



AUTOMATED
BALL PICKING
ROBOT ARM

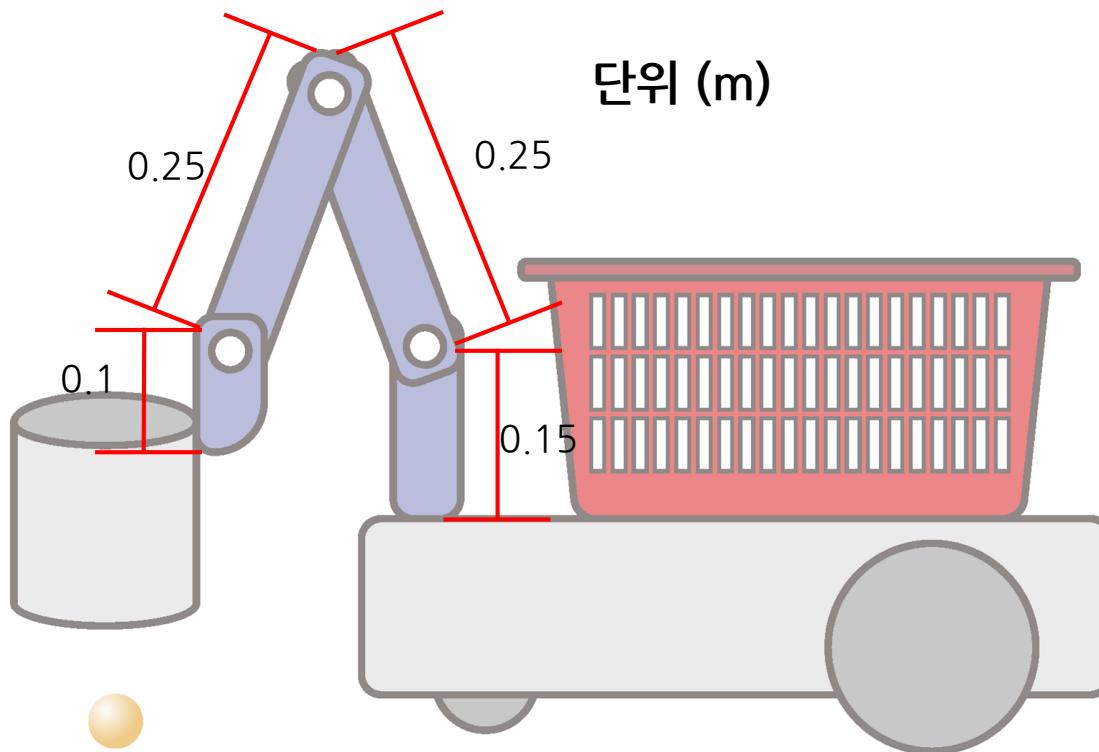
탁구공 줍는 자동화 로봇 팔

과목	담당 교수님	학번	이름
응용로봇설계	김정엽 교수님	17100179	이진우

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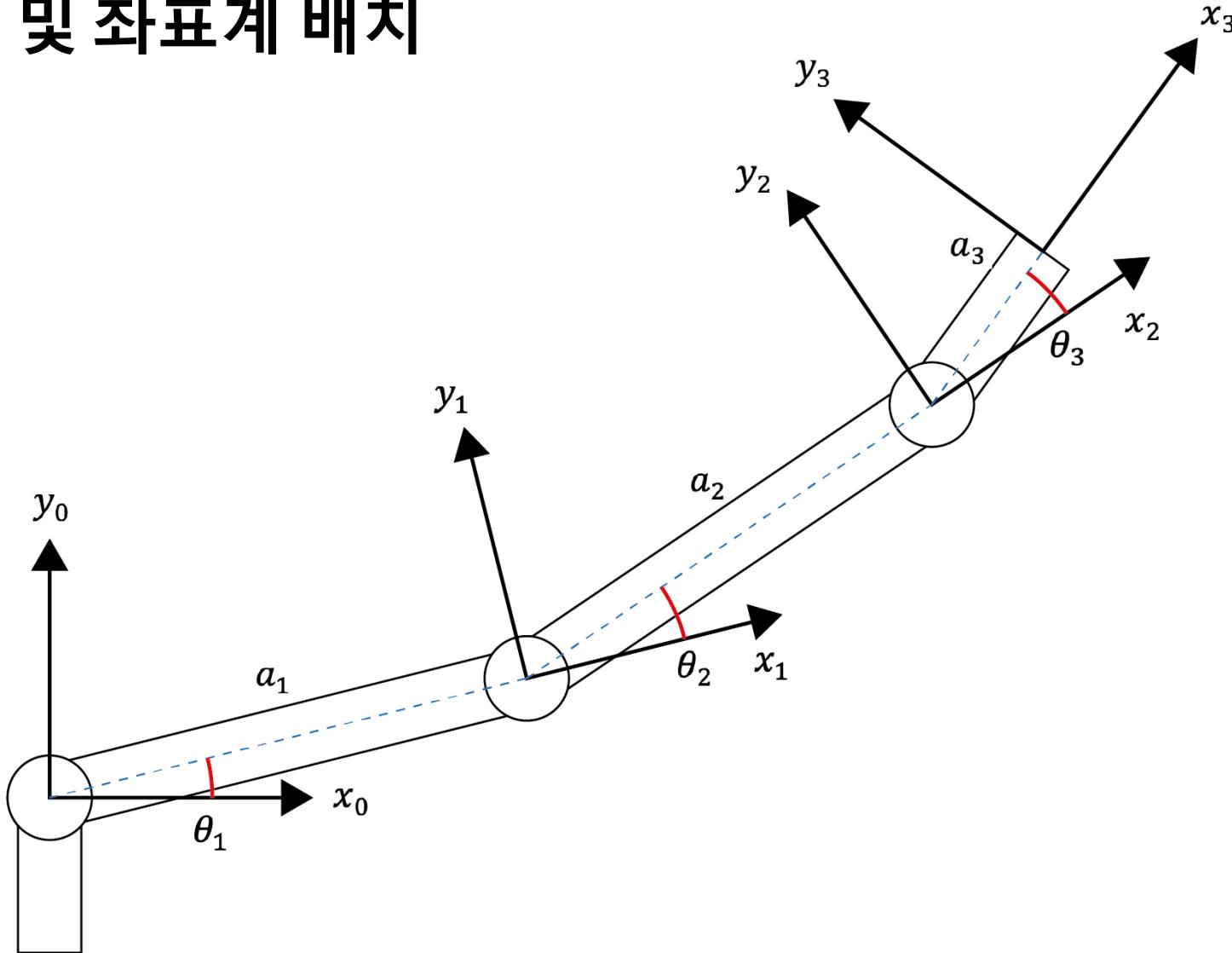
- 1 로봇 팔 치수, 중량, 자유도 및 좌표계
- 2 Forward Kinematics (FK), Inverse Kinematics (IK)
 유도
- 3 End-effector 의 위치 및 방위 궤적 제안
 및 IK 를 이용한 관절 변수 궤적 도출
- 4 Dynamic equation 을 이용한 주어진 관절 궤적에
 따른 관절 힘, 속도 및 절대 파워 그래프 도출

1 로봇 팔의 각 링크 치수/중량 설정



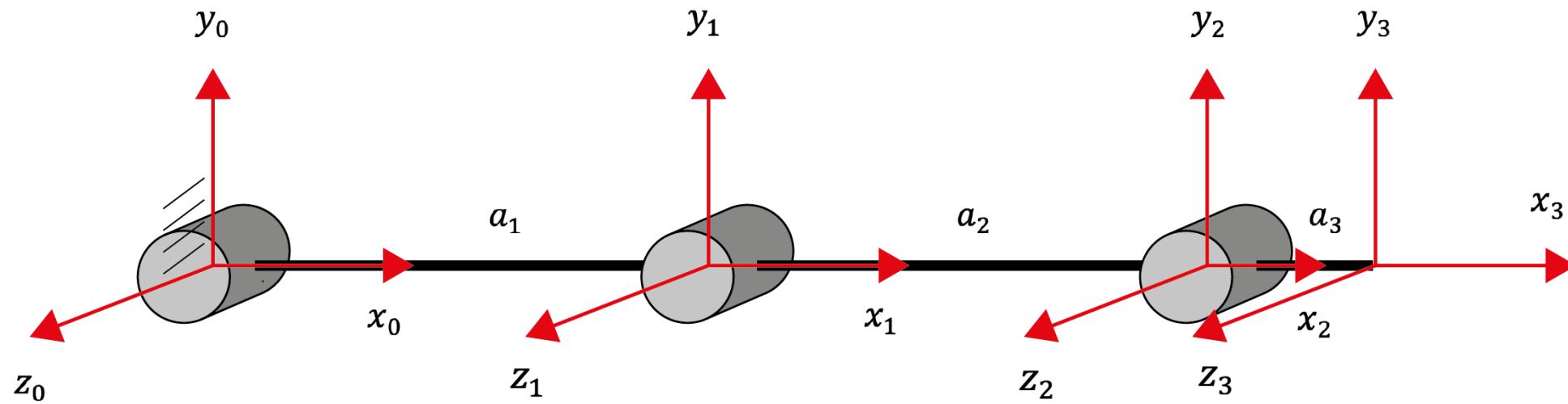
각 링크의 중량은 모두 **2kg**이다.
(마지막 링크는 가반 하중을 포함)

1 관절 배치 및 좌표계 배치



직접 그렸습니다!

1 관절 구조도



직접 그렸습니다!

1 D-H 파라미터 표

Link	a_i	α_i	d_i	θ_i
1	0.25m	0	0	θ_1
2	0.25m	0	0	θ_2
3	0.10m	0	0	θ_3

2 FK 유도

각 관절에서의 Homogeneous Transformation

$$A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0.25\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0.25\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0.25\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0.25\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0.1\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0.1\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 FK 유도

$$A_1 A_2 A_3 =$$

$$\begin{bmatrix} \cos(\theta_1 + \theta_2 + \theta_3) & -\sin(\theta_1 + \theta_2 + \theta_3) & 0 & 0.25\cos\theta_1 + 0.25\cos(\theta_1 + \theta_2) + 0.1\cos(\theta_1 + \theta_2 + \theta_3) \\ \sin(\theta_1 + \theta_2 + \theta_3) & \cos(\theta_1 + \theta_2 + \theta_3) & 0 & 0.25\sin\theta_1 + 0.25\sin(\theta_1 + \theta_2) + 0.1\sin(\theta_1 + \theta_2 + \theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 IK 유도

Geometric Approach IK

$$\cos\theta_2 = \frac{r^2 - a_1^2 - a_2^2}{2a_1a_2} = D$$

$$\theta_2 = \text{Atan2}(\pm\sqrt{1 - D^2}, D)$$

$$r = \sqrt{x_2^2 + y_2^2}$$

$$x_2 = x_3 - a_3 \cos\theta_d$$

$$y_2 = y_3 - a_3 \sin\theta_d$$

$$\theta_1 = \text{Atan2}(y_2, x_2) - \text{Atan2}(a_2 \sin\theta_2, a_1 + a_2 \cos\theta_2)$$

$$\theta_3 = \theta_d - \theta_1 - \theta_2$$

2 특정 자세에 대해서 IK 를 풀어서 검증

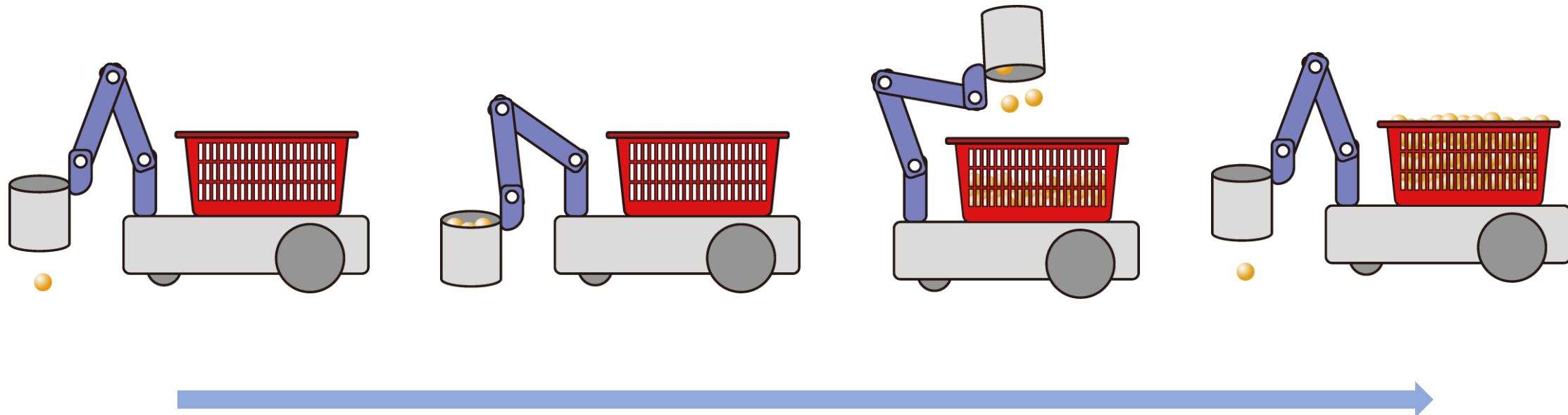
$$x_3 = 0.6m, y_3 = 0m, \theta_d = 0 \rightarrow \theta_1 = 0, \theta_2 = 0, \theta_3 = 0$$

$$x_3 = 0m, y_3 = 0.6m, \theta_d = \frac{\pi}{2} \rightarrow \theta_1 = \frac{\pi}{2}, \theta_2 = 0, \theta_3 = 0$$

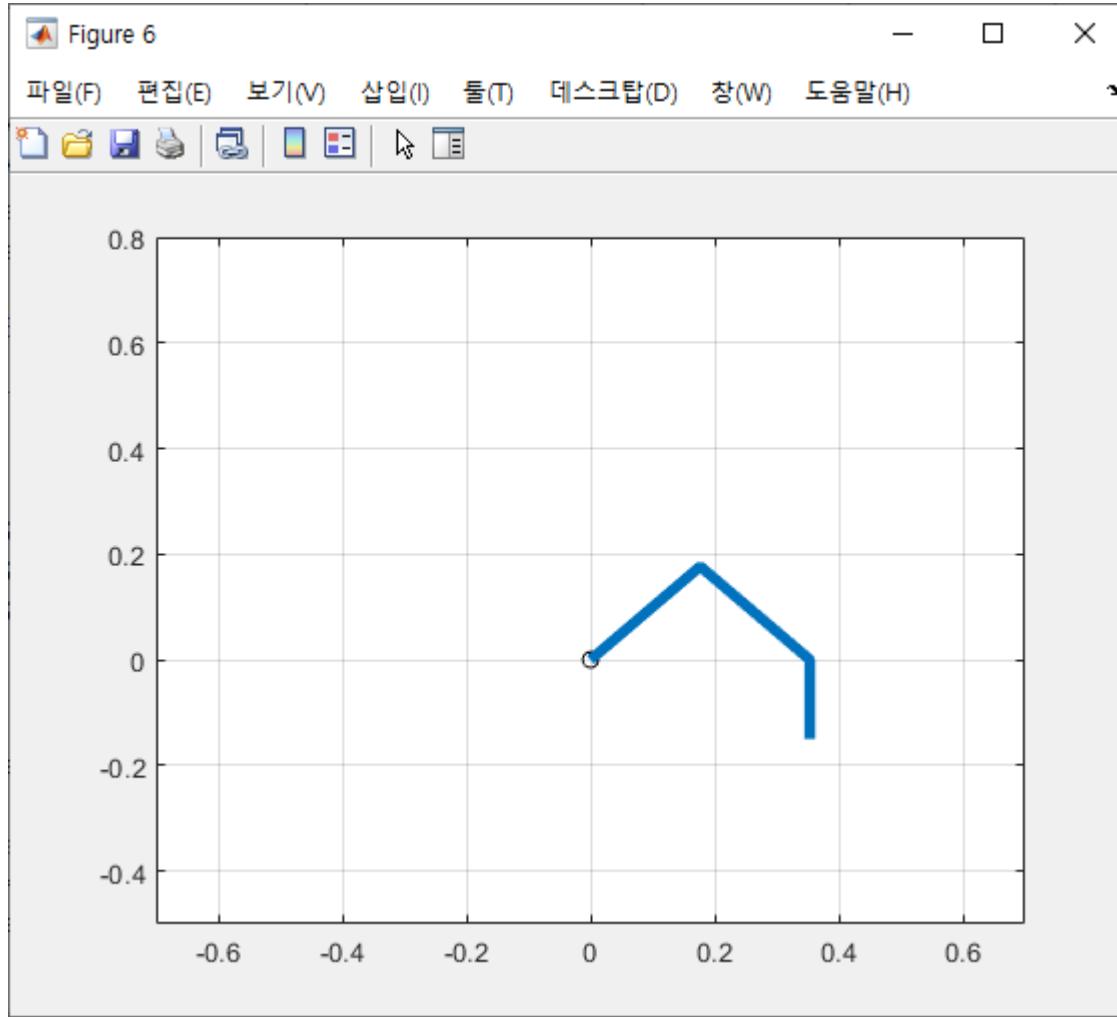
$$x_3 = 0.25\sqrt{2}m, y_3 = -0.1m, \theta_d = -\frac{\pi}{2} \rightarrow \theta_1 = \frac{\pi}{4}, \theta_2 = -\frac{\pi}{2}, \theta_3 = -\frac{\pi}{4}$$

