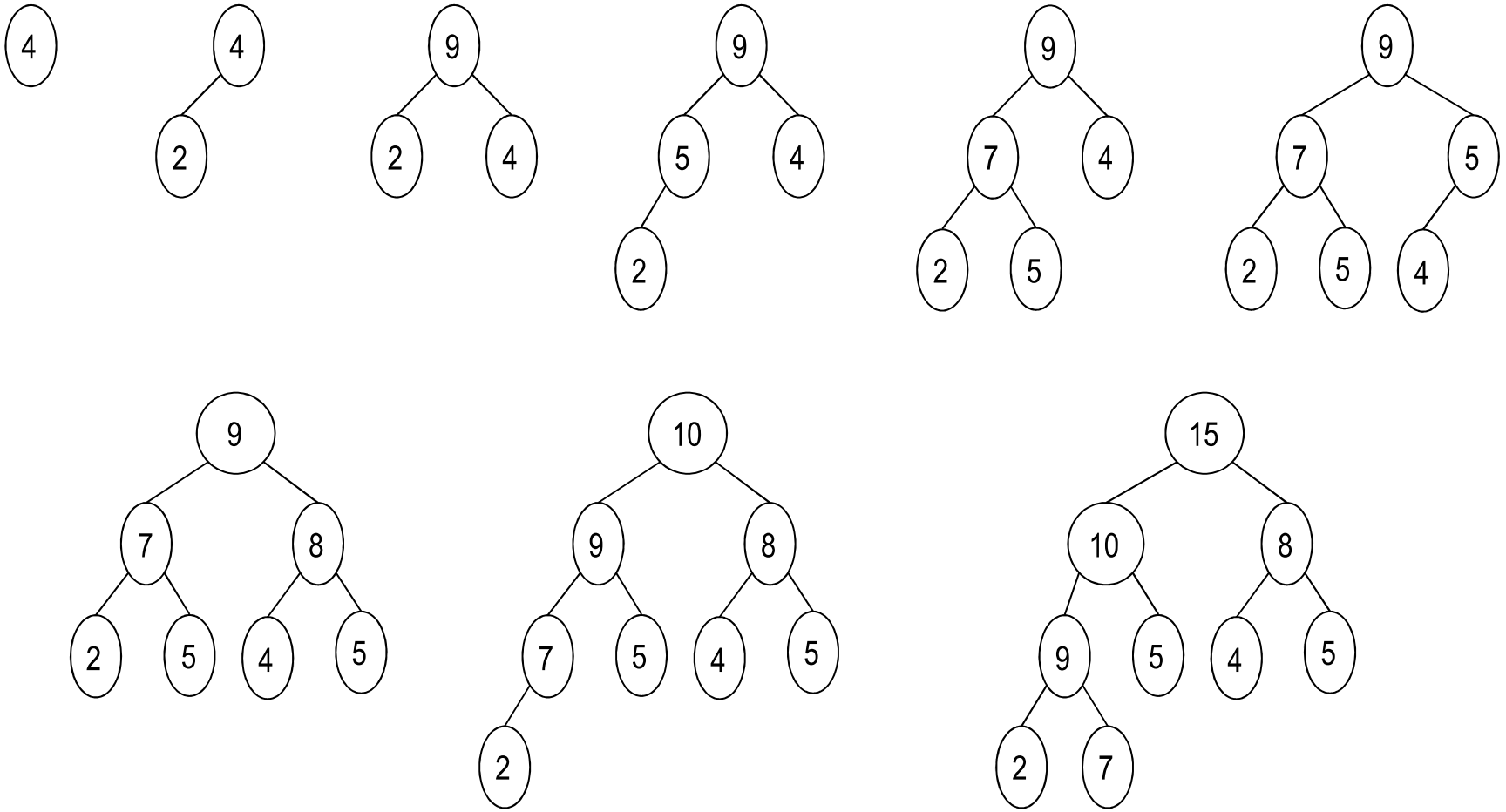
부모노드와 키 값 비교하여
최대힙조건 불만족 시 교환



이진 힙(binary heap): 삽입

https://en.wikipedia.org/wiki/Binary_heap, CC-BY-SA

4, 2, 9, 5, 7, 5, 8, 10, 15 순으로 최대이진힙에 자료 삽입



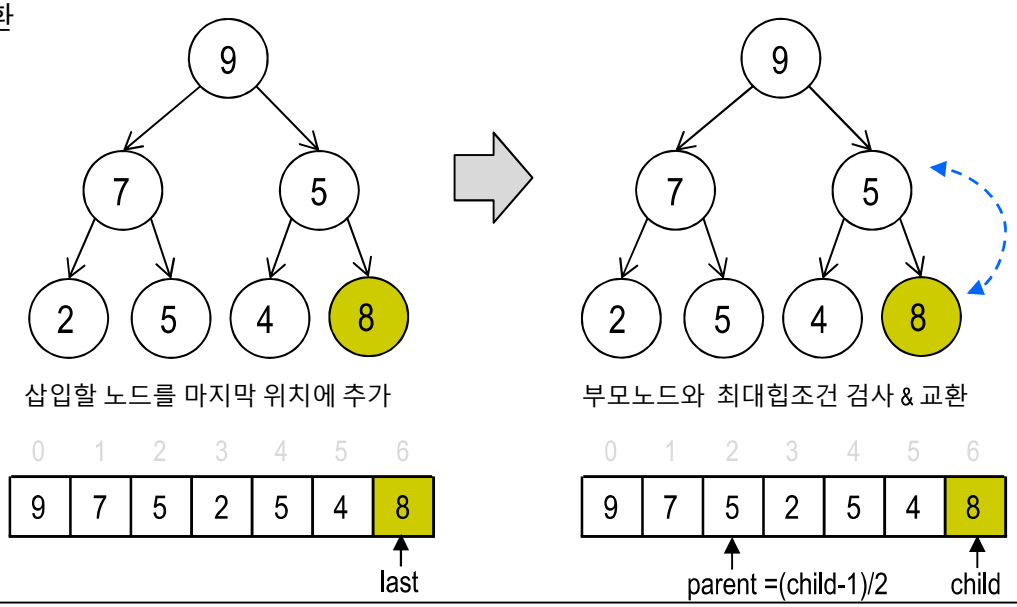
이진 힙(binary heap): 삽입

https://en.wikipedia.org/wiki/Binary_heap, CC-BY-SA
https://algs4.cs.princeton.edu/24pq/MaxPQ.java.html, GPLv3

```
public class SimpleHeap {
        int     last=-1, MaxHeapSize=4;
        int     heap[]=new int[MaxHeapSize];
        private void swap(int m, int n) { int temp=heap[m]; heap[m]=heap[n]; heap[n]=temp; }
        private void resize() {
                MaxHeapSize*=2;
                heap=Arrays.copyOf(heap, MaxHeapSize);
        }
        public void add(Integer data) {
                if(last+1==MaxHeapSize) resize();
                heap[++last]=data; // heap의 마지막 노드 다음 위치에 새 자료 삽입
