Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Engineering, Mathematics and Science

School of Computer Science & Statistics

ST2004: Applied Probability I

14 DEC 2021 **9:30-11:30**

Dr James Ng

Instructions to Candidates:

Answer all 4 questions. Questions carry unequal marks.

1. (a) Two six-sided dice are rolled. Let A be the event that sum of two dice equals to 3. Let B be the event that sum of two dice equals to 7. Let C be the event that at least one of the dice shows a 1.

    i. What is the conditional probability $\mathbb{P}(A|C)$? (5 marks)

    ii. What is the conditional probability $\mathbb{P}(B|C)$? (5 marks)

    iii. Are A and C independent? Are B and C independent? (5 marks)

   (b) Suppose a biased coin with probability p of heads is tossed, while a fair coin is tossed at the same time. Let X be the number of tosses until both coins simultaneously show the same outcome.

    i. What is the probability distribution of X? (6 marks)

    ii. Compute the expected value and standard deviation of X? (4 marks)

2. (a) Suppose there are 10 cards face-down numbered 1 through 10. You can pick one card. If the number on the card is at least 5 your payoff is the dollar value on the card, otherwise your payoff is \$0.5.

    i. What is the expected value of your payoff? (6 marks)

    ii. What is the standard deviation of your payoff? (4 marks)

   (b) Consider the following strategy for betting. I start by betting \$1; if I lose, I double my bet to \$2, and if I lose I double my bet again. I continue to double my bet until I win. Suppose further that the probability of winning each game is $p = 0.5$. This strategy results in a guaranteed net win of \$1. Let X be the amount of money bet on the last game (the one I win).

    i. Determine the probability mass function for X. (6 marks)

    ii. Show that $\mathbb{E}(X) = \infty$. That is, the expected value of X is not finite. (4 marks)

    iii. Comment on what is wrong with this strategy. (2 marks)

3. (a) Suppose 15% of football players are left footed. 11 players including one goalkeeper are picked at random to form a football team.

    i. Find the probability that exactly one left footed player on the football team. (5 marks)

ii. Find the probability that less than three players on the team are left footed. (5 marks)

iii. Suppose the goalkeeper is left footed. What is the probability that at most two of the remainder of the team are left footed? (5 marks)

(b) Claims come into an insurance company at the rate of 2 per day according to a Poisson process. In a given 5 days week. Find:

i. the probability of 2 claims every day (5 marks)

ii. the probability of no claims on exactly 2 of the 5 days (7 marks)

4. (a) Let X be a discrete random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{5}, & 0 \leq x < 2 \\ \frac{2}{5}, & 2 \leq x < 4 \\ 1, & x \geq 4 \end{cases}$$

Determine the probability mass function of X. (6 marks)

(b) Let X, Y be two continuous random variable with joint probability density function

$$f(x, y) = cx^2y(1 + y)$$

for $0 \leq x \leq 3$ and $0 \leq y \leq 3$, and $f(x, y) = 0$ otherwise.

i. Find the value of c. (4 marks)

ii. Compute the joint probability $\mathbb{P}(\{1 \leq X \leq 2\} \cap \{0 \leq Y \leq 1\})$. (4 marks)

iii. Determine the joint cumulative distribution function of X and Y. (4 marks)

iv. Determine the marginal probability density function of X. (4 marks)

v. Are X and Y independent? (4 marks)