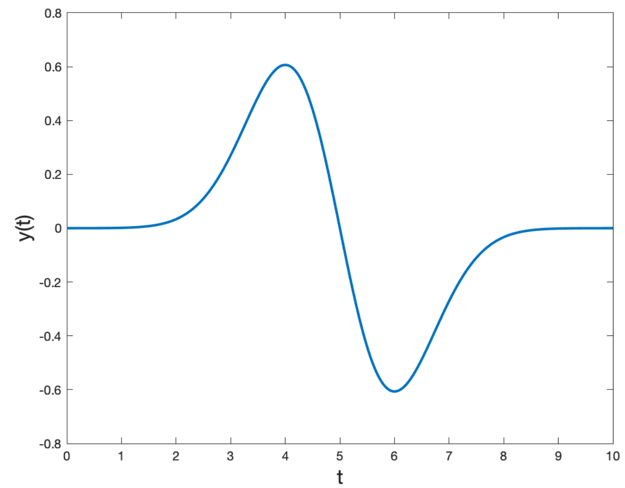
Wavelets

Wave-like oscillation *localized* in space (time)

Example:



$$-(x-b)e^{\frac{-(x-b)^2/(2a^2)}{\sqrt{2\pi}a^3}}$$

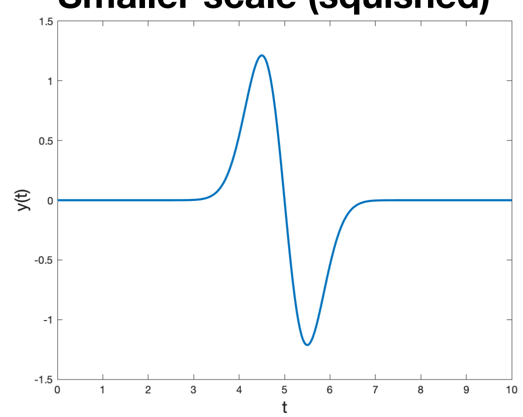
First derivative of Gaussian Function

[1] Two basic properties:

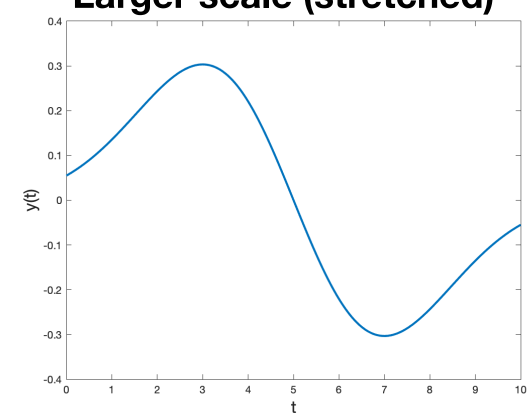
- **Scale (dilation)** - how “stretched” or “squished” wavelet is (related to frequency)

- **Location** - where wavelet is positioned in time

Smaller scale (squished)

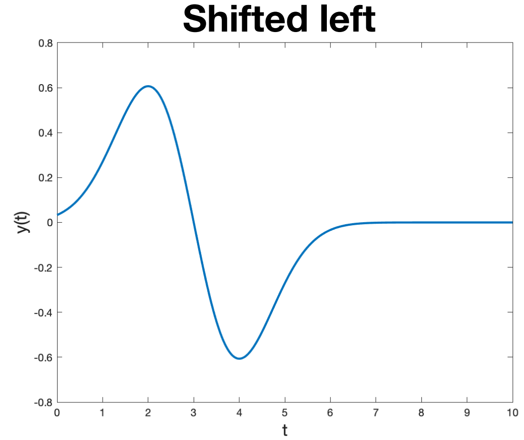


Larger scale (stretched)



