

MAU22C00 - TUTORIAL 3

1) (From the 2016-2017 Annual Exam) Let Q denote the relation on the set \mathbb{Z} of integers, where integers x and y satisfy xQy if and only if

$$x - y = (x - y)(x + 2y).$$

Determine the following:

- (i) Whether or not the relation Q is *anti-symmetric*;
- (ii) Whether or not the relation Q is a *partial order*.

Justify your answers.

2) Let A be a set, and let $\mathcal{A} = \{A_\alpha \mid \alpha \in I\}$, where I is an indexing set, be any partition of the set A . Define a relation R on A as follows: $x, y \in A$ satisfy xRy iff $x, y \in A_\alpha$ for some $\alpha \in I$. In other words, xRy iff x and y belong to the same set of the partition. Prove that R is an equivalence relation and that the partition R defines on A is precisely the given partition \mathcal{A} .

(Hint: Recall we discussed in lecture the one-to-one correspondence between partitions and equivalence relations, and this is the proof direction I sketched in lecture without providing the details.)