CSU1001 Linear Algebra Formulae

- The cross product of two vectors in $\mathbb{R}^3$ is defined by

$$\mathbf{x} \times \mathbf{y} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} x_2y_3 - x_3y_2 \\ x_3y_1 - x_1y_3 \\ x_1y_2 - x_2y_1 \end{pmatrix}.$$

  where $\mathbf{x}$ and $\mathbf{y}$ are elements in $\mathbb{R}^3$.

- Given a point P (with coordinates (p_1, p_2, p_3)) in a plane and a vector $\mathbf{n}$ in $\mathbb{R}^3$ that is normal to the plane, the implicit equation of the plane is

$$n_1(x - p_1) + n_2(y - p_2) + n_3(z - p_3) = 0.$$

- Given a point P with coordinates (p_1, p_2, p_3) in a plane and two vectors $\mathbf{u}$ and $\mathbf{v}$ that lie in the plane, the parametric equation of the plane is

$$\begin{aligned} x &= p_1 + u_1 s + v_1 t \\ y &= p_2 + u_2 s + v_2 t \\ z &= p_3 + u_3 s + v_3 t \end{aligned}$$

  where $-\infty < s < \infty$ and $-\infty < t < \infty$.

- Given a point P with coordinates (p_1, p_2, p_3) on a line and a vector $\mathbf{u}$ that is parallel to the line, the implicit equation of the line is given by the symmetric equations

$$\frac{x - p_1}{u_1} = \frac{y - p_2}{u_2} = \frac{z - p_3}{u_3}$$

- Given a point P with coordinates (p_1, p_2, p_3) on a line and a vector $\mathbf{u}$ that is parallel to the line, the parametric equation of the line is

$$\begin{aligned} x &= p_1 + u_1 t \\ y &= p_2 + u_2 t \\ z &= p_3 + u_3 t \end{aligned}$$

  where $-\infty < t < \infty$.

- Orthogonal projection of $\mathbf{u}$ on $\mathbf{a}$:

$$\mathrm{proj}_{\mathbf{a}}\mathbf{u} = \frac{\mathbf{u}.\mathbf{a}}{||\mathbf{a}||^2} \; \mathbf{a}$$

  Vector component of $\mathbf{u}$ orthogonal to $\mathbf{a}$:

$$\mathbf{u} - \mathrm{proj}_{\mathbf{a}}\mathbf{u} = \mathbf{u} - \frac{\mathbf{u}.\mathbf{a}}{||\mathbf{a}||^2} \; \mathbf{a}$$