a

Shifted left



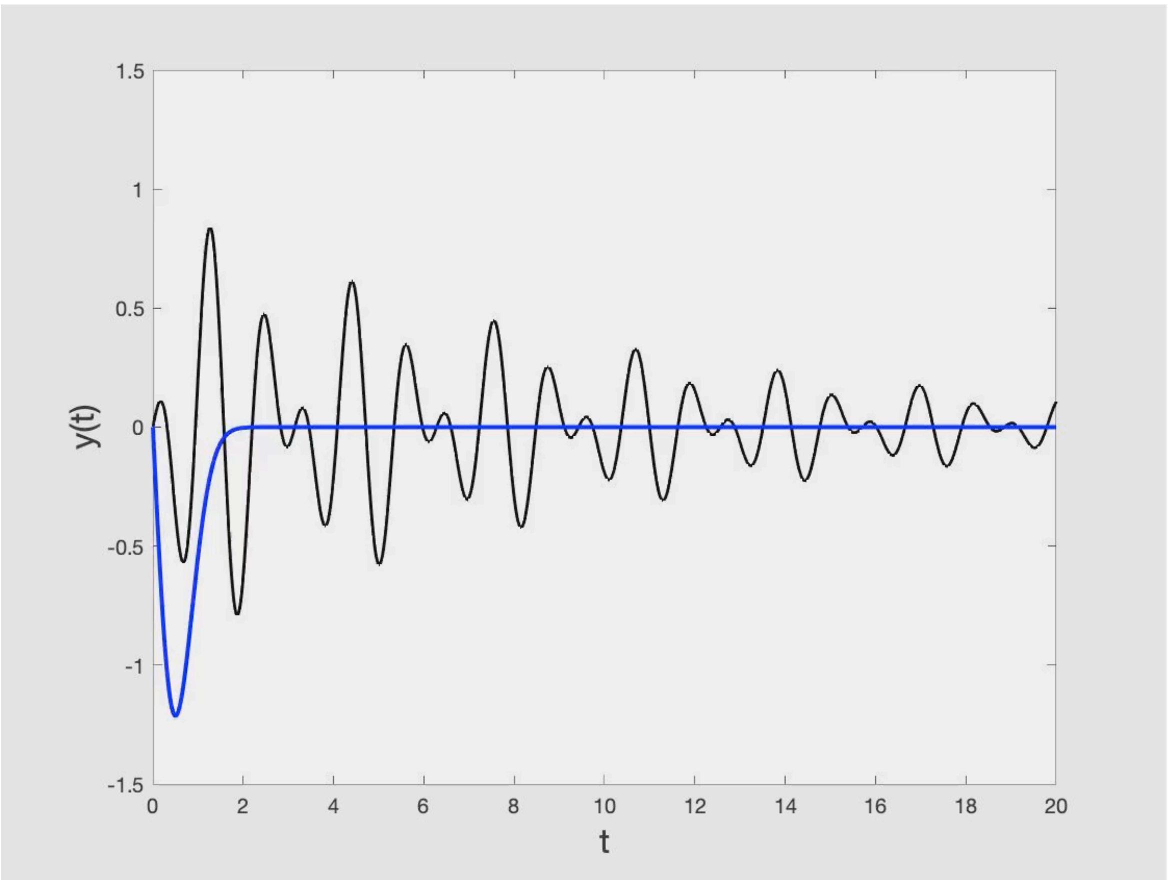
Shifted right and squished

b

[1] Addison, P.S. (2005). Wavelet transforms and the ecg: A review. Physiological Measurement, 26 (5), p. R155

Wavelet Transform

Decomposition of a signal using wavelets of varying scale and location



Basic idea: compute *how much* of wavelet is *in* the signal for a particular scale and location
i.e. evaluate convolution of signal and wavelet at varying *scales*

Why wavelets?

- Traditional **Fourier transform** (decomposition using sine and cosine) gives global average over entire signal, thus may obscure local information
- **Wavelet transforms** can extract local spectral **and** temporal information simultaneously
- **Variety of wavelets** to choose from

Addison, P.S. (2005). Wavelet transforms and the ecg: A review. Physiological Measurement, 26 (5), p. R155

Wavelet Transform

Two methods

Continuous Wavelet Transform (CWT)

"Mother" (basis) wavelets are defined everywhere

$$\psi = \psi(t)$$

Transform must be discretized for implementation

$$\boxed{T(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\, \psi^* \frac{(t-b)}{a} dt}$$