                for (int child=last; child>0; ) { // 마지막 노드를 heapify-up
                        int     parent=(child-1)/2;
                        if(heap[child]<=heap[parent]) break; // 부모 키와 최대힙조건 검사
                        swap(child, parent); // 조건 불만족 시 교환
                        child=parent;
                }
        }
        public int remove() { … }
        @Override
        public String toString() { return Arrays.toString(heap); }
}
```

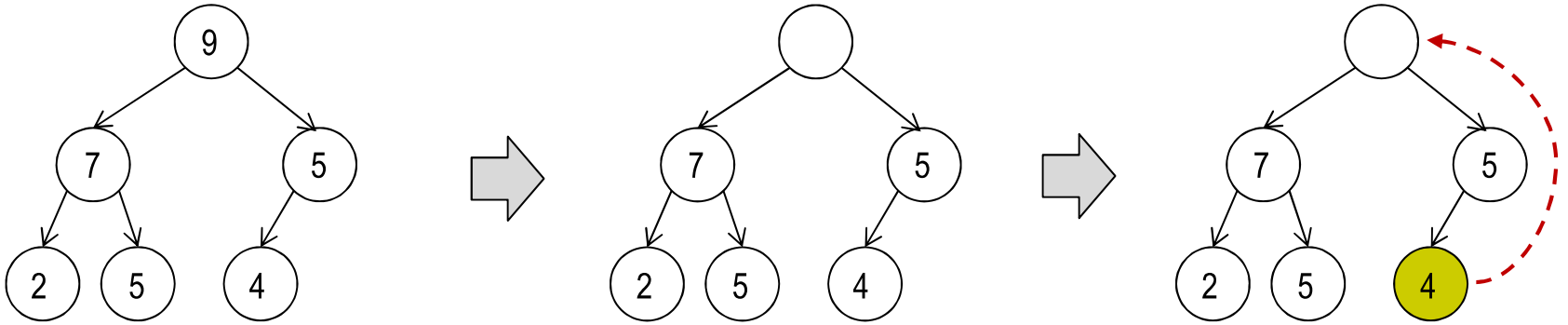
```
public class Test {
        public static void main(String[] args) {
                SimpleHeap    heap=new SimpleHeap();
                for (int i = 0; i < 10; i++) {
                        heap.add(i);
                        System.out.println(heap);
                }
                for (int i = 0; i < 10; i++) {
                        System.out.println(heap.remove()+"=>"+heap);
                }
        }
}
```

