MAU22C00: TUTORIAL 15 PROBLEMS
MINIMAL SPANNING TREES AND DIRECTED GRAPHS

1) Prove that any subgraph (V', E') of a connected graph (V, E) is contained in some spanning tree of $(V, E) \iff (V', E')$ contains no circuits.

2) (Annual Exam Trinity Term 2018) Consider the connected undirected graph with vertices A, B, C, D, E, F, G, H, I, J, K, and L, and with edges listed with associated costs in the following table:

CF	JK	IJ	AD	CH	EI	BL	CE	HG	BH
2	2	3	3	3	4	5	6	6	7
EF	FJ	GK	CD	DE	HL	AC	FH	EJ	AB
8	8	9	9	10	10	10	11	12	14

Determine the minimum spanning tree generated by Prim's Algorithm, starting from the vertex F, where that algorithm is applied with the queue specified in the table above. For each step of the algorithm, write down the edge that is added.

3) (Annual Exam Trinity Term 2018)

(a) How many distinct directed graphs with three vertices $V = \{a, b, c\}$ are there? Justify your answer.

(b) Using the one-to-one correspondence between directed graphs and relations, draw a directed graph that corresponds to a relation on $V = \{a, b, c\}$ that is reflexive **but neither** symmetric **nor** transitive. Justify your answer.

(c) Using the one-to-one correspondence between directed graphs and relations, draw a directed graph that corresponds to a relation on $V = \{a, b, c\}$ that is symmetric **but neither** reflexive **nor** transitive. Justify your answer.

(d) Using the one-to-one correspondence between directed graphs and relations, draw a directed graph that corresponds to a relation on $V = \{a, b, c\}$ that is transitive **but not** reflexive **nor** symmetric. Justify your answer.