$T(a,b)$ may have redundant information since same feature may be *captured* by multiple scales

Discrete Wavelet Transform (DWT)

"Mother" (basis) wavelets are **only defined** on discrete grid

$$\psi = \psi_{m,n}(t)$$

Transform is already discrete!

$$\boxed{T_{m,n} = \int_{-\infty}^{\infty} x(t)\, \psi_{m,n}(t)\, dt}$$

Coefficients ($T_{m,n}$) do not have redundant information since discretized wavelets can be defined to be orthonormal

$$\int_{-\infty}^{\infty} \psi_{m,n}(t)\psi_{m',n'}(t)\,\mathrm{d}t = \begin{cases} 1 & \text{if} \quad m = m' \text{ and } n = n' \\ 0 & \text{otherwise.} \end{cases}$$

Addison, P.S. (2005). Wavelet transforms and the ecg: A review. Physiological Measurement, 26 (5), p. R155

Example

```
clear; close all; clc
```

Code to use Maximal Overlap Discrete Wavelet Transform (MODWT) to Analyze ECG Data

Code authored by: Shawhin Talebi

The University of Texas at Dallas

Mutli-scale Integrated Remote Sensing and Simulation (MINTS)

Load Data

```
load('ECG.mat');
```

Perform Wavelet Transform

Use the maximal overlap discrete wavelet transform (MODWT) to enhance the R peaks in the ECG waveform. The MODWT is an undecimated wavelet transform, which handles arbitrary sample sizes.

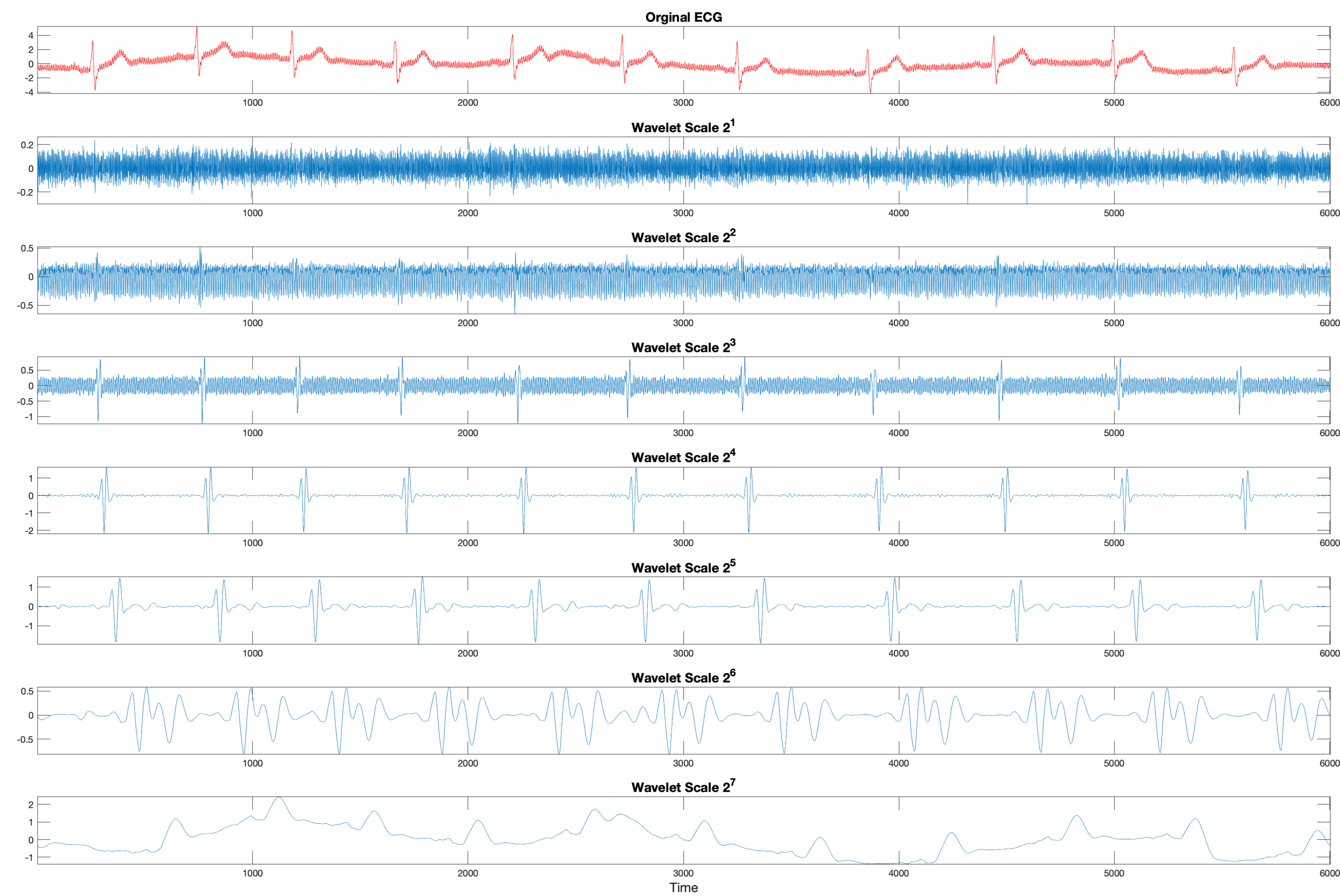
Perform MODWT for 6 different scales. The rows of wt give the wavelet (detail) coefficents for each scale. Here we are using Symlets wavelet with 4 vanishing moments