시간복잡도 $O(\log n)$



이진 힙(binary heap): 삭제

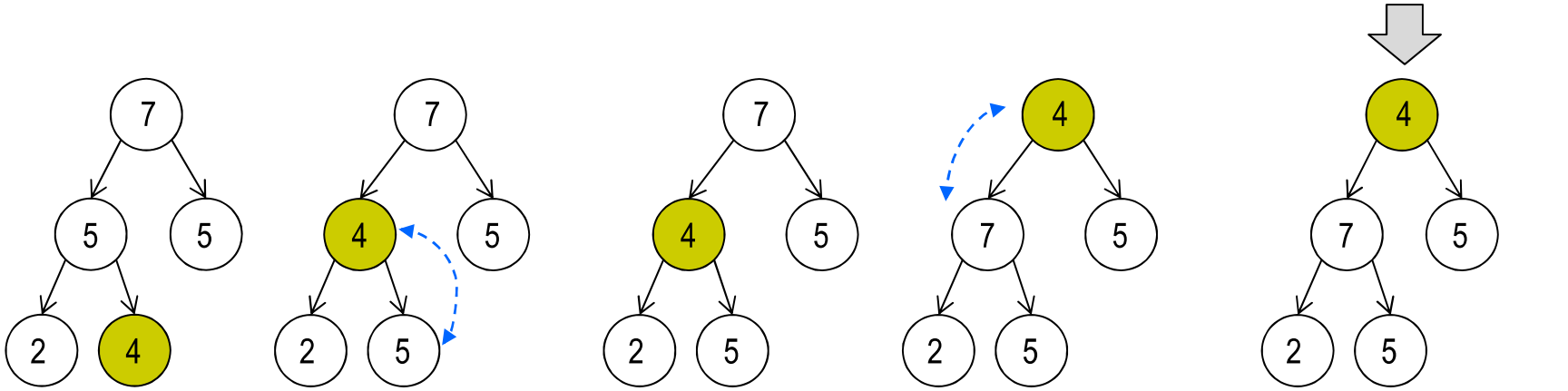
https://en.wikipedia.org/wiki/Binary_heap, CC-BY-SA
https://algs4.cs.princeton.edu/24pq/MaxPQ.java.html, GPLv3



최대힙에서 최대값 삭제

루트 노드 값 삭제

마지막 노드를 루트로 이동



두 자식노드 키 값 중 큰 값과 비교 후
최대힙조건 불만족 시 교환

두 자식노드 키 값 중 큰 값과 비교 후
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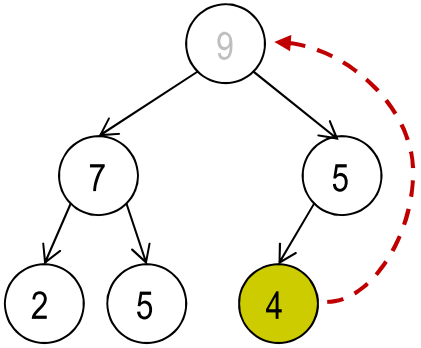
마지막 노드를 루트로 이동

이진 힙(binary heap): 삭제

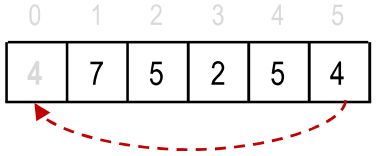
https://en.wikipedia.org/wiki/Binary_heap, CC-BY-SA
https://algs4.cs.princeton.edu/24pq/MaxPQ.java.html, GPLv3

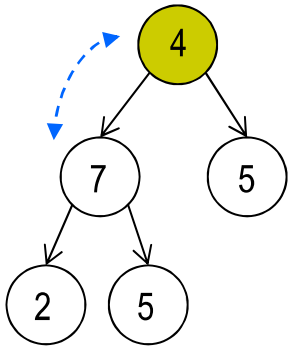
시간복잡도 $O(\log n)$

```
public class SimpleHeap {
        int     last=-1, MaxHeapSize=4;
        int     heap[]=new int[MaxHeapSize];
        private void swap(int m, int n) { int      temp=heap[m]; heap[m]=heap[n]; heap[n]=temp; }
        private void resize() { …}
        public void add(Integer data) { … }
        public int remove() {
                if(last<0) throw new RuntimeException("heap empty");
                int     data=heap[0]; // root 노드 자료 추출
                heap[0]=heap[last--]; // 마지막 노드 root로 이동 & 크기 1 감소
                for (int parent=0, child=2*parent+1; child<=last; parent=child, child=2*parent+1) { // root를 heapify-down
                        if(child<last && heap[child]<heap[child+1]) child++;
                        if(heap[child]<=heap[parent]) break;
                        swap(child,parent);
                }
                return data;
        }
        @Override
        public String toString() { return Arrays.toString(heap); }
}
```

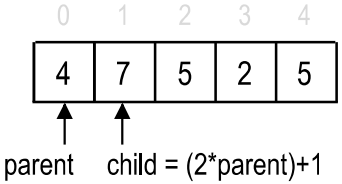


마지막 노드를 루트로 이동





키 값이 큰 자식노드와 비교 & 교환



이진 힙 vs. 균형탐색트리

- 최소힙
  - 생성: O(n)
  - 최소값 탐색: O(1)
  - 최소값 삭제: O(log n)
  - 삽입: O(log n)
  - 균형탐색트리보다 공간 효율적 (child link 불필요)
- 균형탐색트리
  - 생성: O(n log n)
  - 삽입: O(log n)
  - 삭제: O(log n)
  - 탐색: O(log n)

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