Maths

$y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$ .... (i)

$\frac{dy}{dx} = \frac{1}{2}((x+6)^2 + 25)^{\frac{-1}{2}} \times 2(x+6)) + \frac{1}{2}((x-6)^2 + 121)^{\frac{-1}{2}} \times 2(x-6))$

$\frac{dy}{dx} = \frac{x+6}{(x+6)^2+25)^{\frac{1}{2}}} + \frac{x-6}{(x+6)^2+25)^{\frac{1}{2}}} = 0$

$(x+6)((x-6)^2 + 121)^{\frac{1}{2}} + ((x-6)(x+6)^2 + 25)^{\frac{1}{2}} = 0$

$(x+6)((x-6)^2 + 121)^{\frac{1}{2}} = -(x-6)((x-6)^2 + 25)^{\frac{1}{2}}$

$(x+6)^2((x-6)^2 + 121) = (x-6)^2((x+6)^2 + 25)$

$(x+6)^2(x-6)^2 + 121(x+6)^2 = (x-6)^2((x+6)^2 + 25(x-6)^2$

$121(x+6)^2 - 25(x-6)^2 = (x-6)^2(x+6)^2 - (x-6)^2((x+6)^2$

$121(x^2 + 12x + 36) - 25(x^2 - 12x + 36) = 0$

$96x^2 + 1752x + 3456 = 0$

$x = \frac{-1752 \pm \sqrt{1752^2 - 4(96)(3456)}}{2(96)}$

$x = \frac{-1752 \pm 1320}{192}$

$x = -2.25 \text{ or } -16$

Substituting into equation (i)

$y = 20 \text{ or } 35.78$

$y_{min} = 20 \text{ at } x = -2.25$