Other wavelet families can be used. use waveletfamilies('f') to view all

```
% define number of levels to use
numLevels = 6;

% perform maximal overlap discrete wavelet transform (MODWT)
wt = modwt(ECG, 'sym4', numLevels);
```

Compare Coefficents for all MODWT Scales

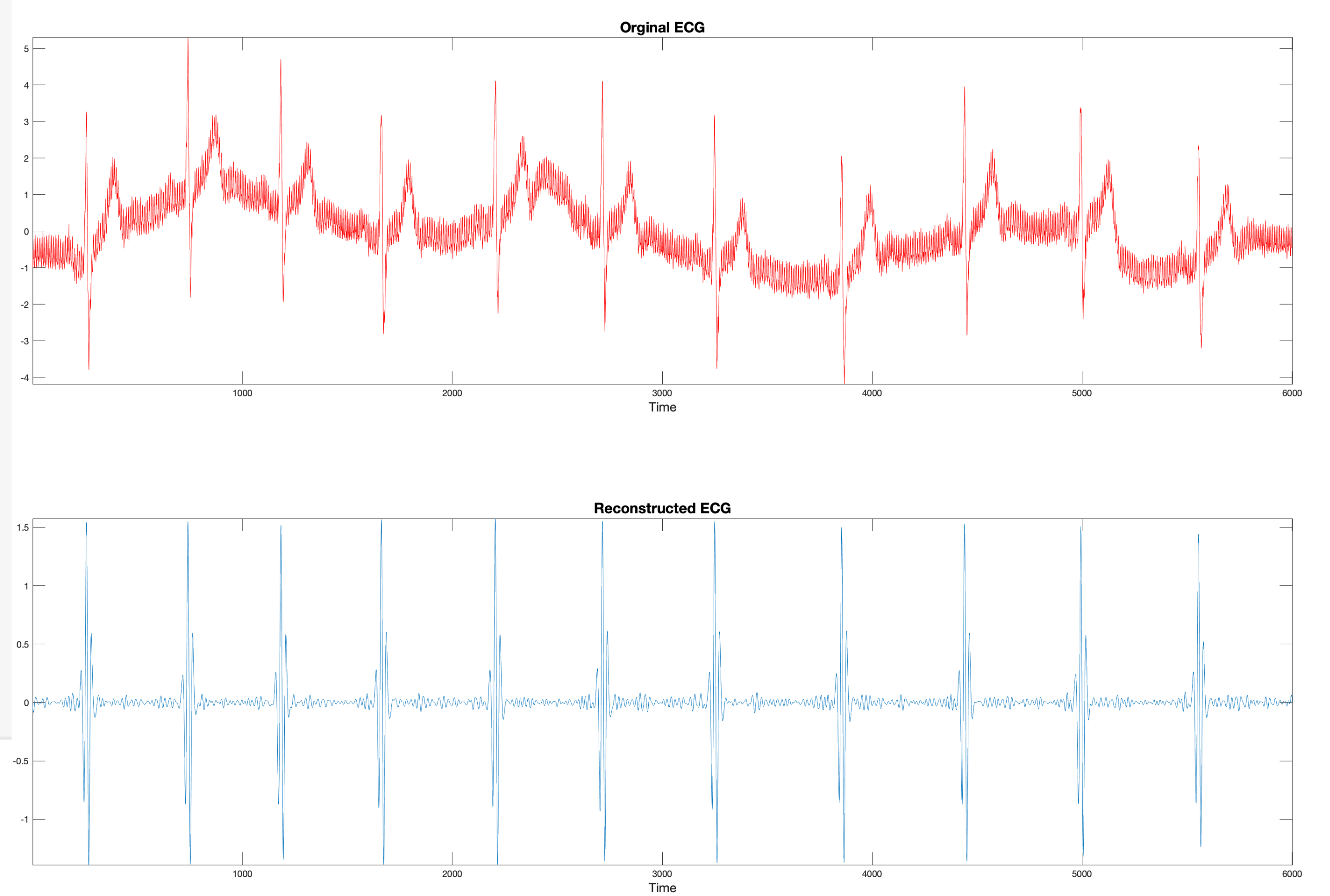
```
% get number of Wavelet Scales
[numScales, ~] = size(wt);

% create figure
fig = figure(1);
fig.Units = 'normalized';
fig.Position = [0 0 1 1];

% plot original signal
subplot(numScales+1,1,1)
plot(ECG, 'r-')
title('Orginal ECG', 'FontSize', 14)
axis tight

% plot wavelet scale coefficents
for i=2:numScales+1

    subplot(numScales+1,1,i)
    plot(wt(i-1,:))
    title(strcat('Wavelet Scale 2^', string(i-1)), 'FontSize', 14)
    axis tight
end

xlabel('Time', 'FontSize', 14)

% print figure to file
print('waveletTransform_signal_decomposition', '-dpng');
```



