STU22005 Applied Probability II
Professor Caroline Brophy
Continuous Assessment Sheet 1

Due date: submit before 4pm Friday 11th February 2022.

For this assignment, you must do and submit question 1. All other questions should also be done for practice and preparation for the live quiz during the week following submission, but are not handed up.

- Question 1 is to be done on your own and not in consultation with anyone else, do not discuss or show your answers to anyone else and do not look at anyone else's answers. You are welcome to discuss your answers to the additional questions with other students.

To submit the assignment, you will submit one single pdf file:

- File to be submitted: Fill in the answer sheet that accompanies this assignment. Fill in the answers on the pdf document, save the file, and upload to Blackboard.

1. Ten measurements of the concentration of nitrate ions (in μg/mL) in a specimen of water were recorded as follows:

   0.513 0.524 0.529 0.481 0.492 0.499 0.518 0.490 0.494 0.501

   (a) Construct a 99% confidence interval for the true mean. State the critical value and give the confidence interval (correct to two decimal places).

   (b) In one sentence, interpret the confidence interval.

   (c) In one sentence, explain the distributional assumption required for validity of the confidence interval. In a second sentence, explain how the distributional assumption could be checked. (Note you do not need to check it here, just explain how it could be checked.)

Below here are additional questions that do not need to be handed up.

2. For the nitrate ions example above:

   (a) Create an appropriate graph (by hand) to check the assumption given in part (c).

   (b) Describe what the sampling distribution of the mean is in the context of this example.

3. Carry out the following hypothesis tests assuming independent normally distributed data. Find the test statistic, the critical value and state your conclusion. Note that the variance is estimated here, not known. The t-distribution should be used in each case to find the critical values.

   (a) $n = 10$, $\bar{y} = 124$, $s = 10$, $H_0 : \mu = 110, H_A : \mu \neq 110$, $\alpha = 0.01$.

   (b) $n = 8$, $\bar{y} = 0.6$, $s = 0.2$, $H_0 : \mu = 0.5, H_A : \mu \neq 0.5$, $\alpha = 0.05$.

   (c) $n = 25$, $\bar{y} = 33.4$, $s = 6.8$, $H_0 : \mu = 30, H_A : \mu > 30$, $\alpha = 0.1$.

4. We showed in class that if $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed (IID) as $\mathrm{N}(\mu, \sigma^2)$ that

$$E(\bar{Y}) = \mu \qquad \text{and} \qquad Var(\bar{Y}) = \frac{\sigma^2}{n}$$

   Verify that this is true even if the Y_i are not normally distributed, so long as they are IID with $E(Y_i) = \mu$ and $Var(Y_i) = \sigma^2$, $i = 1, \ldots, n$.

5. The distribution of scores for an examination are normally distributed with mean $\mu = 55$ and variance $\sigma^2 = 100$ and scores are independent of each other. Any mark in excess of 70 is a first. Let Y_i denote the random variable representing the score of a student selected at random.

   (a) What is the probability a student selected at random gets a first?

   (b) Suppose 10 students are selected at random. What is the probability exactly one of the 10 students will get a first?

   (c) Suppose 10 students are selected at random. What is the probability that their average mark will be above 60?