3 Task 모션 제안

작동 순서



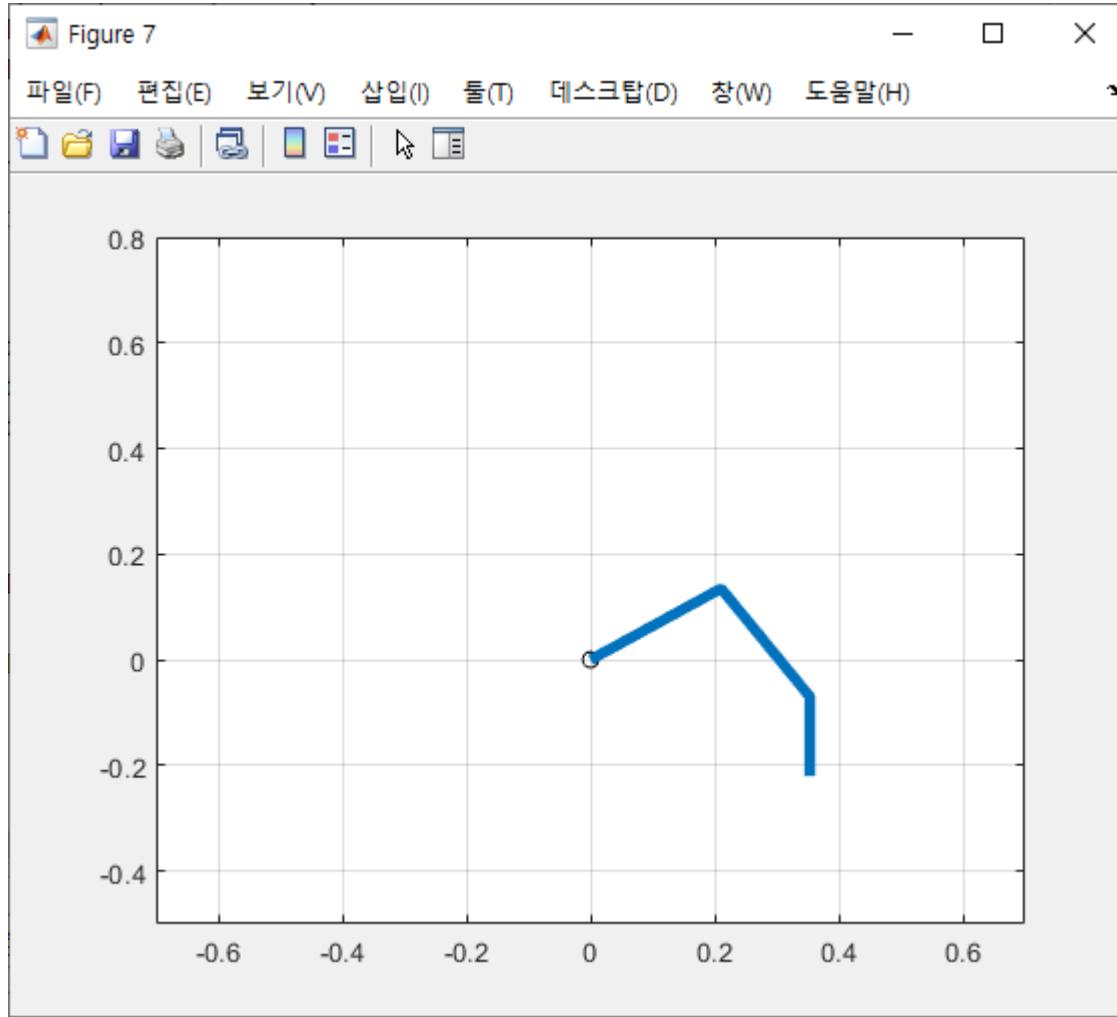
3 Task 모션 제안



초기자세 :

$$(x_3, y_3) = (0.25\sqrt{2}, -0.1), \theta_d = -\frac{\pi}{2}$$

3 Task 모션 제안

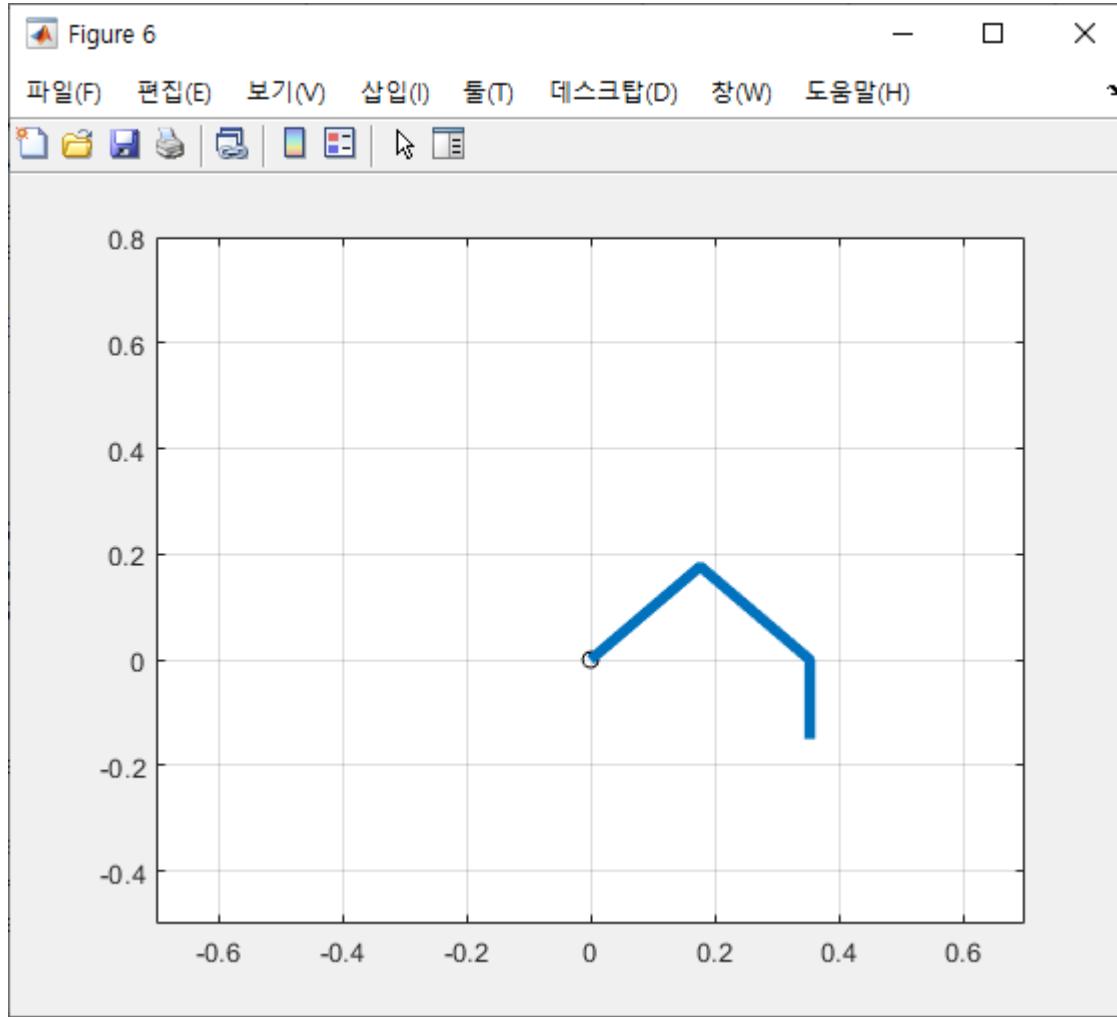


Step 1. 공을 주을 때

초기자세 : $(x_3, y_3) = (0.25\sqrt{2}, -0.1)$, $\theta_d = -\frac{\pi}{2}$

바닥으로 0.07m 이동 : $(0.25\sqrt{2}, -0.17)$, $-\frac{\pi}{2}$

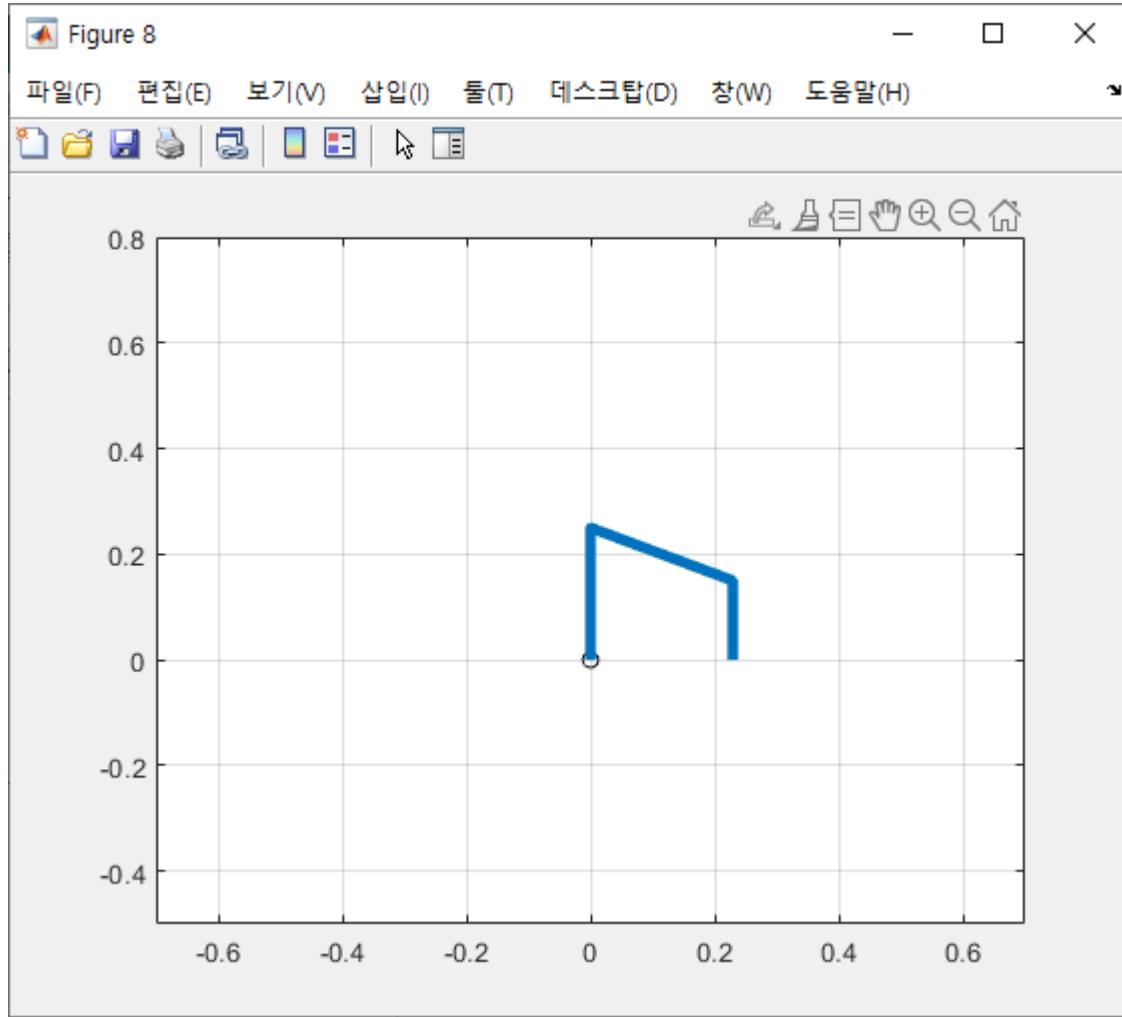
3 Task 모션 제안



Step 2. 복귀

초기자세 복귀 후 $\theta_3 = -\frac{\pi}{2}$ 로 고정한다.

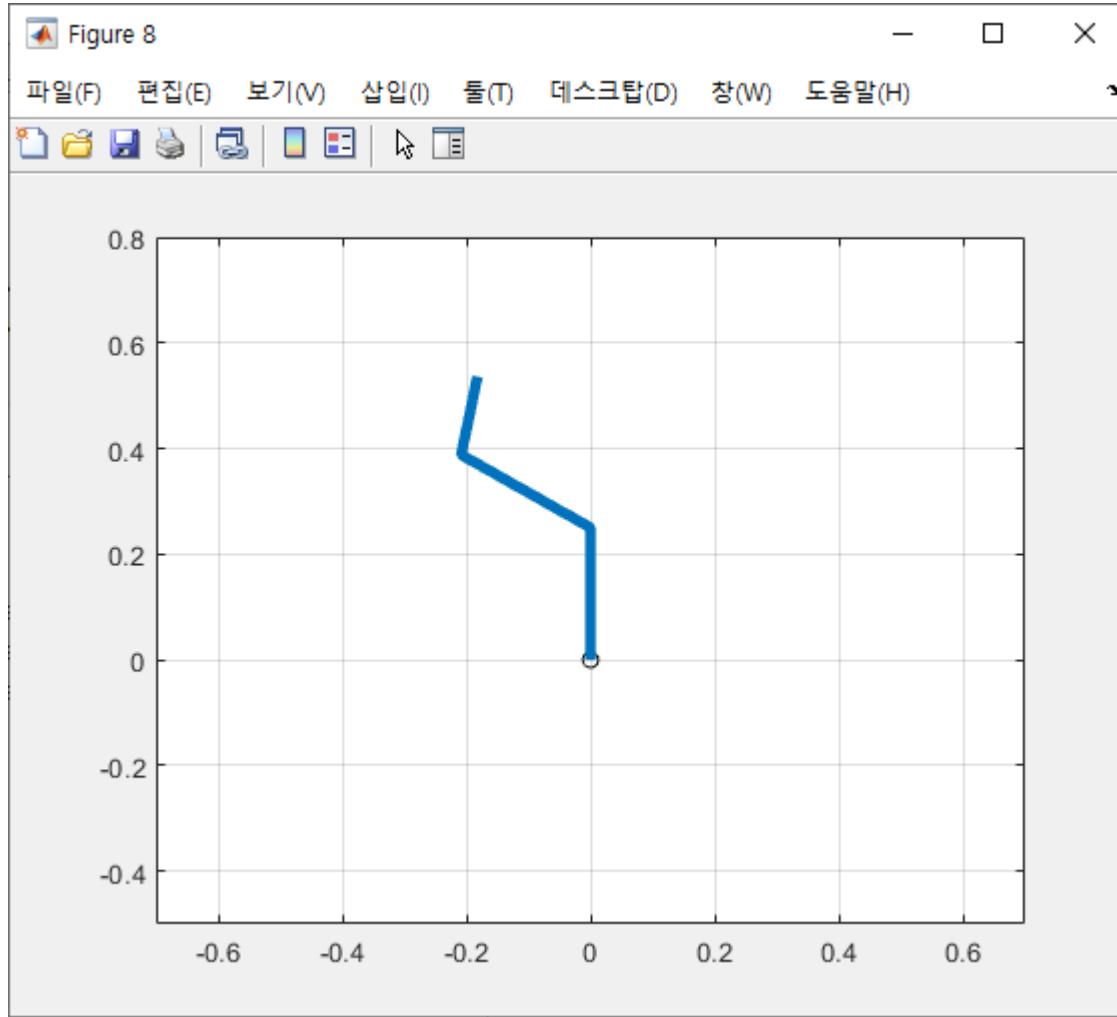
3 Task 모션 제안



Step 3. 준비 자세

2번 링크 $\theta_2 = -\frac{\pi}{2}$ 도 될 수 있도록 x위치를
 $\sqrt{0.25^2 - (0.25 - 0.1)^2}$ 로 조정

3 Task 모션 제안



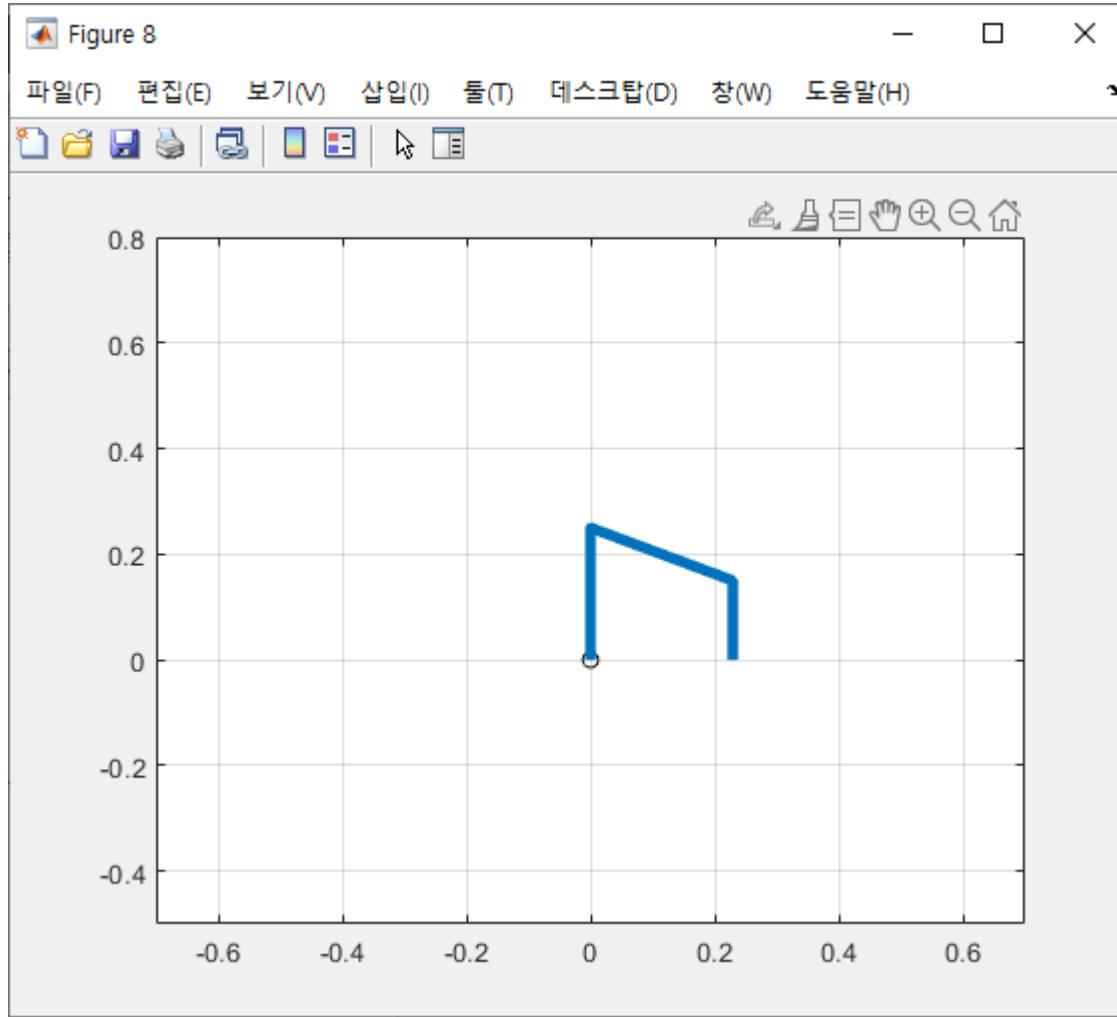
Step 4. 공 넘기기

$(\sqrt{0.25^2 - (0.25 - 0.1)^2}, 0)$ 에서 θ_1 의 반시계 방향으로 돌린다.