Example

Reconstruct Signal via Inverse MODWT

```
% create figure
fig = figure(2);
fig.Units = 'normalized';
fig.Position = [0 0 1 1];

% initialize an array to store the coefficents corresponding to scale 2^4
recwt = zeros(size(wt));

% store coefficents corresponding to scale 4 in array
recwt(4,:) = wt(4,:);

% reconstruct signal using only scale 4 with inverse transform
recECG = imodwt(recwt,'sym4');

% compare original signal and reconstructed
% plot original signal
subplot(2,1,1)
plot(ECG, 'r-')
title('Orginal ECG', 'FontSize', 16)
xlabel('Time', 'FontSize', 14)
axis tight
% plot reconstructed signal
subplot(2,1,2)
plot(recECG)
title('Reconstructed ECG', 'FontSize', 16)
xlabel('Time', 'FontSize', 14)
axis tight

% print figure to file
print('waveletTransform_reconstructed_signal', '-dpng');
```



Example

Find R Peaks Using Reconstructed Signal

```
% define array of time indicies
t = 1:length(ECG);

% square reconstructed signal
recECG = abs(recECG).^2;
% find peaks in squared reconstructed signal
[qrspeaks,locs] = findpeaks(recECG, t, 'MinPeakHeight', 0.35, 'MinPeakDistance',100);

% create figure
fig = figure(3);
fig.Units = 'normalized';
fig.Position = [0 0 1 1];

% plot signal with detected r peaks using reconstructed signal from DWT
plot(t,ECG)
hold on
plot(locs,ECG(locs),'ro')
xlabel('Time', 'FontSize', 14)
title('R Peaks Detected Using Reconstructed Signal from Wavelet Transform', ...
    'FontSize', 16)
axis tight
print('waveletTransform_rpeak_annotation', '-dpng');
```