$$\int_a^b H_0'(k|s-t|) f(t)\,dt$$

d

input: $a, b, k, s \rightarrow$ number

$f \rightarrow$ function that we ca

$s \notin (a,b]$ midpt rule

Simplest case $f \equiv 1$

$$I(a,b,k,s) = \int_a^b H_0'(k|s-t|)\,dt$$

$$s \notin (a,b] \rightarrow I(a,b,k,s) = \sum_{j=1}^{N} \int_{a+(j-1)h}^{a+jh} H_0'(k|$$

input a, b, h, s

$$\approx h \sum_{j=1}^{N} H_0'(k|s - a + (j$$

= h*sum(besselh(0, k*

$$s \in [a,b] \quad I(a,b,k,s) = \int^b H_0'(k|s-t|)\,d$$

$s \in [a,b]$

$f \in L^2(a,b)$

$u.$

$s - t) \, dt$

$h = \frac{(b-a)}{N}$

$- \frac{1}{2})h))$

j

$\mathrm{abs}(s - a + (\underbrace{[1:N]}_{}-\frac{1}{2}).h))$

lt

$m \; ltilde$

SE[a,b] $I(a,b,k,s) = \int_a^b H_0^1(k|s-t|)\,d$

product integration: $\frac{i}{4}H_0^1(k|s-t|) = -\frac{1}{2\pi}\ln(k|s-t|)\,J_0$

$+\frac{i}{4}H_0^1(k|s-t|) - ($

$$\chi(t) := \begin{cases} 1, & |t| \leq 1 \\ 1 + \exp\left(\frac{\pi-1}{\pi-|t|} - \frac{1-\pi}{1-|t|}\right) \\ 0 & \pi \leq t \end{cases}$$

m'little using weighted quadrature

$$\int_a^b -\frac{1}{2\pi}\ln(k|s-t|)\,J_0(k|s-t|)\,\chi(k|s-t|)\,dt$$

$$= \sum_{j=1}^{N}\int_{a+(j-1)h}^{a+jh} -\frac{1}{2\pi}\ln(k|s-t|)\,J_0(k|s-t|$$

$$\approx \sum_{j=1}^{N}\Big(-\frac{1}{2\pi}J_0(k|s-\text{midpt}|)\,\chi(k|s-$$

$$\frac{i}{4}\int_a^b H_0^1(k|s-t|)\,dt = \int_a^b \log(k|s-t|)\,m(t)\,d$$

$$\approx \sum_{j=1}^{N}\int_{a+(j-1)h}^{a+jh}\log(k|s-t|)\,dt\,\Big(m(a+(j-\tfrac{1}{2})h)\Big) +$$

dt

m 1 tilde

$o\,(h\,|s-t|)\,Z\,(h\,|s-t|)$ → simple

non-singular

m2

standard midpt

$1 \le |t| \le \pi$

$)\;Z\,(h\,|s-t|)\;dt$

$\text{midpt})\Big]\displaystyle\int_{a+(j-1)h}^{a+jh} \ln(h\,|s-t|)\;dt$

$dt \;+\; \displaystyle\int_a^b m_2(t)\,dt$

$h\,m_2\,(a+(j-\tfrac{1}{2})h)$

$$\approx \sum_{j=1} \underbrace{\int_{a+(j-1)h} \log(h|s-t|)\,dt}_{w_j} \; m\left(a+\left(j-\tfrac{1}{2}\right)h\right) + \ldots$$

$$\int_a^b \log(h|s-t|)\,dt$$

case (i) $\quad s > b \qquad u = h(s-t)$

$$= -\frac{1}{h}\int_{h(s-a)}^{h(s-b)} \log(u)\,du = -\frac{1}{h}\left[u\log(u) - \ldots\right.$$

$$= (b-s)\left[\log(h\ldots\right.$$

$$-(a-s)\left[\ldots\right.$$

$$= (b-s)\log(h(s-b)) - (a-s)\log(h(s\ldots$$

case (ii) $\quad s < a \qquad u = h(t-s) \Rightarrow \frac{du}{dt} = \ldots$

$$\int_a^b \log(h|s-t|)\,dt = \frac{1}{h}\int_{h(a-s)}^{h(b-s)} \log(u)\,du = \ldots$$

$$= (b-s\ldots$$

$$-\ldots$$

Same w/ Log = abs

$$= (b-s)\log(h(b-s)) - (a-s)\log h(a-s\ldots$$

case (iii) $\quad a < s < b$

$$\int_a^b \log(h|s-t|)\,dt = \int_a^s \log(h|s-t|)\,dt + \ldots$$

$$\int_a^s \log(h|s-t|)\,dt \qquad u = h(s-t) \qquad \frac{du}{dt} = -h$$

$-h m_2 (a + (j - \frac{1}{2})h)$

$t) \Rightarrow \frac{du}{dt} = -h$

$u \Big]_{h(s-a)}^{h(s-b)}$

$(s-b)) - 1]$

$[\log(h(s-a)) - 1]$

$-a)) - (b-a)$

h

$\frac{1}{h} \Big[u \log(u) - u \Big]_{h(a-1)}^{h(b-1)}$

$) [\log(h(b-1)) - 1]$

$(a-s) (\log(h(a-1)) - 1]$

$)) - (b-a)$

$\int_s^b \log(h(s-t))\,dt$

$h(s-a)$

$$\int_a \log(k(s-t))\,dt \qquad \frac{du}{dt}$$

$$= -\frac{1}{k}\int_{k(s-a)}^{0} \log u\,du = \frac{1}{k}\int_0^{k(s-a)} \log u\,du =$$

$$= (s-$$

$$\int_s^b \log(k(s-t))\,dt \qquad u = k(t-s) \qquad \frac{du}{dt} = k$$

$$= \frac{1}{k}\int_0^{k(b-s)} \log u\,du = \frac{u}{k}(\log u - 1)\Big|_0^{k(b}$$

$$\text{So integral} = (b-s)\log k(b-s) - b \cancel{+s} + (s-$$

$$= (b-s)\log(k(b-s)) - (a-s)\log(k(s-$$

in each case integral =

$$\boxed{(b-s)\log(\mathrm{abs}(k(b-s))) - (a-s)\log(\mathrm{a}}$$

$$= \frac{u}{k}(\log u - 1)\Big|_0^{k(s-a)}$$

$$a)(\log k(s-a) - 1)$$

$$-s) = (b-s)(\log k(b-s) - 1)$$

$$a)\log k(s-a) - \cancel{s} + a$$

$$a)) - (b-a)$$

$$\boxed{bs(k(s-a))) - (b-a)}$$