Desired 위치로 가면 회전을 멈춘다.

(des_x = -0.1)

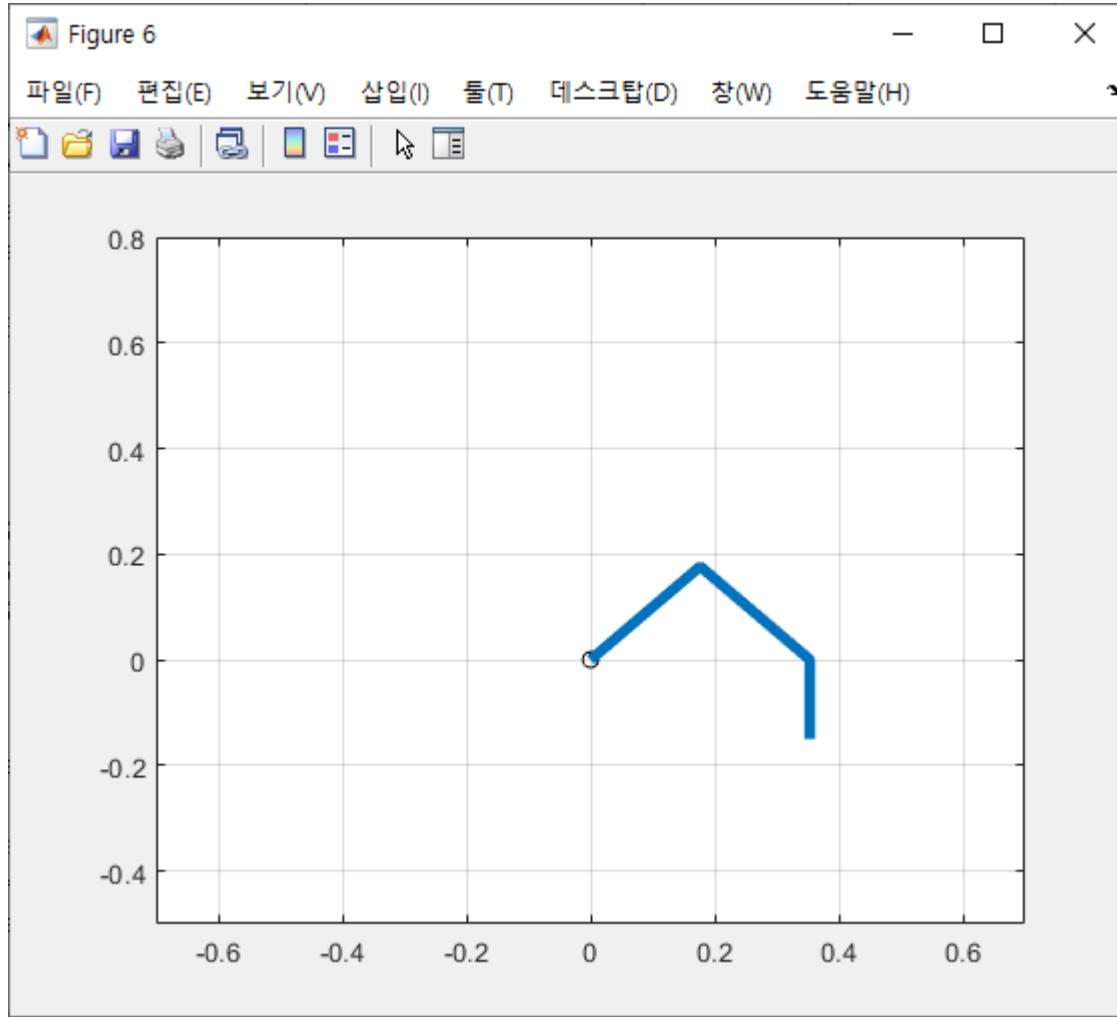
3 Task 모션 제안



Step 5. 준비 자세 복귀

준비자세 복귀 후 $\theta_3 = -\frac{\pi}{2}$ 고정한다.

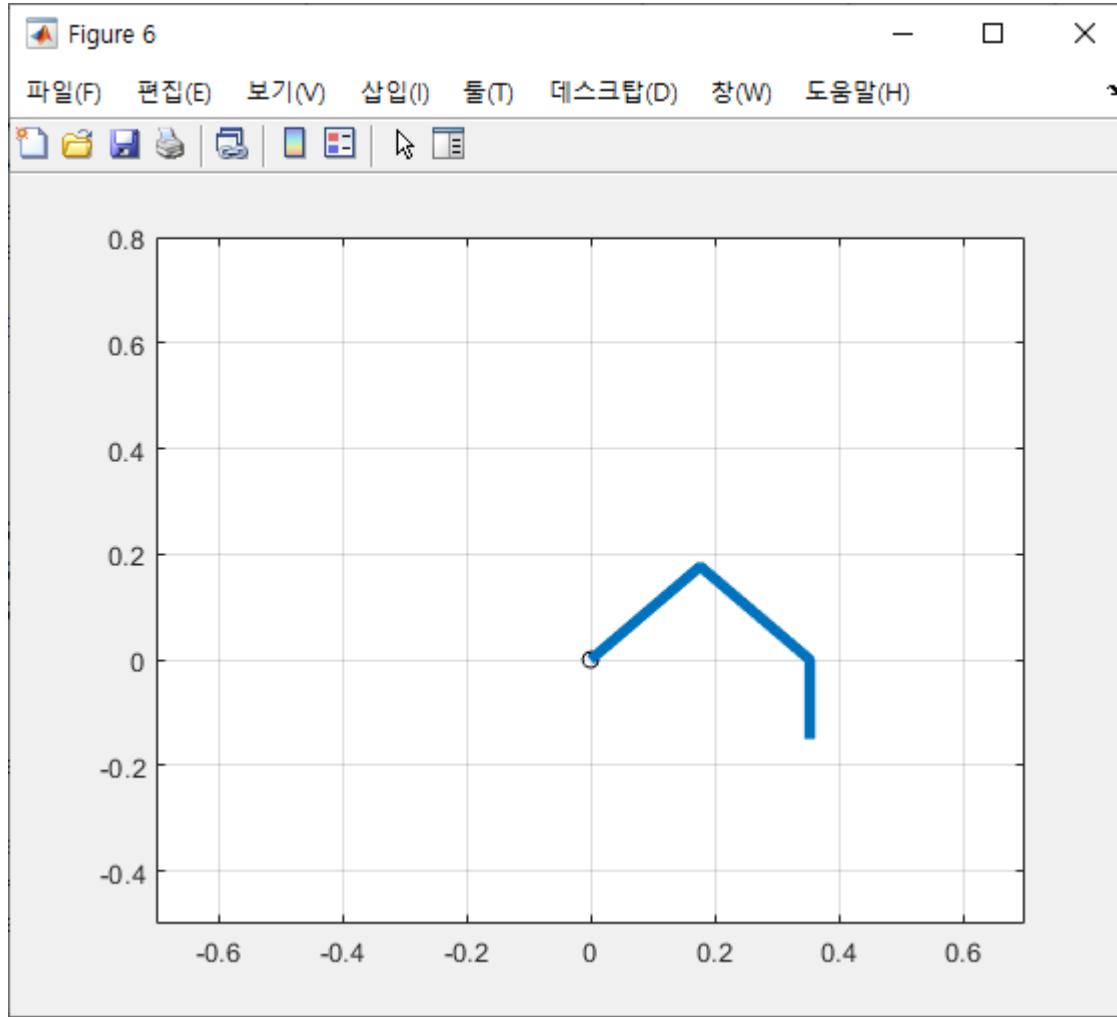
3 Task 모션 제안



Step 6. 완전 복귀

초기자세 : $(x_3, y_3) = (0.25\sqrt{2}, -0.1)$, $\theta_d = -\frac{\pi}{2}$
로 복귀한다.

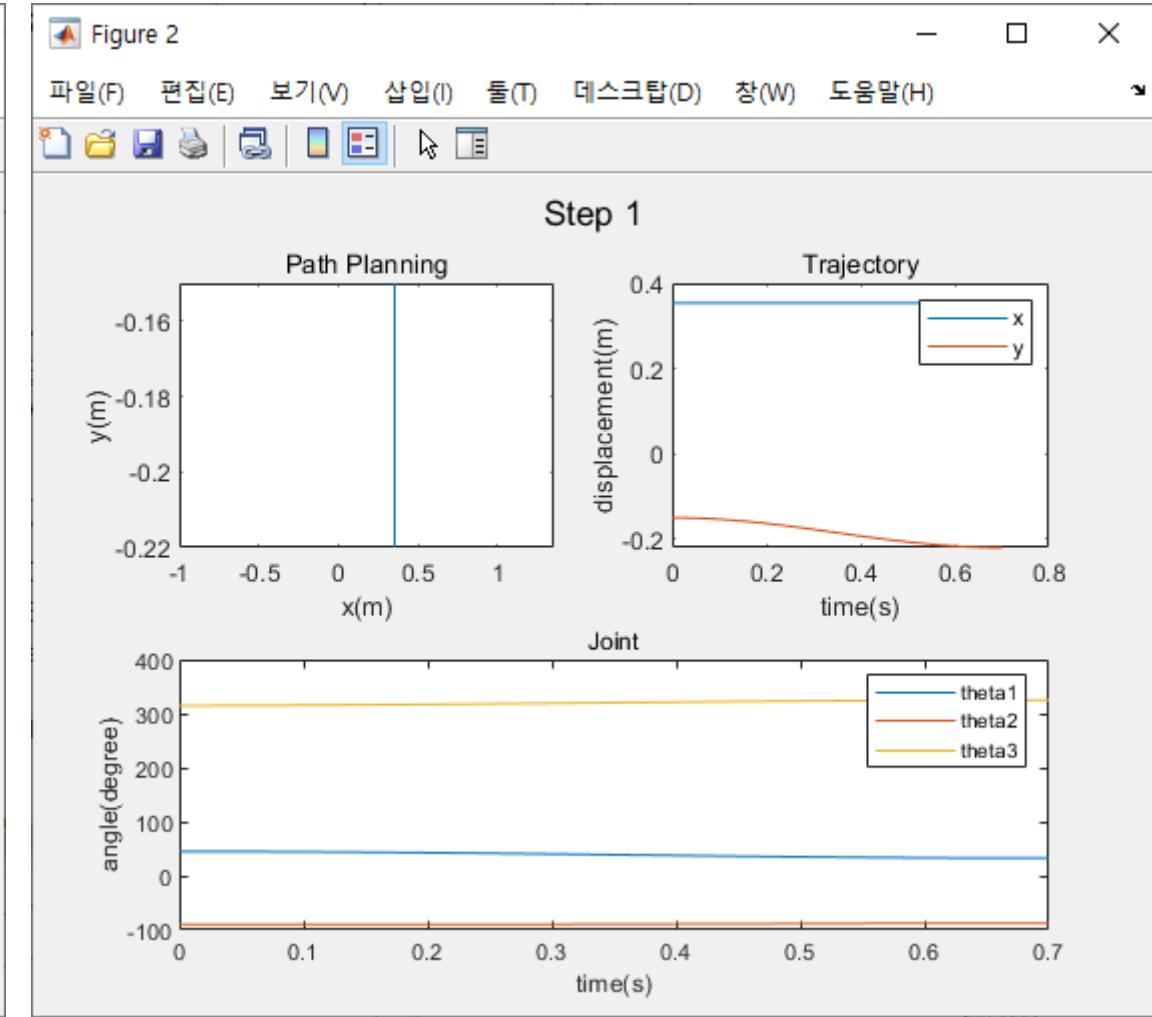
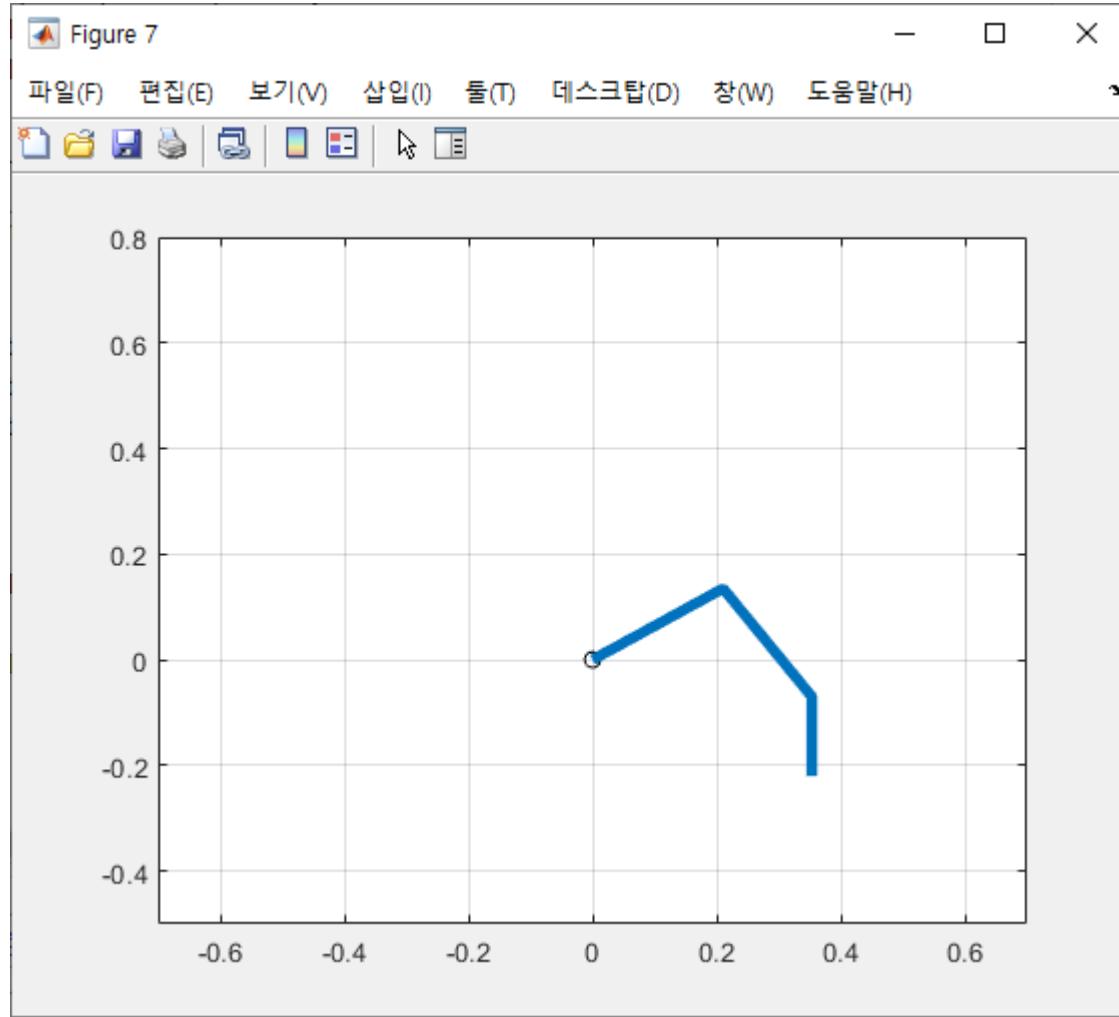
3 End-effector 경로 계획 및 궤적 생성 & 각 관절 변수 궤적



