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2 Predicate logic and Quantifiers

Task: Understand enough predicate logic to make sense of quantified statements.

In predicate logic, propositions depend on variables x, y, z, so their truth value may change depending on which values these variables assume: $P(x), Q(x, y), R(x, y, z)$

2.1 Introduce quantifiers

2.1.1 $\exists$ existential quantifier

Syntax: $\exists x P(x)$

Definition: $\exists x P(x)$ is true if $P(x)$ is true for some value of x. It is false otherwise.

2.1.2 $\forall$ universal quantifier

Syntax: $\forall x P(x)$

Definition: $\forall x P(x)$ is true if $P(x)$ is true for all allowable values of x. It is false otherwise.

2.1.3 $\exists!$ for one and only one (additional quantifier standard in maths)

Syntax: $\exists! x P(x)$

Definition: $\exists! x P(x)$ is true if $P(x)$ is true for exactly one value of x and false for all other values of x; otherwise, $\exists! x P(x)$ is false.

Example: $P(x)$: x is/was the pope and x is Argentine.
(Compound statement; two sentences with connector $\wedge$ between them)
$\exists! x P(x)$ is true with x being Pope Francis.
Now, set $Q(x)$: x is/was the pope and x is Brazilian.
$\exists! x Q(x)$ is false as there has not been a Brazilian pope so far.
In fact, $\exists x Q(x)$ is also false.

2.2 Alternation of Quantifiers

$\forall x \exists y \forall z \quad P(x, y, z)$

NB: The order cannot be exchanged as it might modify the truth value of the statement (think of examples with two quantifiers).

2.3 Negation of Quantifiers

$$\neg(\exists x P(x)) \quad \leftrightarrow \quad \forall x \neg P(x)$$
$$\neg(\forall x P(x)) \quad \leftrightarrow \quad \exists x \neg P(x)$$