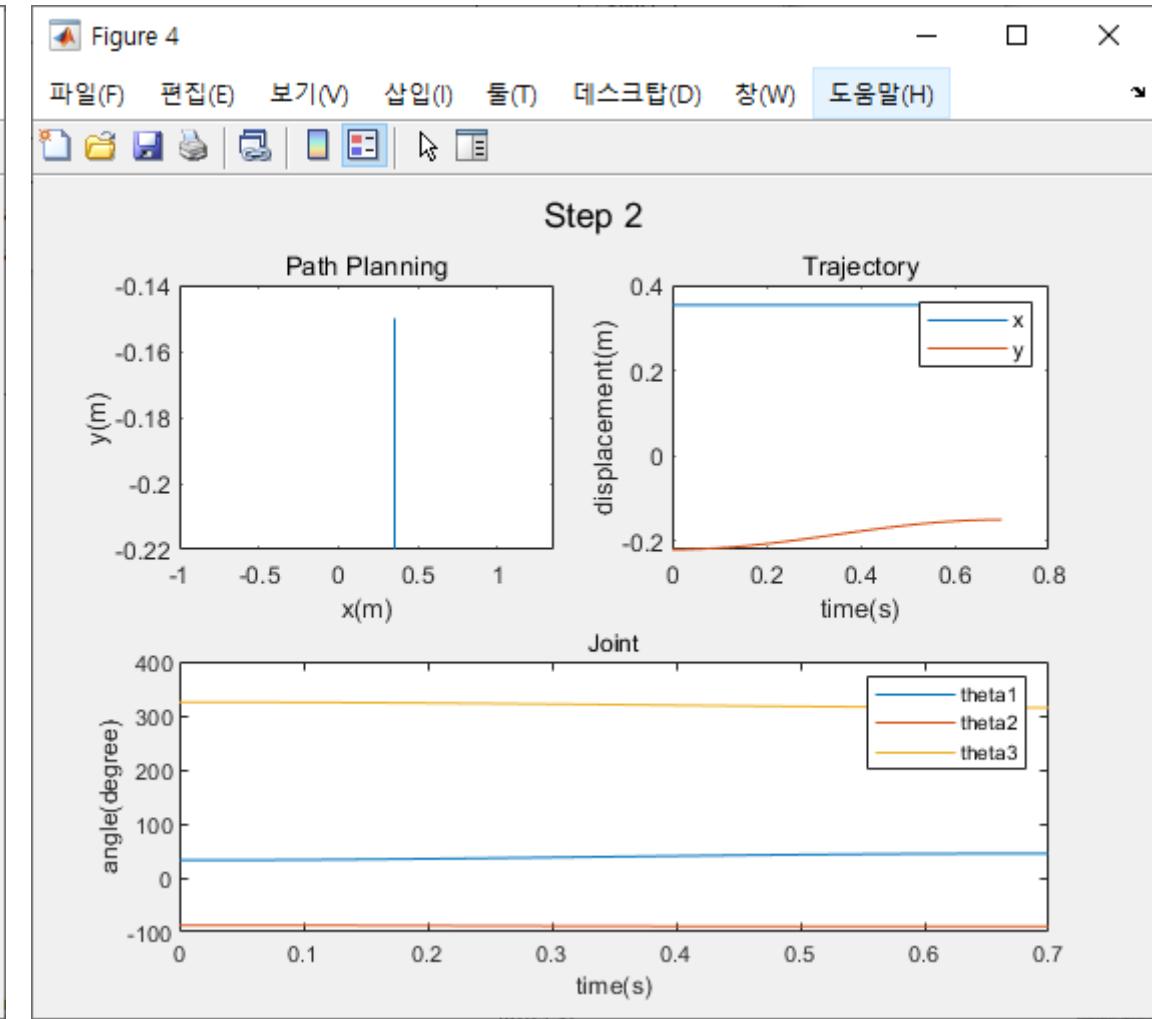
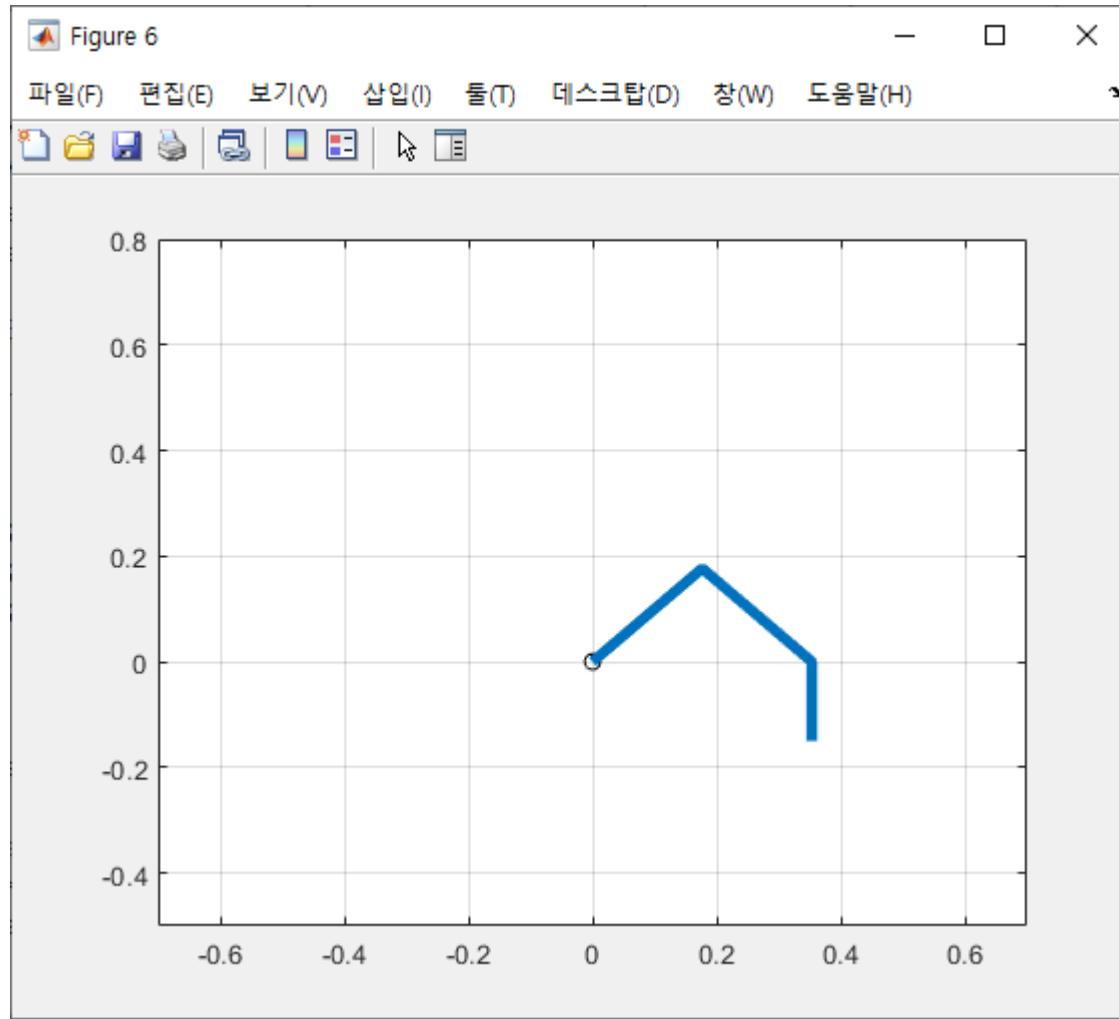
초기자세 :

$$(x_3, y_3) = (0.25\sqrt{2}, -0.1), \theta_d = -\frac{\pi}{2}$$

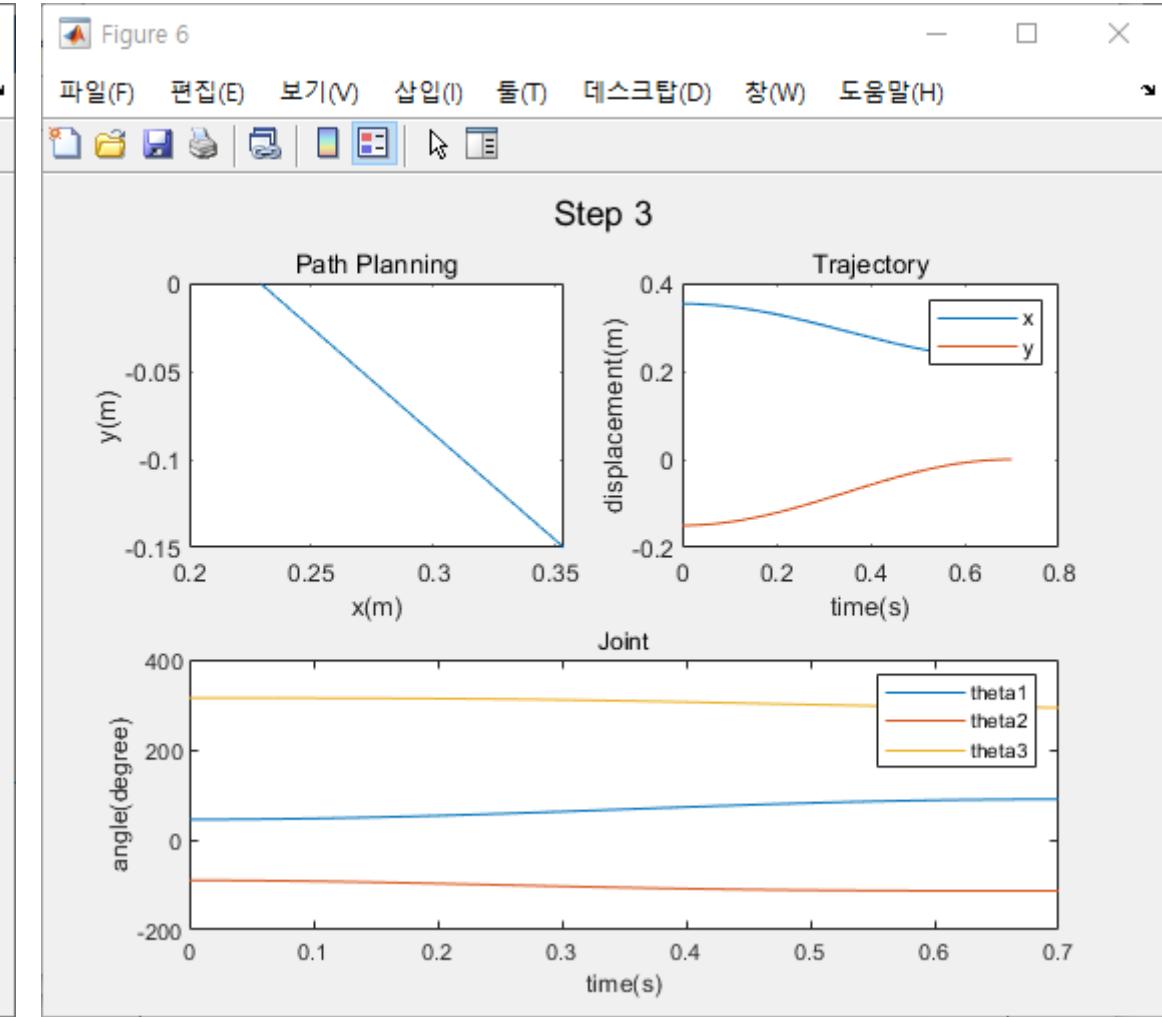
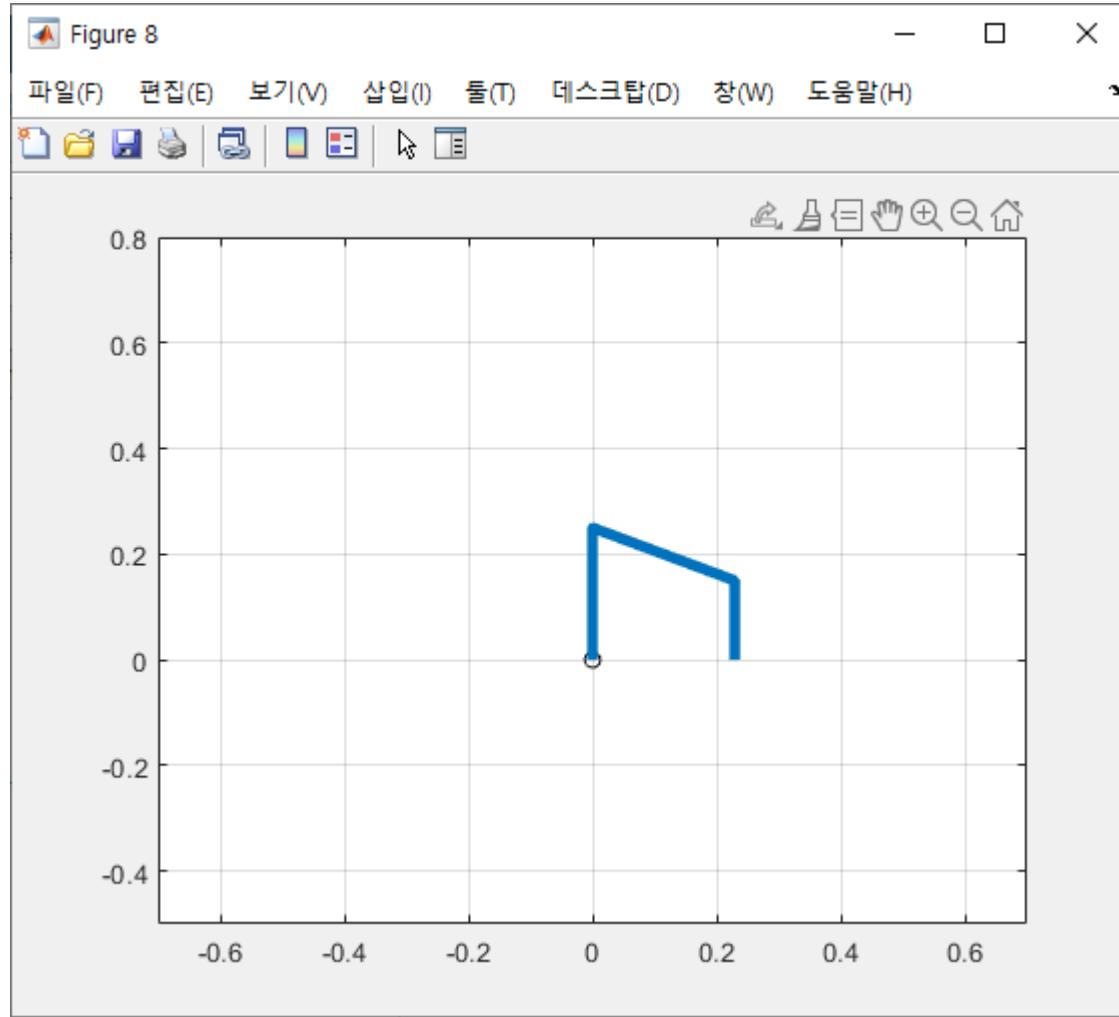
3 End-effector 경로 계획 및 궤적 생성 & 각 관절 변수 궤적



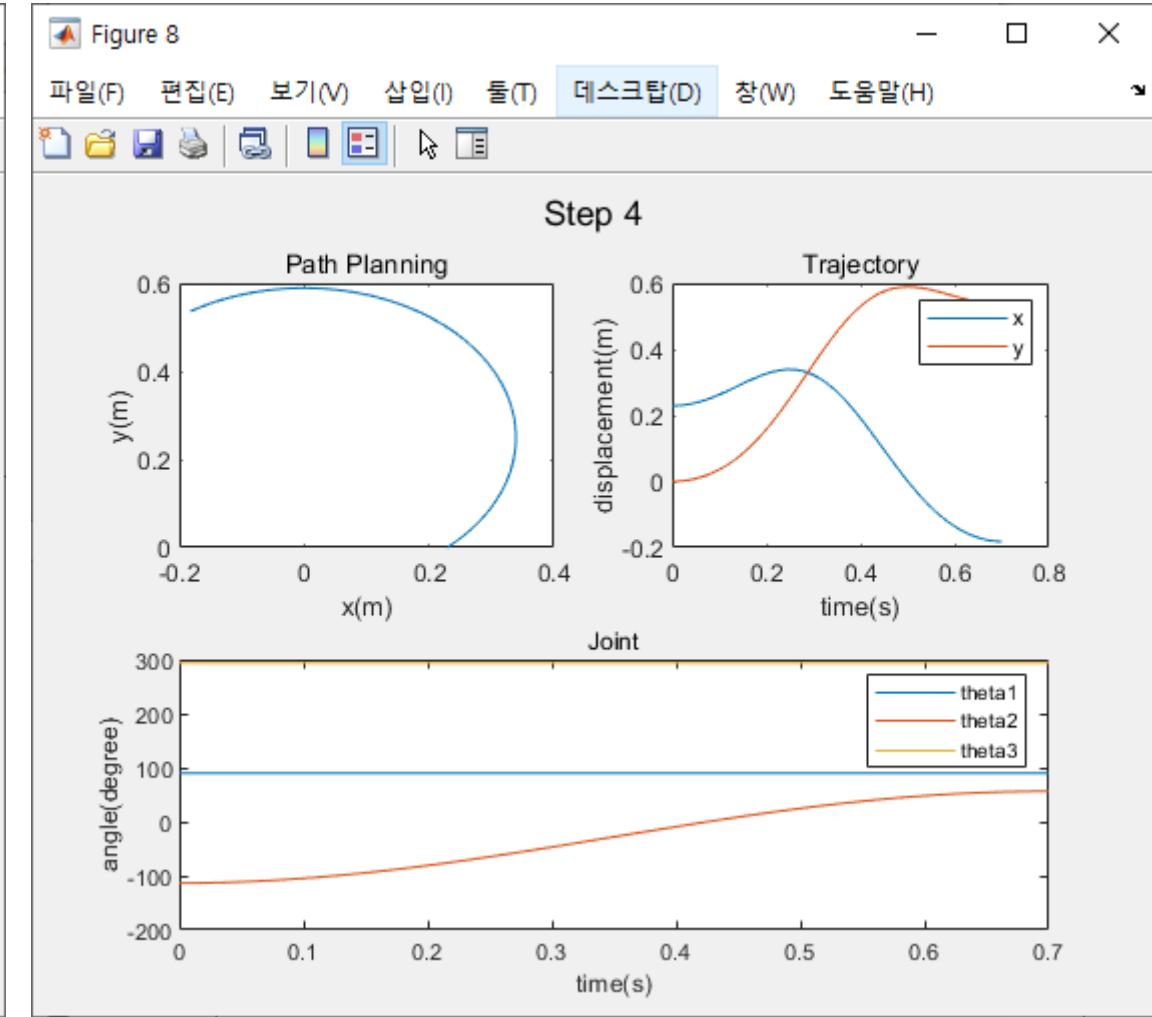
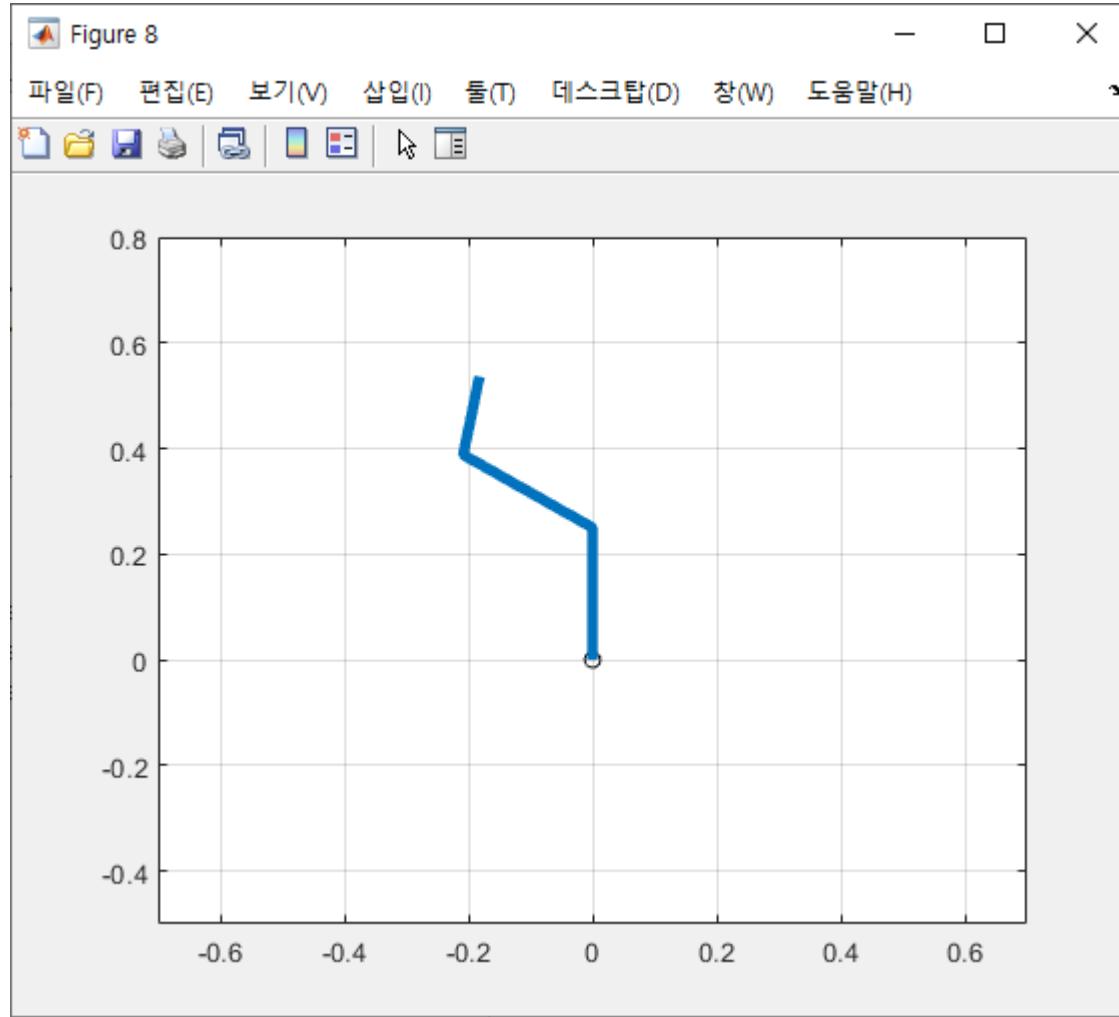
3 End-effector 경로 계획 및 궤적 생성 & 각 관절 변수 궤적



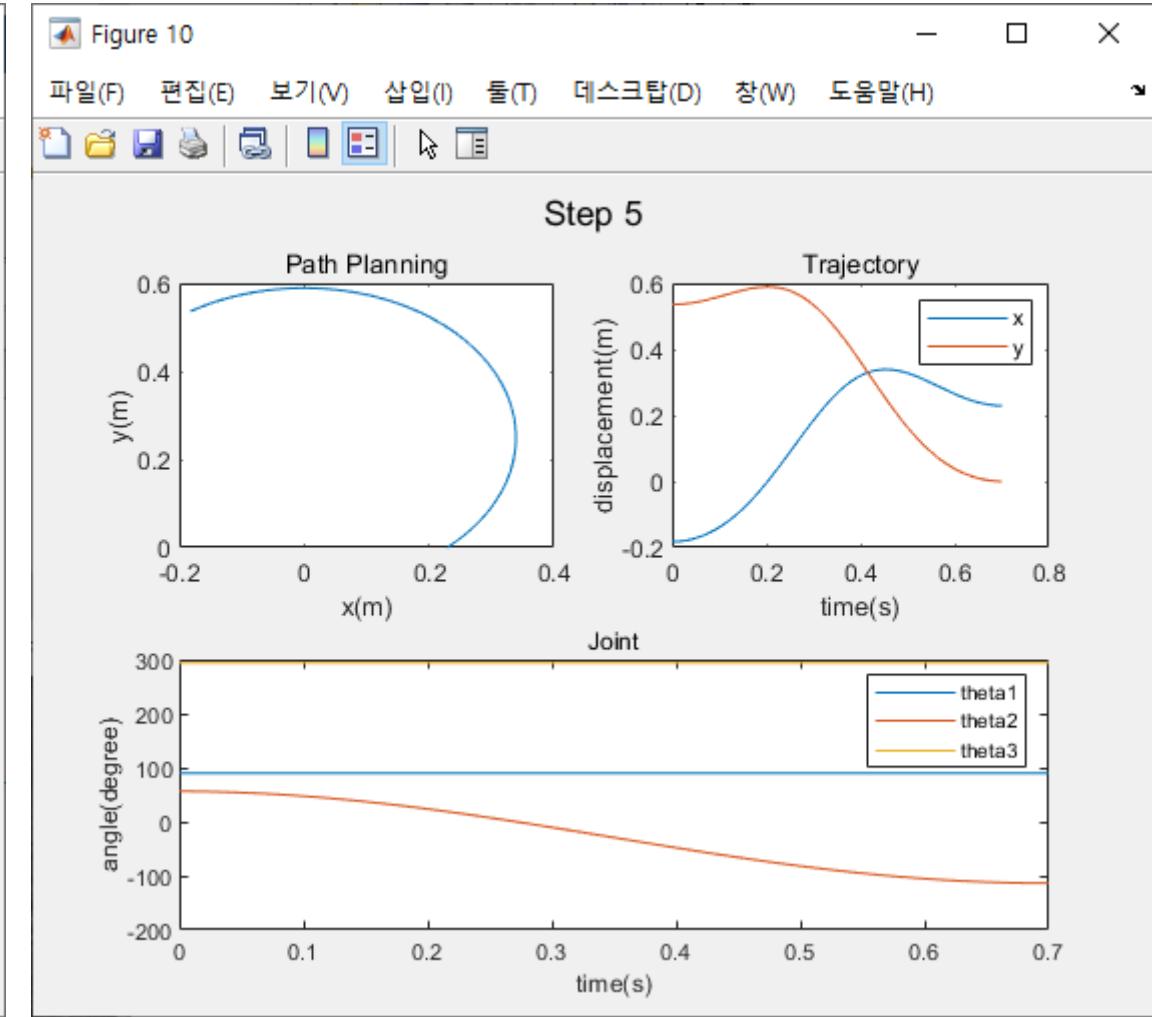
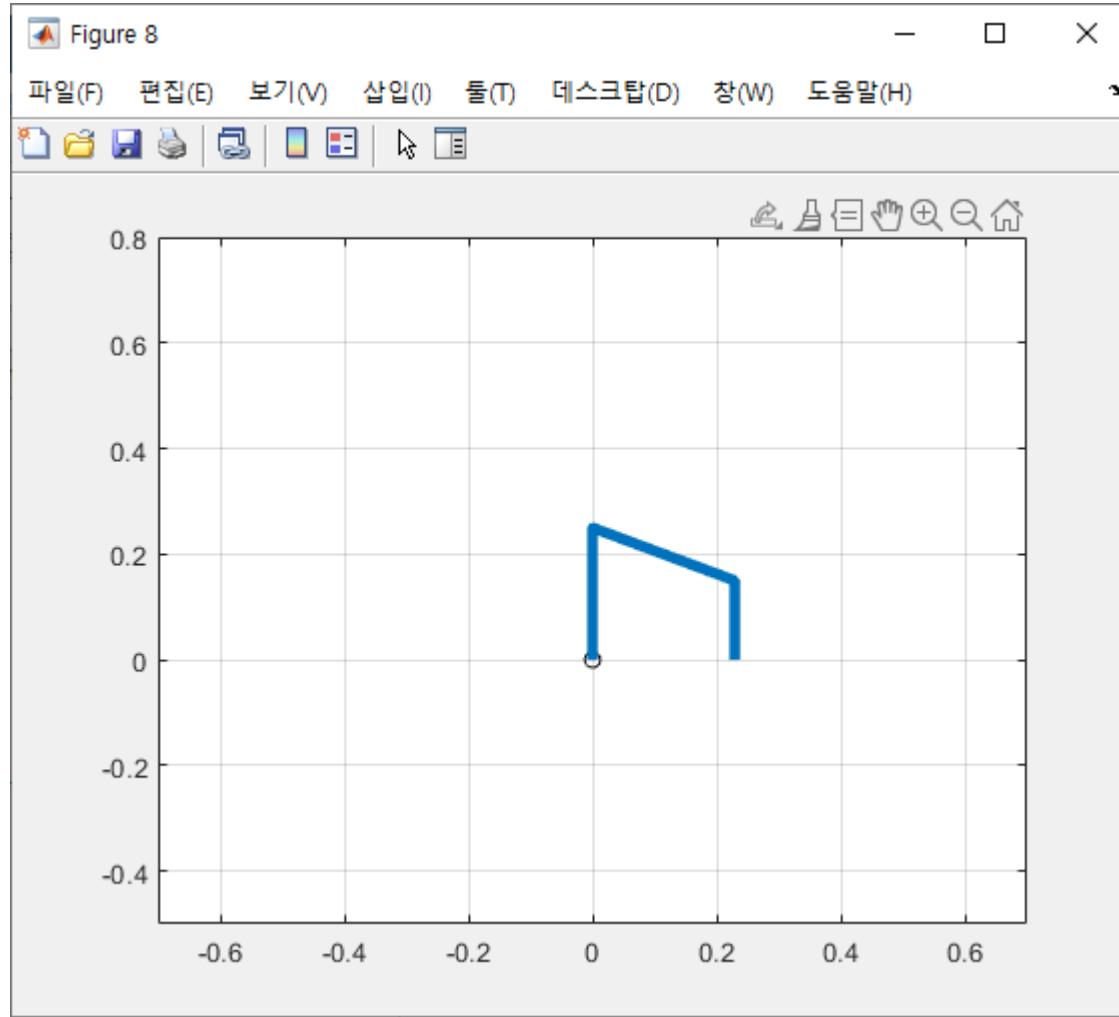
3 End-effector 경로 계획 및 궤적 생성 & 각 관절 변수 궤적



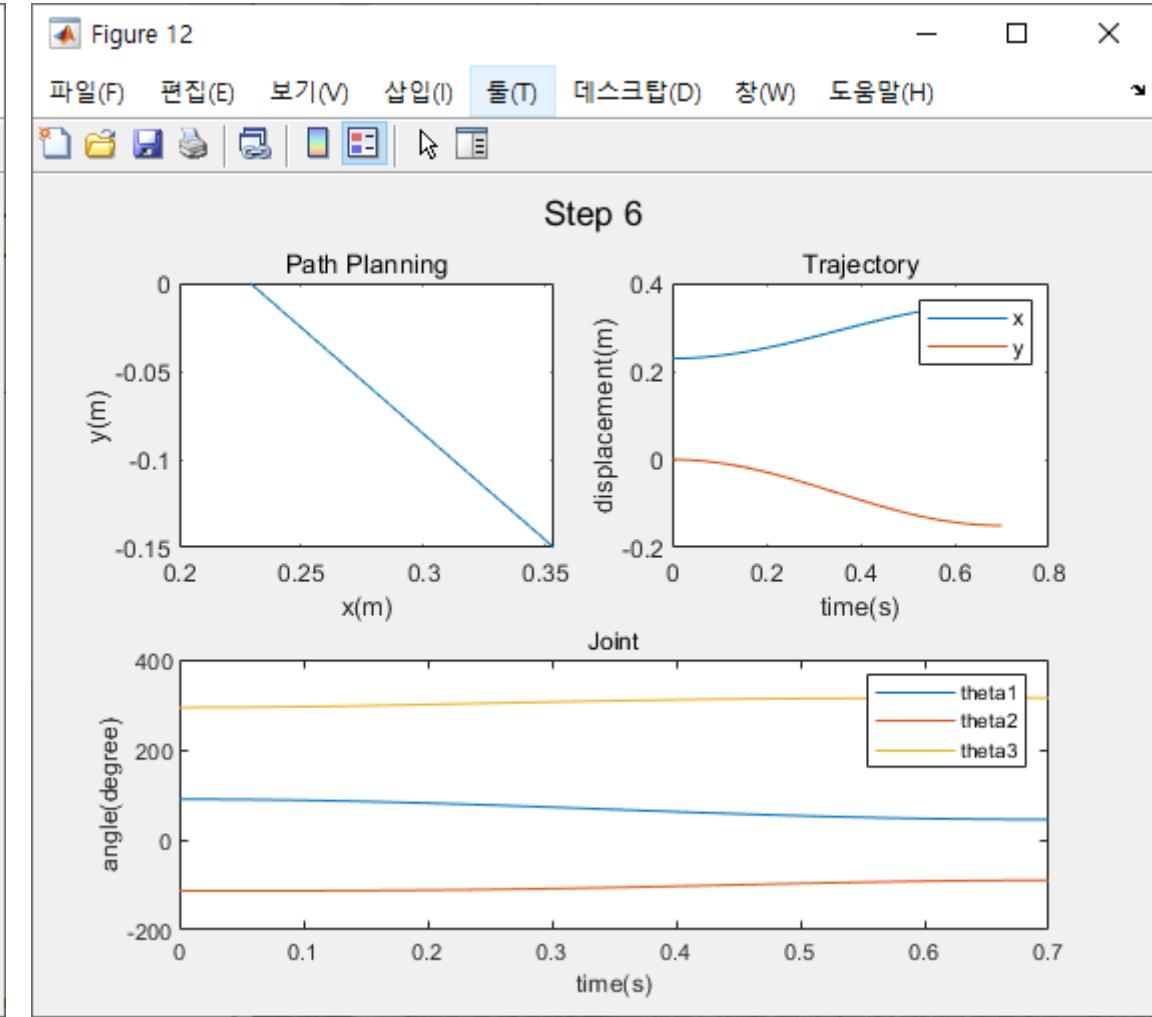
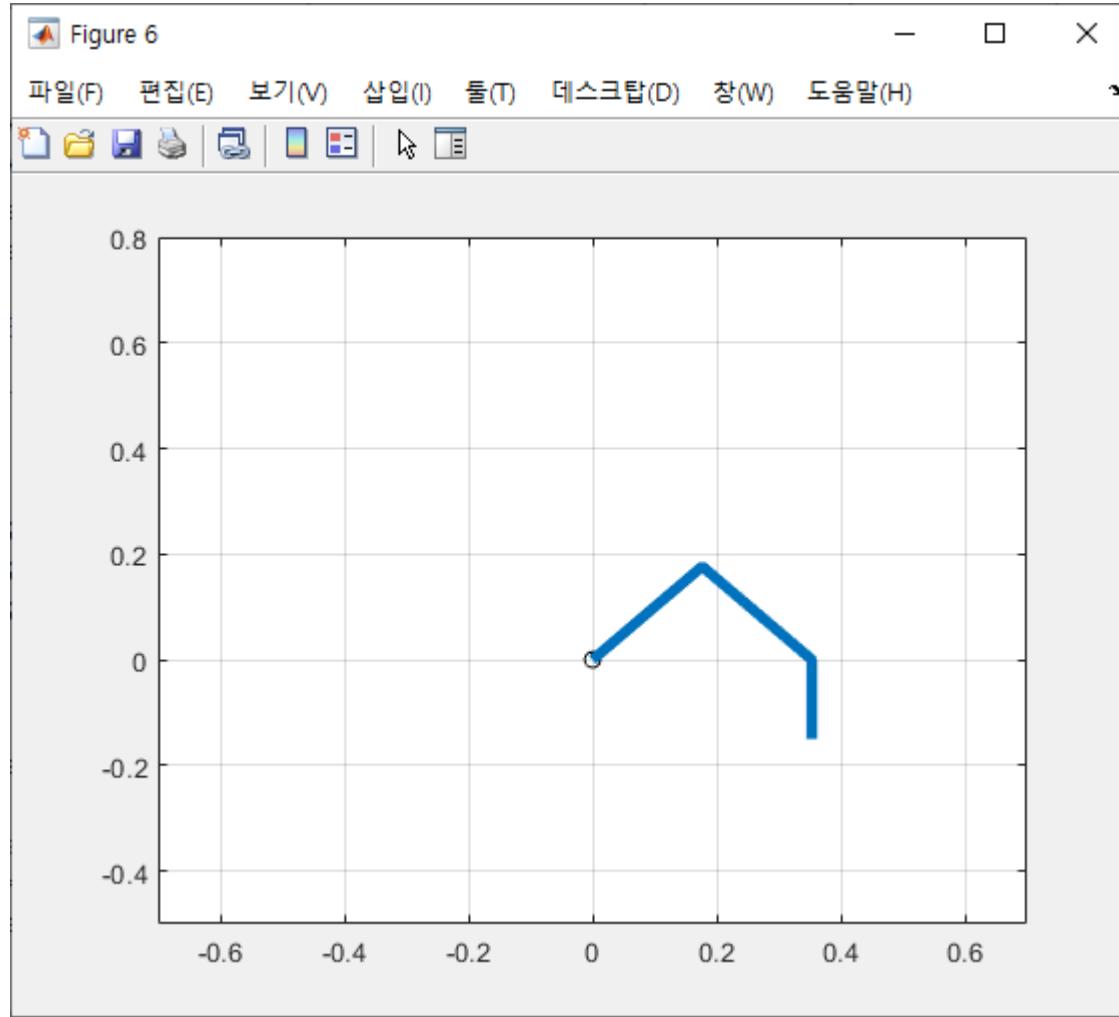
3 End-effector 경로 계획 및 궤적 생성 & 각 관절 변수 궤적



3 End-effector 경로 계획 및 궤적 생성 & 각 관절 변수 궤적



3 End-effector 경로 계획 및 궤적 생성 & 각 관절 변수 궤적



4 Euler-Lagrangian 방법에 의한 운동방정식 제시

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} A_{112}\dot{\theta}_1\dot{\theta}_2 + A_{123}\dot{\theta}_2\dot{\theta}_3 + A_{113}\dot{\theta}_1\dot{\theta}_3 + A_{133}\dot{\theta}_3^2 + A_{122}\dot{\theta}_2^2 \\ A_{211}\dot{\theta}_1^2 + A_{233}\dot{\theta}_3^2 + A_{212}\dot{\theta}_1\dot{\theta}_2 + A_{233}\dot{\theta}_2\dot{\theta}_3 + A_{213}\dot{\theta}_1\dot{\theta}_3 \\ A_{311}\dot{\theta}_1^2 + A_{322}\dot{\theta}_3^2 + A_{312}\dot{\theta}_1\dot{\theta}_2 + A_{323}\dot{\theta}_2\dot{\theta}_3 + A_{313}\dot{\theta}_1\dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}$$

```
T(1:3,i) = M*[theta_1_ddot(i) theta_2_ddot(i)  
theta_3_ddot(i)]' + C + G;  
P(1:3,i) = abs([theta_1_dot(i) theta_2_dot(i)  
theta_3_dot(i)]'.*T(1:3,i));
```

4 Euler-Lagrangian 방법에 의한 운동방정식 제시

```
C1 = cos(theta_1(i));  
C2 = cos(theta_2(i));  
C3 = cos(theta_3(i));  
C12 = cos(theta_1(i)+theta_2(i));  
C13 = cos(theta_1(i)+theta_3(i));  
C23 = cos(theta_2(i)+theta_3(i));  
C123 =  
cos(theta_1(i)+theta_2(i)+theta_3(i));  
S1 = sin(theta_1(i));  
S2 = sin(theta_2(i));  
S3 = sin(theta_3(i));  
S12 = sin(theta_1(i)+theta_2(i));  
S13 = sin(theta_1(i)+theta_3(i));  
S23 = sin(theta_2(i)+theta_3(i));
```

```
% Inertia Matrix  
M11 = m1*lc1^2 + l1 + m2*a1^2 + m2*lc2^2 +  
2*m2*a1*lc2*C2 + l2 + m3*a1^2 + m3*a2^2 + m3*lc3^2 +  
m3*a1*a2*2*C2 + m3*a2*lc3*2*C3 + m3*a1*lc3*2*C23 + l3;  
M12 = m2*lc2^2 + m2*a1*lc2*C2 + l2 + m3*a2^2 + m3*lc3^2  
+ m3*a1*a2*C2 + m3*a2*lc3*2*C3 + m3*a1*lc3*C23 + l3;  
M13 = m3*lc3^2 + m3*a2*lc3*C3 + m3*a1*lc3*C23 + l3;  
M21 = m2*lc2^2 + l3 + m2*a1*lc2*C2 + l2 + m3*a2^2 +  
m3*lc3^2 + m3*a1*a2*C2 + 2*m3*a2*lc3*C3 +  
m3*a1*lc3*C23;  
M22 = m2*lc2^2 + l3 + l2 + m3*a2^2 + m3*lc3^2 +  
2*m3*a2*lc3*C3;  
M23 = m3*lc3^2 + m3*a2*lc3*C3 + l3;  
M31 = m3*lc3^2 + m3*a2*lc3*C3 + m3*a1*lc3*C23 + l3;  
M32 = m3*lc3^2 + m3*a2*lc3*C3 + l3;  
M33 = m3*lc3^2 + l3;
```

4 Euler-Lagrangian 방법에 의한 운동방정식 제시

% Centrifugal and Coloilis Matrix

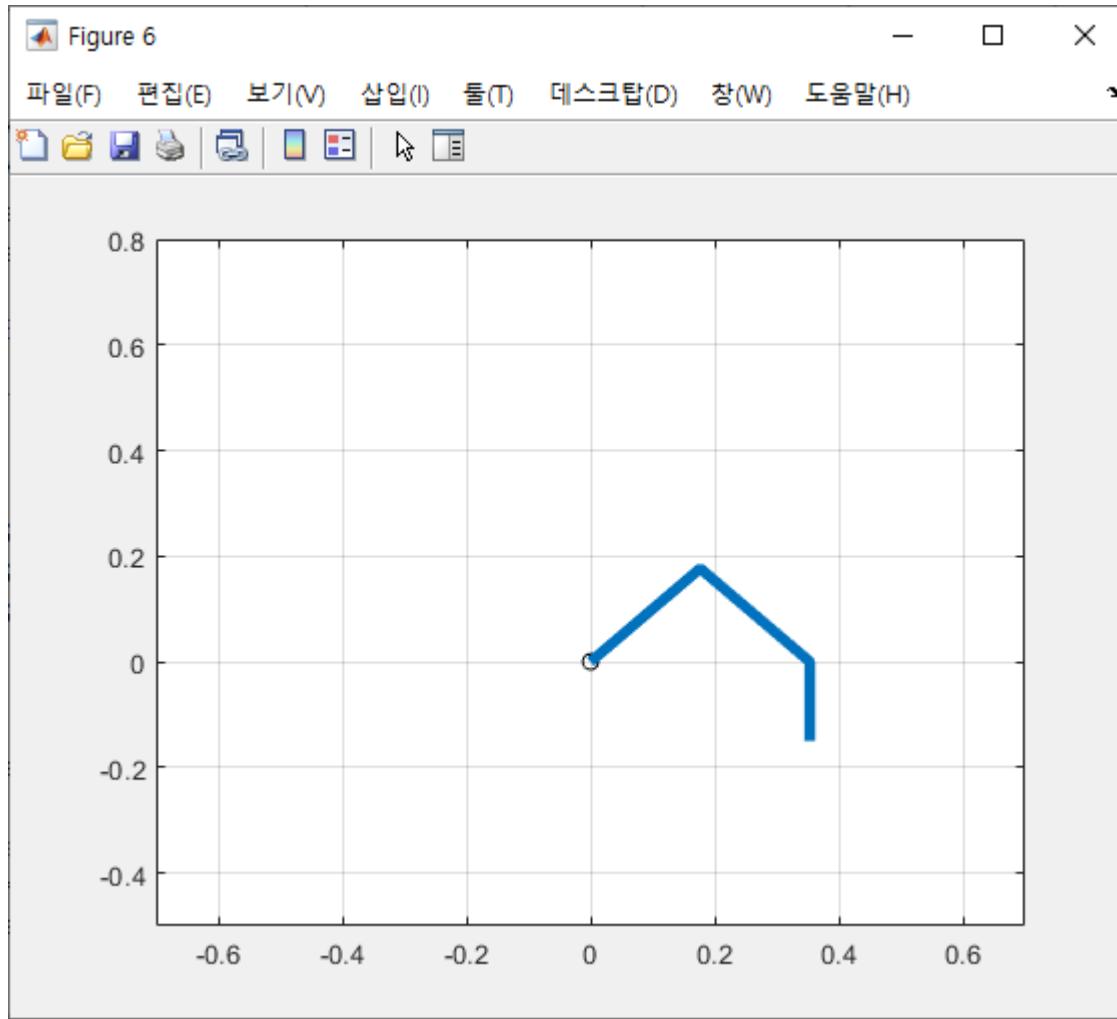
```
A112 = -m2*a1*lc2*2*S2 - m3*a1*a2*2*S2 -  
2*m3*a1*lc3*S23;  
A123 = -m3*a2*lc3*2*S3 - 2*m3*a1*lc3*S23;  
A113 = -2*m3*a2*lc3*S3 - 2*m3*a1*lc3*S23;  
A122 = -m2*a1*lc3*S3 - m3*a1*a2*S2 - m3*a1*lc3*S23;  
A133 = -m3*a1*lc3*S23 - m3*a2*lc3*S3;  
A211 = m2*a1*lc2*S2 + m3*a1*a2*S2 + m3*a1*lc3*S23;  
A233 = -m3*a2*lc3*S3;  
A212 = 0;  
A223 = -m3*a2*lc3*2*S3;  
A213 = -2*m3*a2*lc3*S3;  
A311 = m3*a2*lc3*S3 + m3*a1*lc3*S23;  
A322 = m3*a2*lc3*S3;  
A312 = 2*m3*a2*lc3*S3;  
A323 = 0;  
A313 = 0;
```

```
dot12 = theta_1_dot(i)*theta_2_dot(i);  
dot13 = theta_1_dot(i)*theta_3_dot(i);  
dot23 = theta_2_dot(i)*theta_3_dot(i);  
dot1sq = (theta_1_dot(i))^2;  
dot2sq = (theta_2_dot(i))^2;  
dot3sq = (theta_3_dot(i))^2;
```

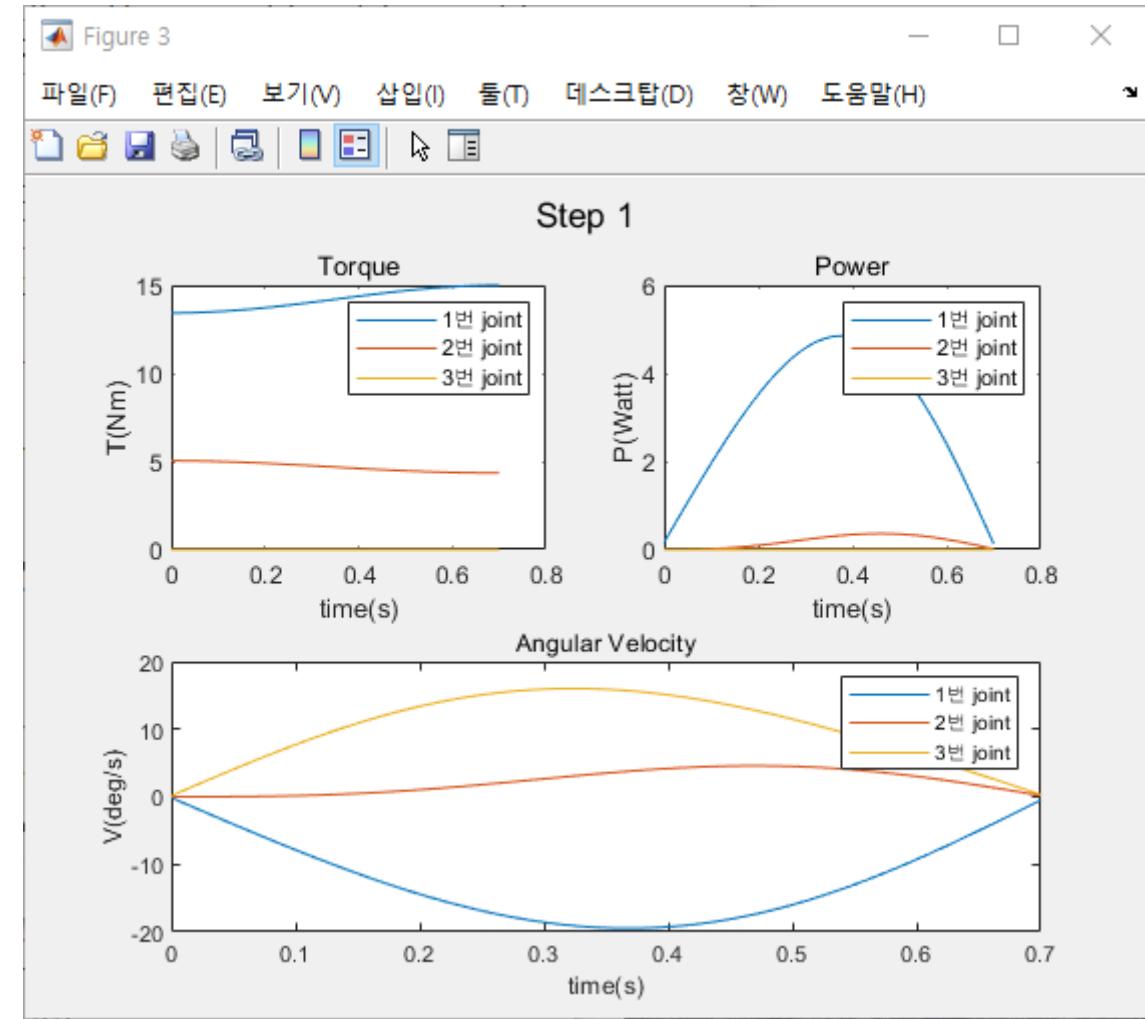
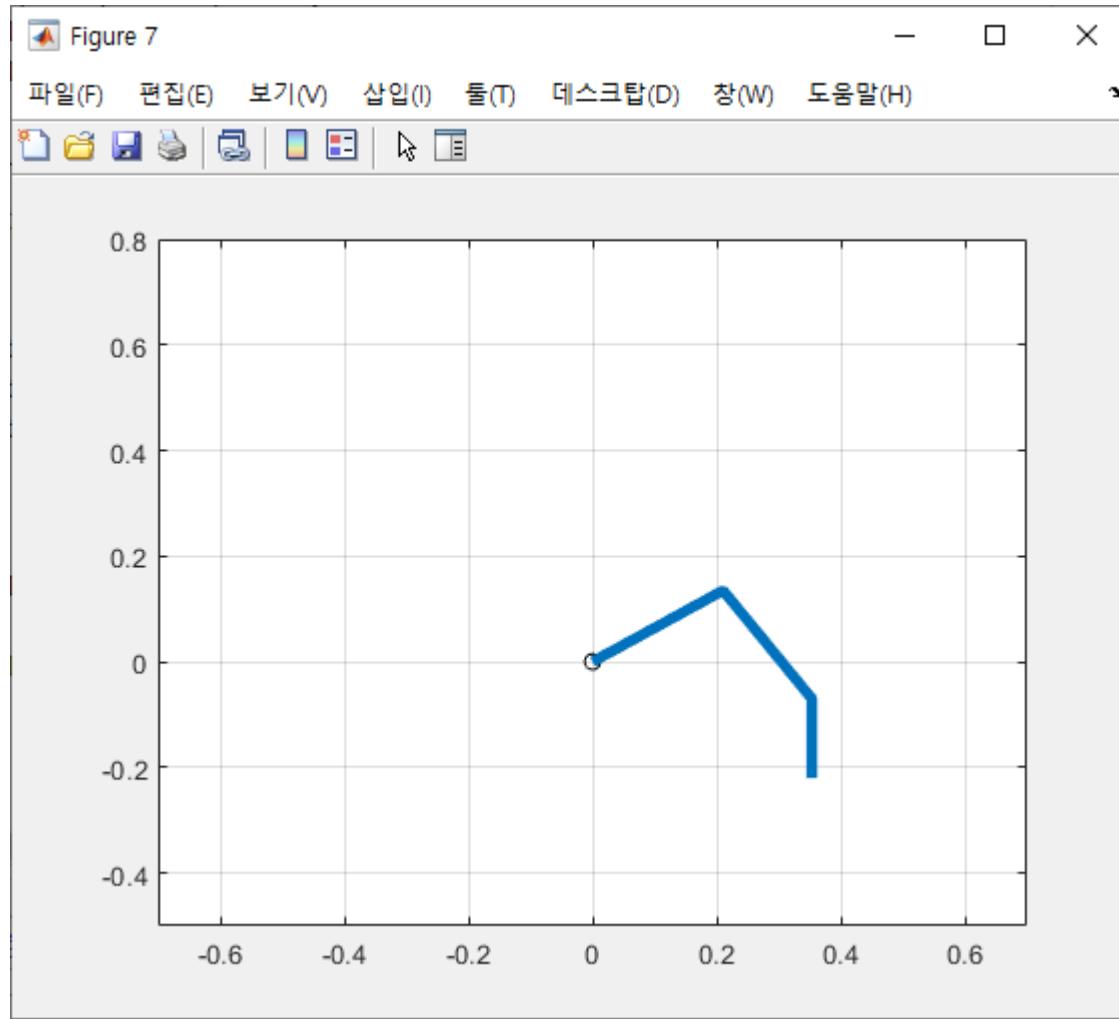
% Gravity

```
G1 = m1*g*lc1*C1 + m2*g*(a1*C1+lc2*C12) + m3*g*(a1*C1  
+ a2*C12 + lc3*C123);  
G2 = m2*g*lc2*C12 + m3*g*(a2*C12 + lc3*C123);  
G3 = m3*g*lc3*C123;  
M = [M11,M12,M13;M21,M22,M23;M31,M32,M33];  
C =  
[A112*dot12+A123*dot23+A113*dot13+A133*dot3sq+A122*  
dot2sq;A211*dot1sq+A233*dot3sq^2+A212*dot12+A223*dot  
23+A213*dot13;A311*dot1sq+A322*dot2sq+A312*dot12+A3  
23*dot23+A313*dot13];  
G = [G1; G2; G3];
```

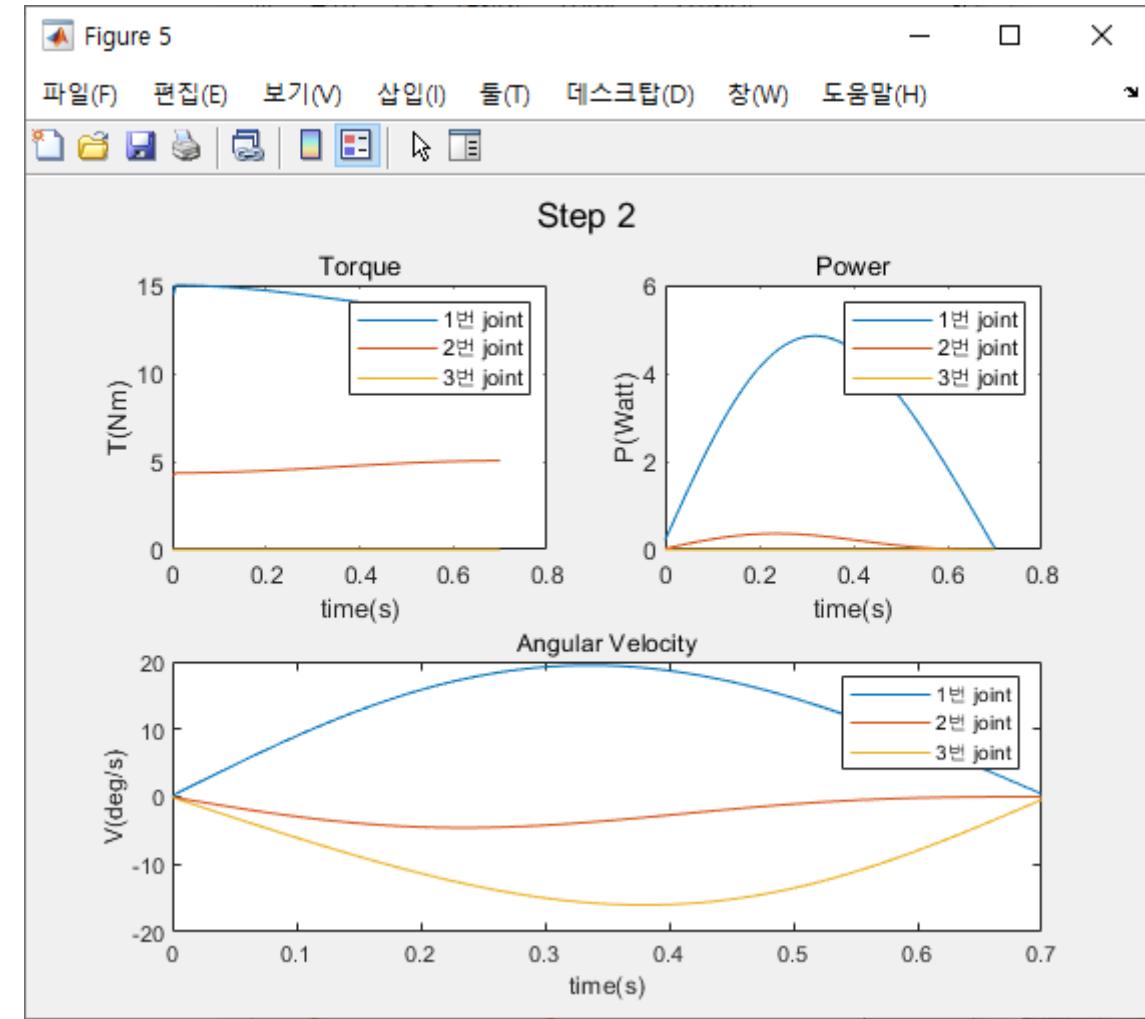
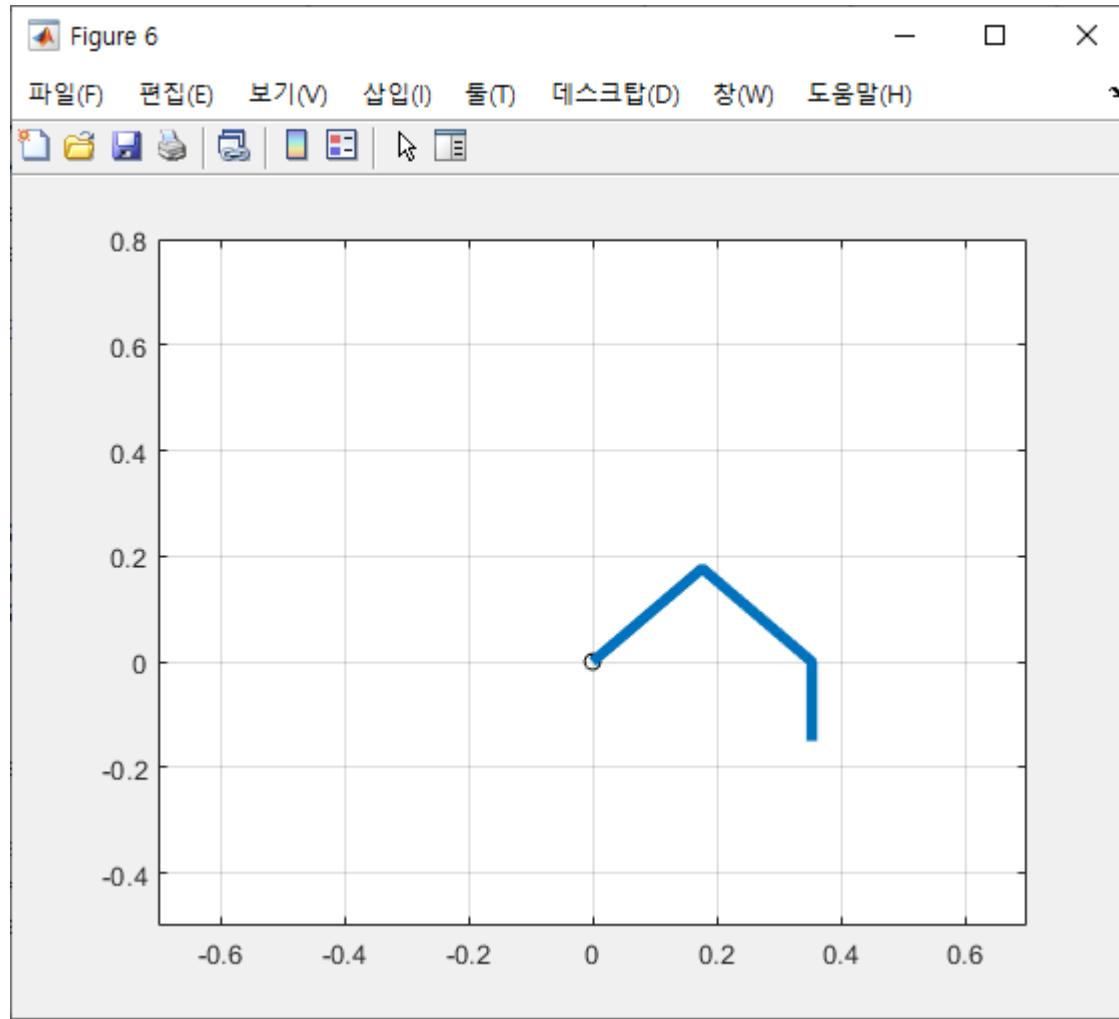
4 힘, 속도 및 절대 파워 그래프



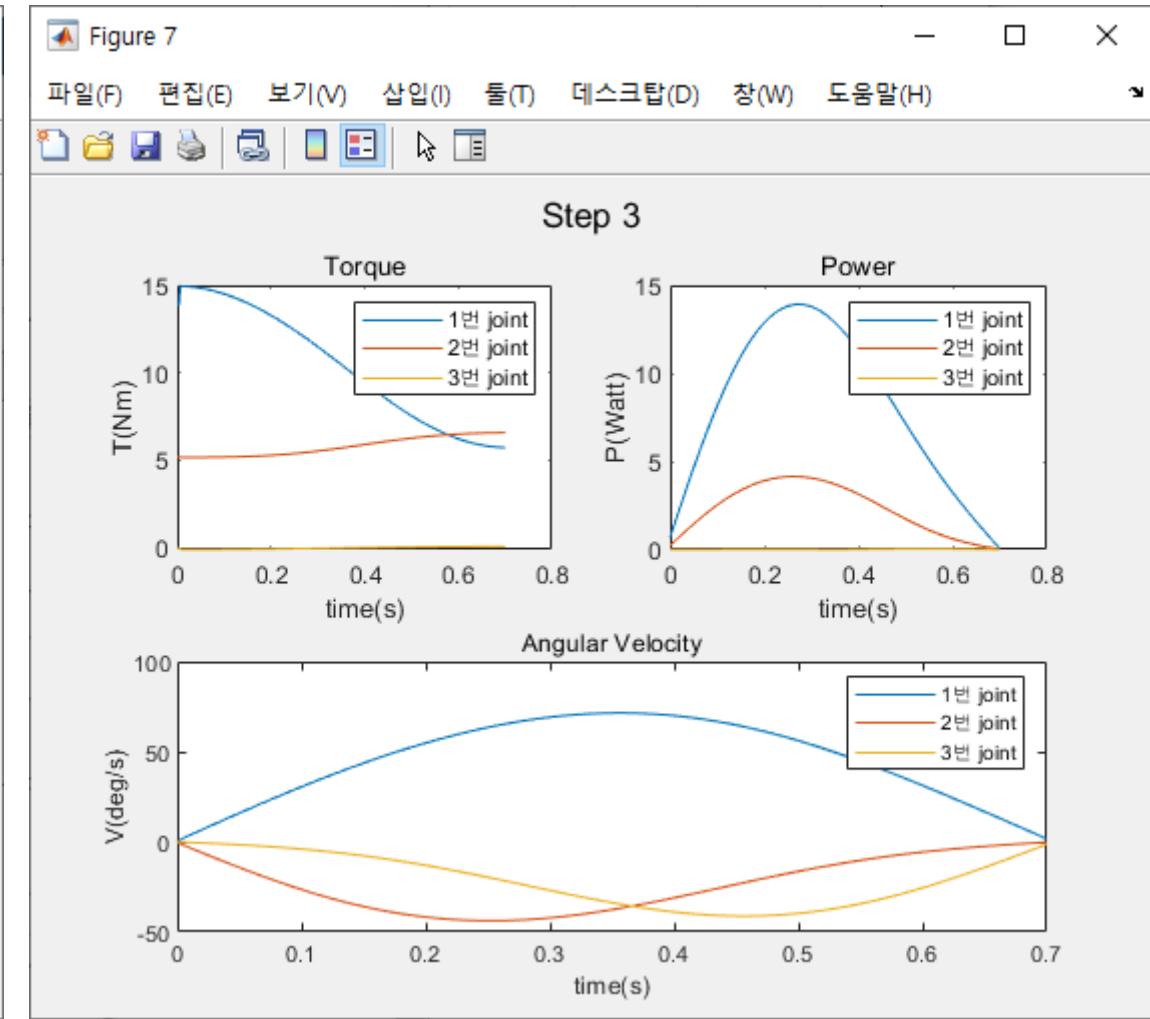
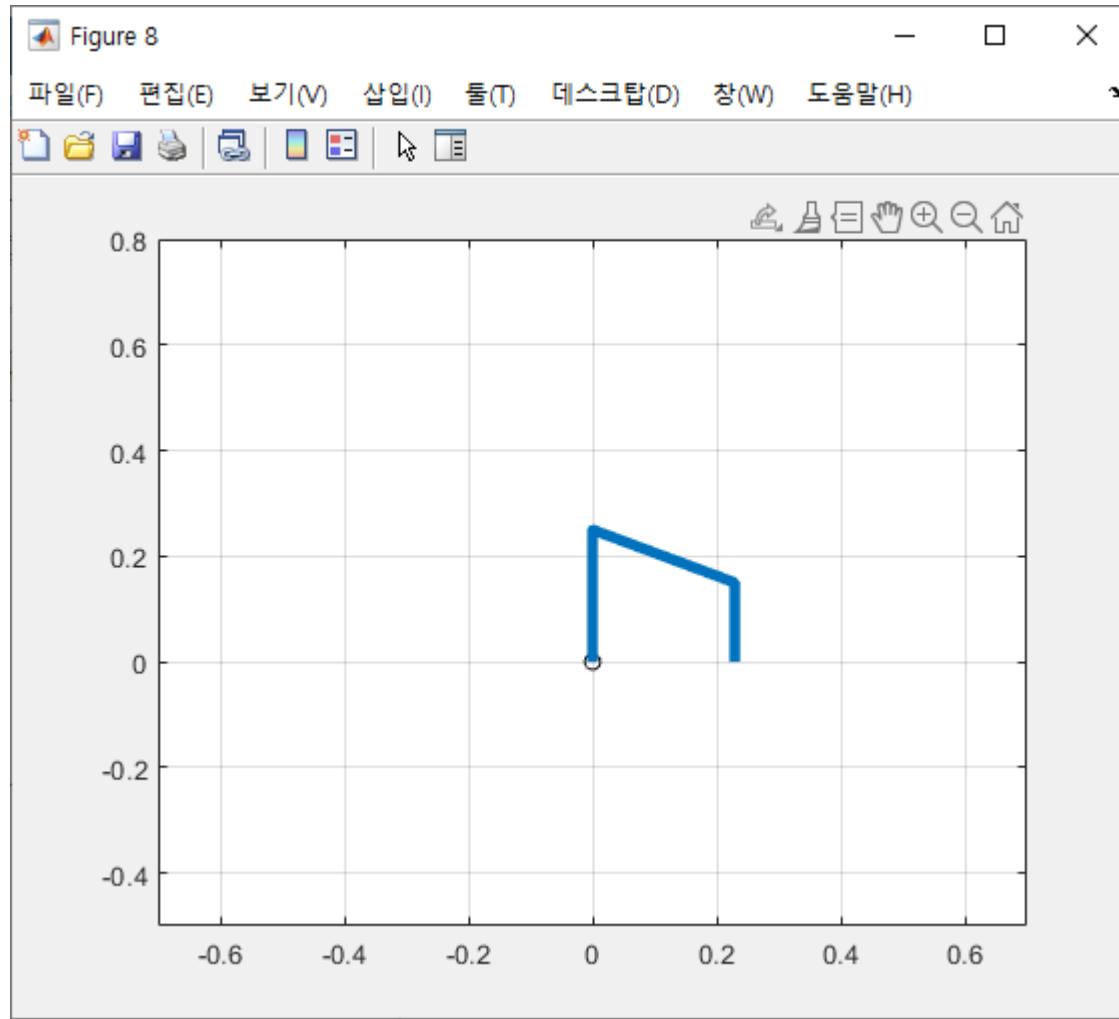
4 힘, 속도 및 절대 파워 그래프



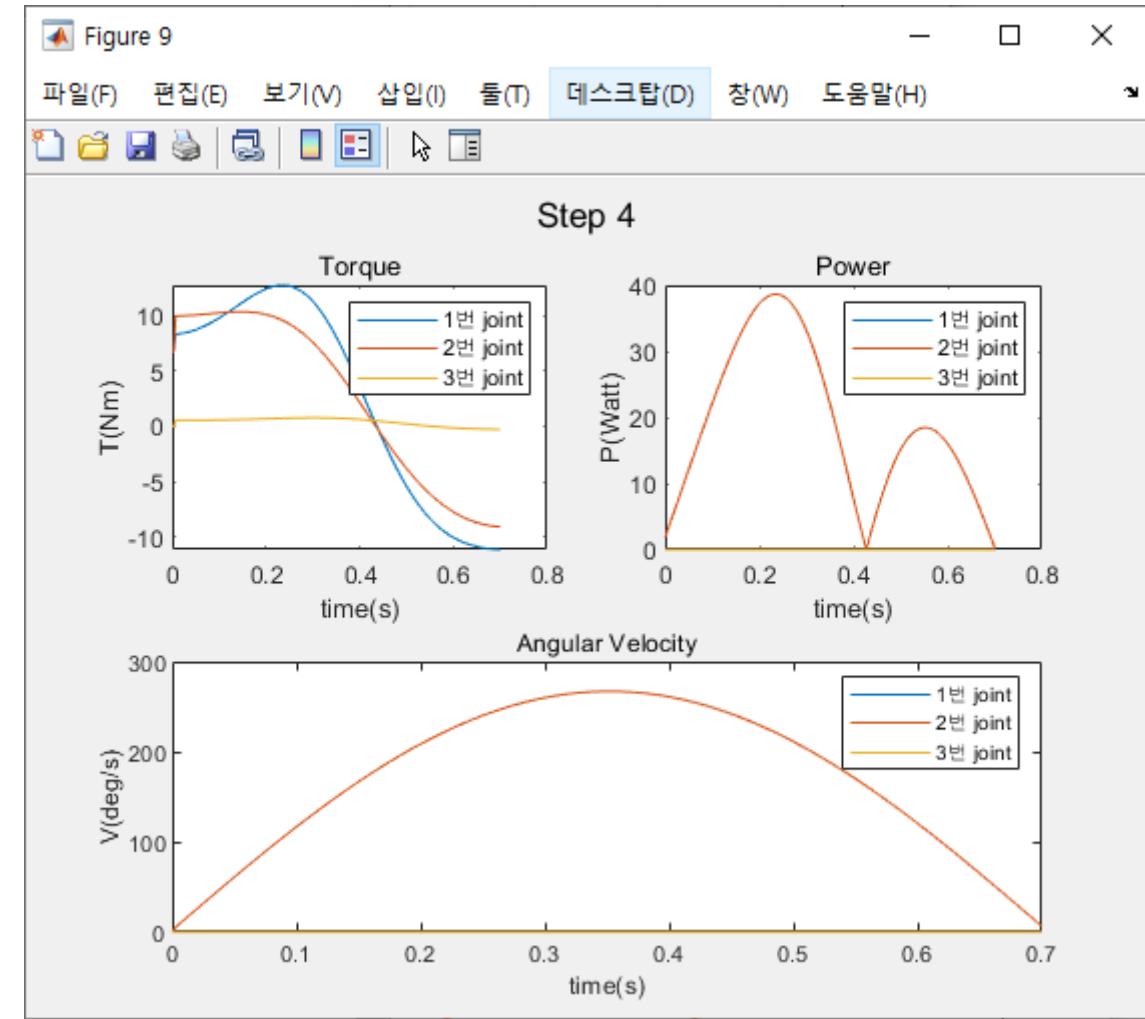
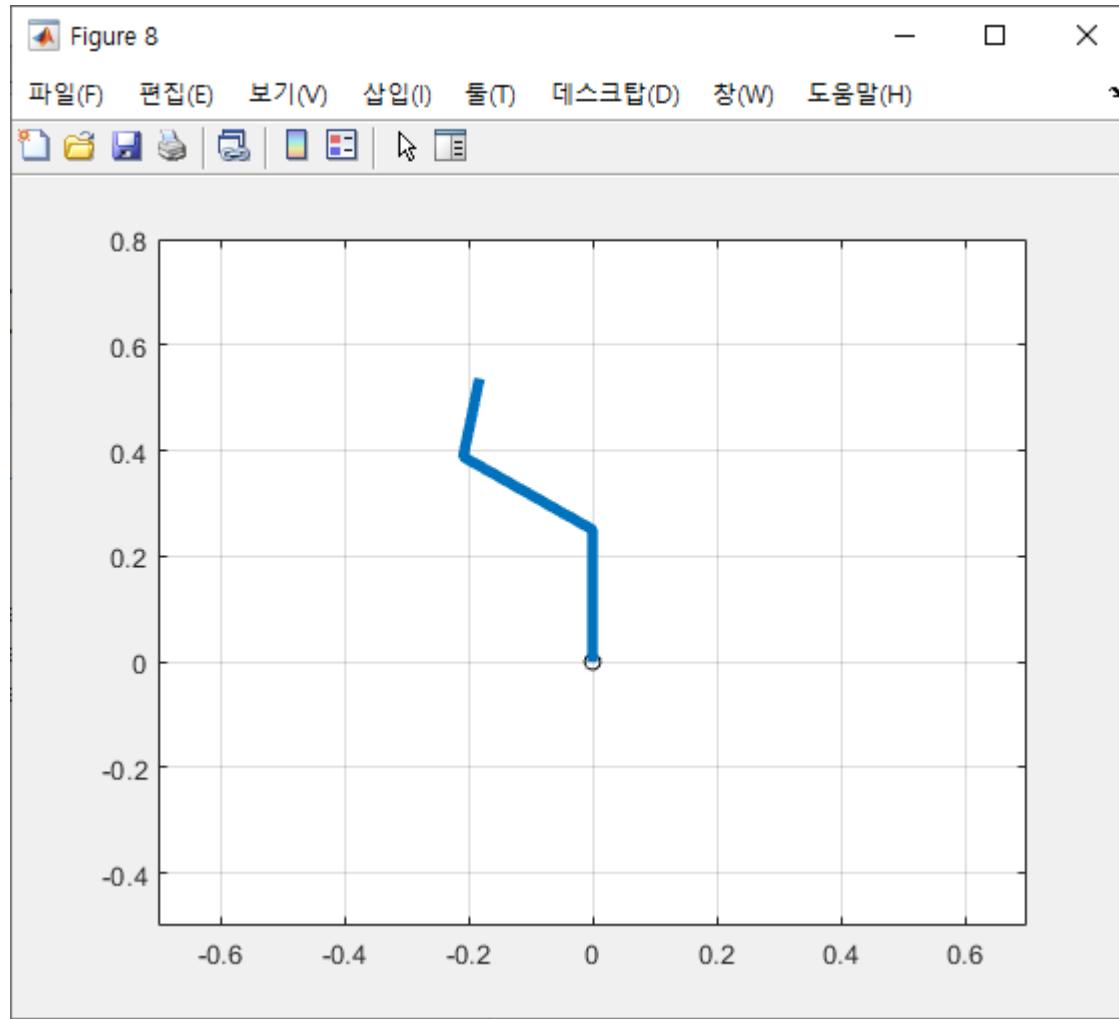
4 힘, 속도 및 절대 파워 그래프



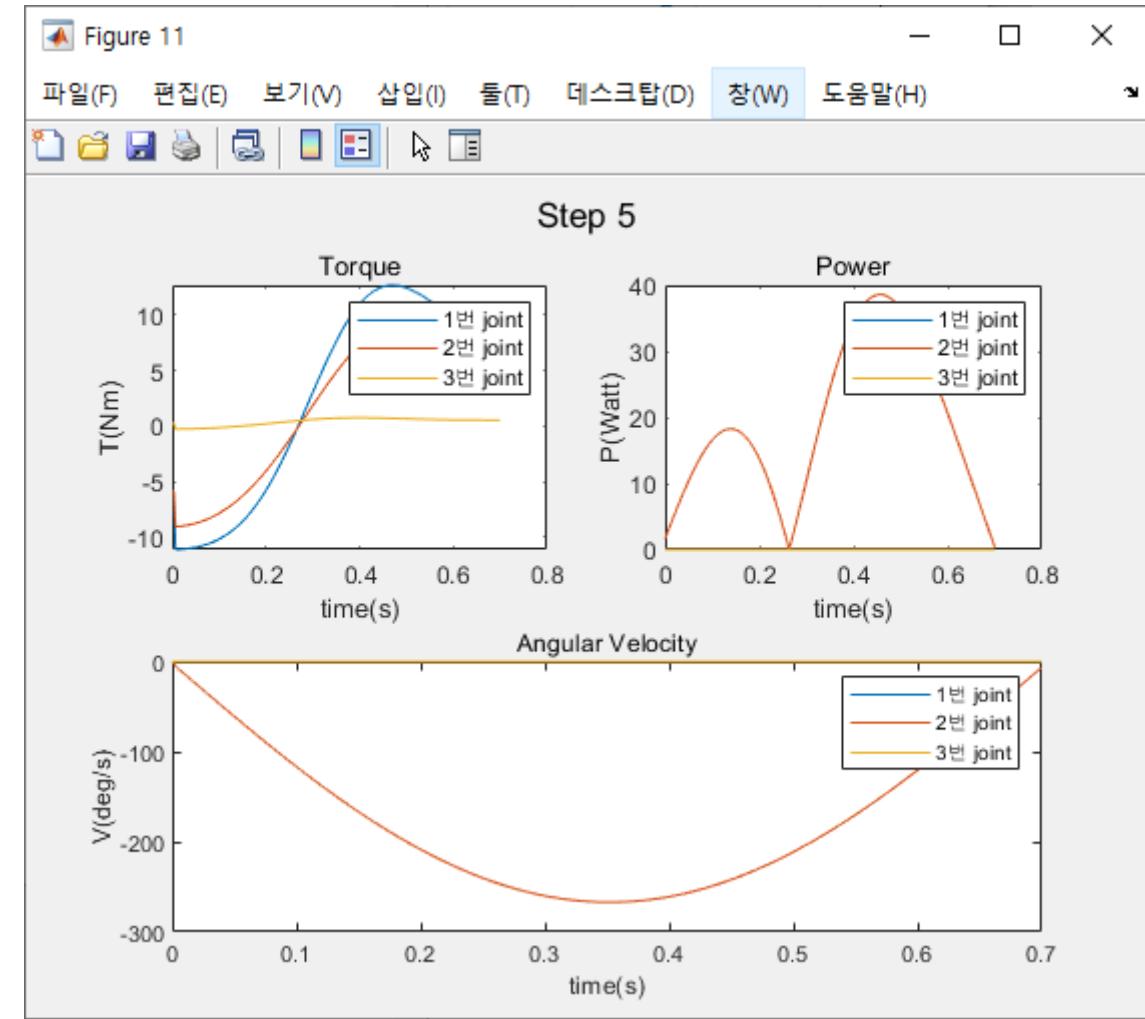
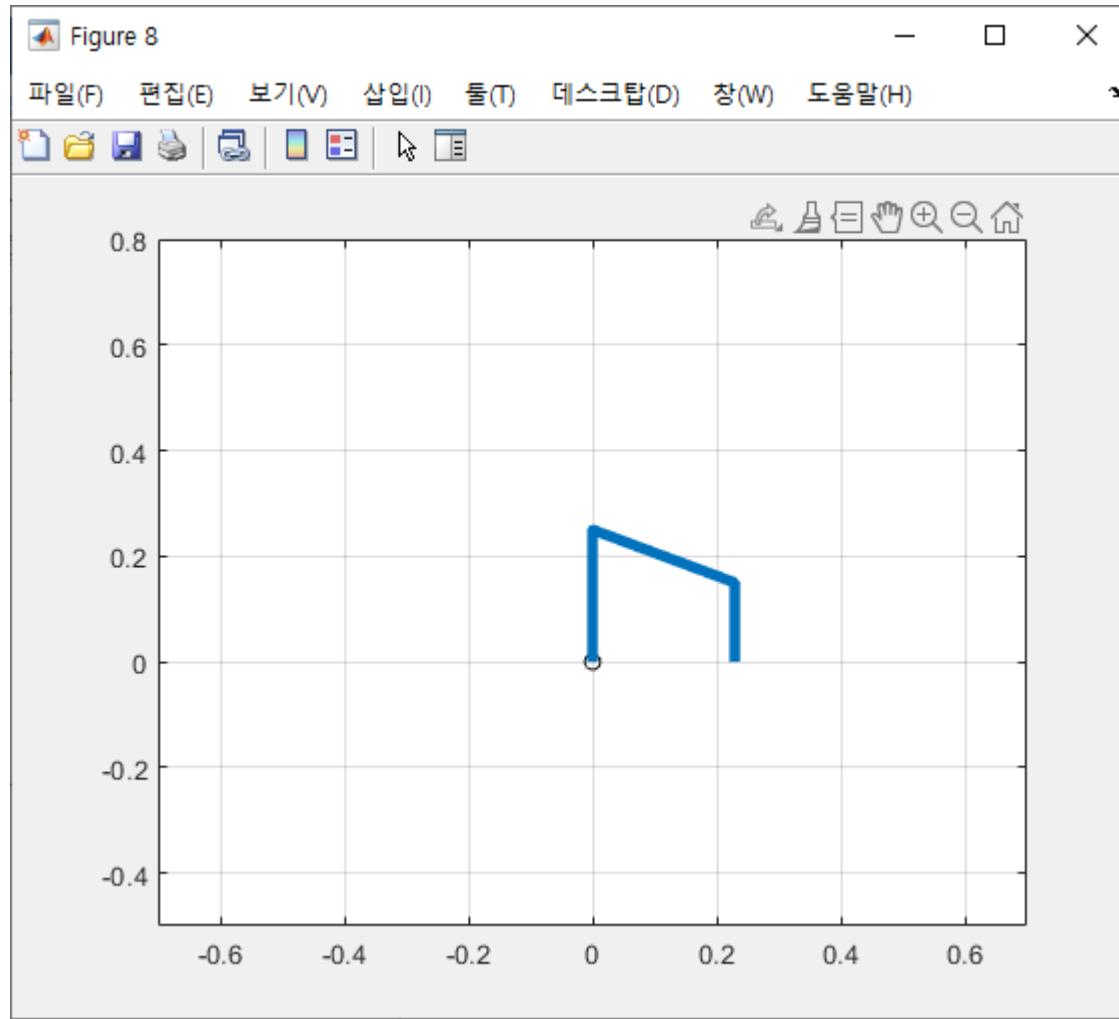
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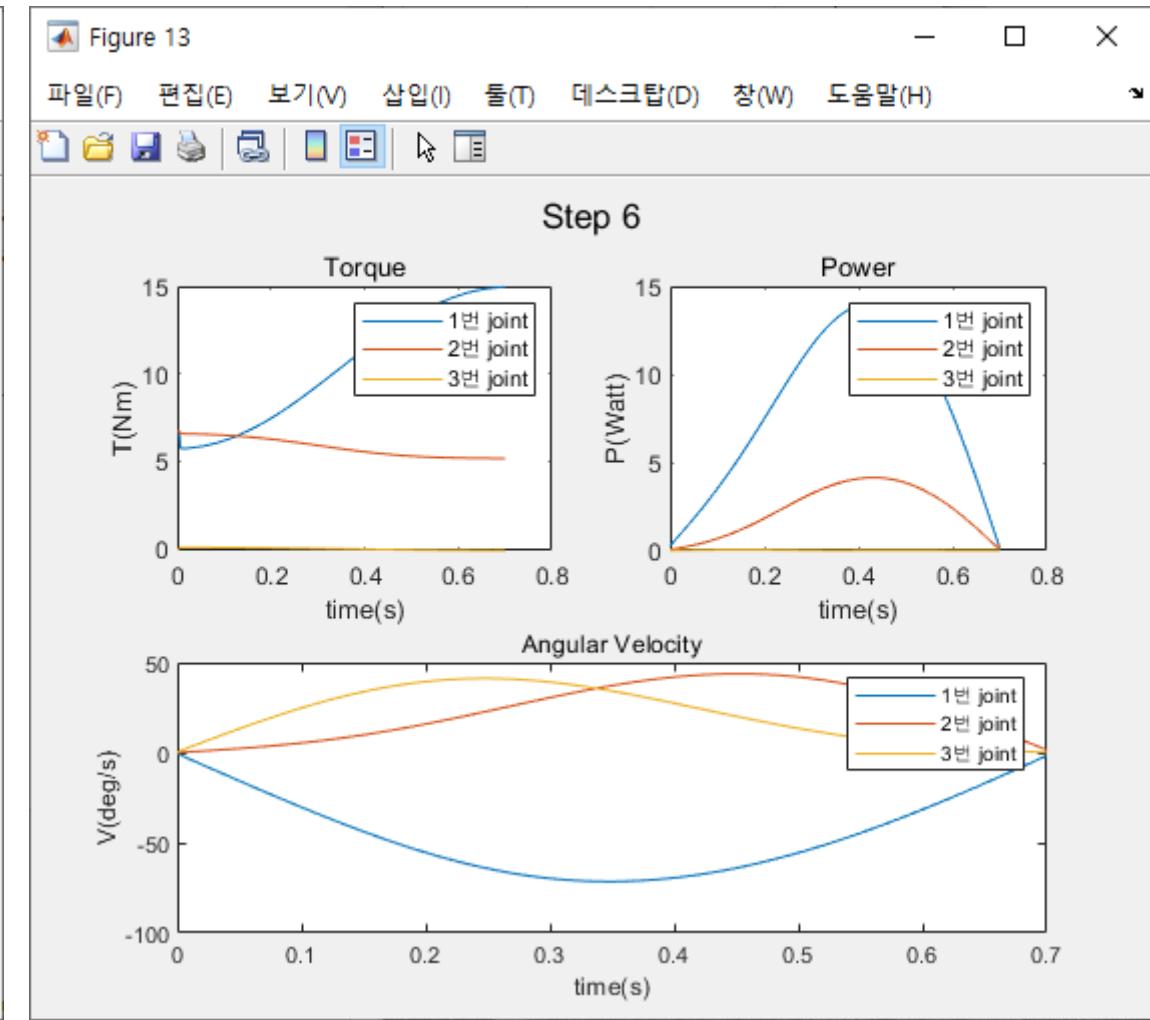
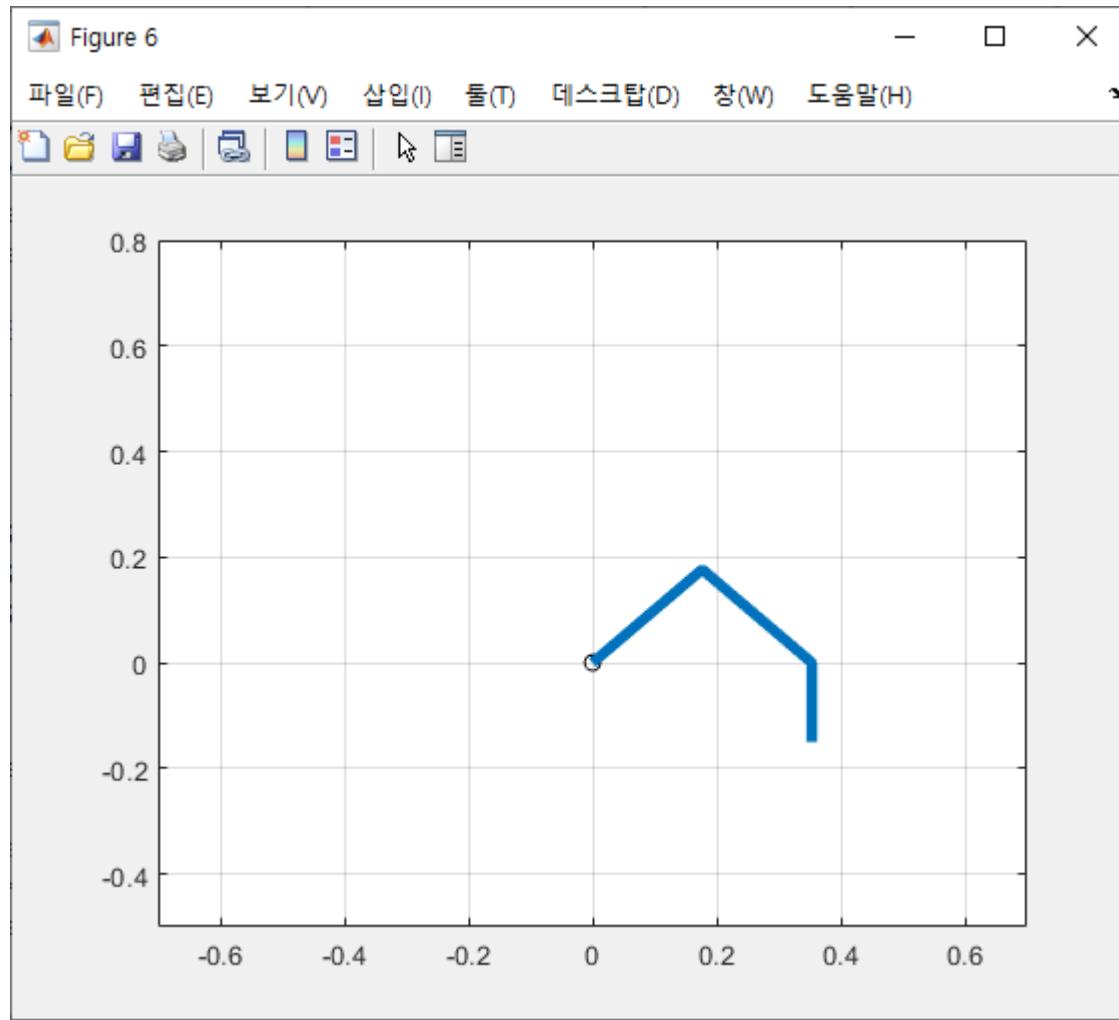
4 힘, 속도 및 절대 파워 그래프



4 힘, 속도 및 절대 파워 그래프



4 힘, 속도 및 절대 파워 그래프



4 최대 속도, 최대 힘 및 최대 파워

	최대 속도 (rad/s)	최대 토크 (Nm)	최대 파워 (Watt)
Joint 1	1.2475	14.9535	13.9109
Joint 2	4.6604	10.0584	38.6486
Joint 3	0.7191	0.7407	0.0110

지나치게 값이 큰 경우가 있어, 시간에 따른 Trajectory를
재조정할 필요가 있어 보인다.

4 계획표에서의 현재 위치

현재 위치 목표

