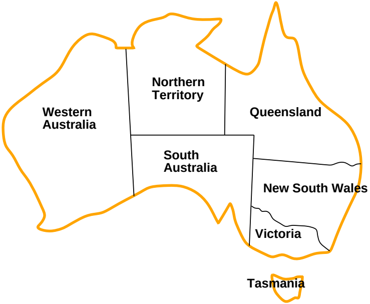
Graph modeling



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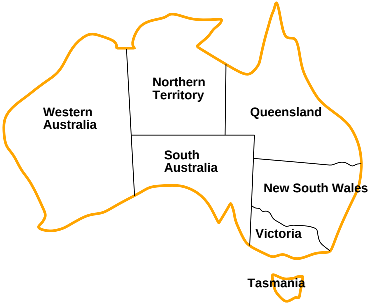
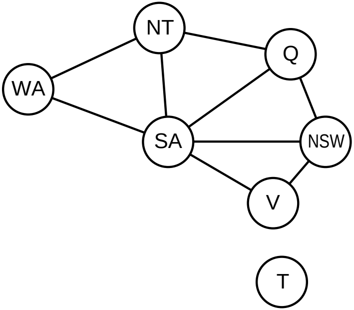
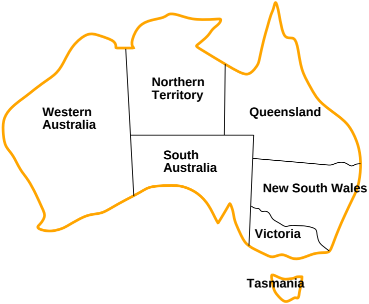
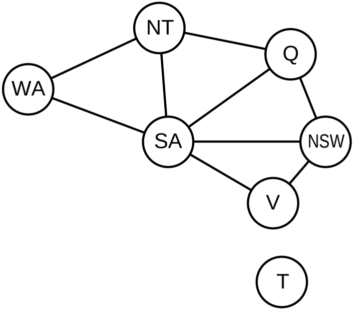




Russell & Norvig

Graph modeling





Russell & Norvig

```
arc(wa,nt).    arc(nt,q).    arc(q,nsw).
arc(wa,sa).    arc(nt,sa).   arc(sa,q).
arc(sa,nsw).   arc(sa,v).    arc(v,nsw).

arc2(X,Y) :- arc(X,Y) ; arc(Y,X).
```

Non-termination (due to poor choices)

```
i :- p,q.                                [i]

i :- r.

p :- i.

r.
```

```
| ?- i.
```

```
prove([],_).

prove([H|T],KB) :- member([H|B],KB), append(B,T,Next),
                   prove(Next,KB).

| ?- prove([i],[[i,p,q],[i,r],[p,i],[r]]).
```

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```
i :- p,q.

i :- r.

p :- i.

r.
________
| ?- i.
```

$$\begin{tikzcd}[column sep=small] & {[i]} \arrow[dl] \arrow[dr] & \\ {[p,q]} & & {[r]} \end{tikzcd}$$

```
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```
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p :- i.

r.
________

| ?- i.
```

$$
\begin{array}{ccc}
 & \texttt{[i]} & \\
\swarrow & & \searrow \\
\texttt{[p,q]} & & \texttt{[r]} \\
\downarrow & & \downarrow \\
\texttt{[i,q]} & & \texttt{[]}
\end{array}
$$

```
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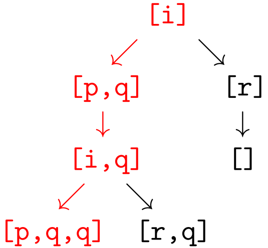


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r.
________
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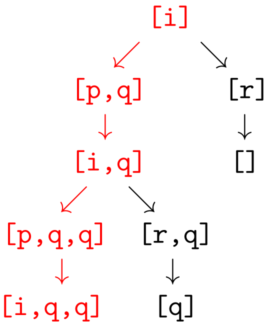
```

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r.
________

| ?- i.
```

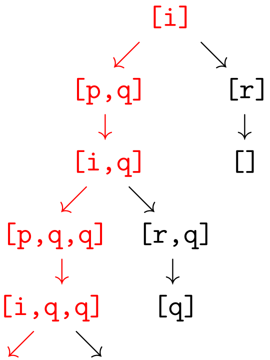


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                   prove(Next,KB).

| ?- prove([i],[[i,p,q],[i,r],[p,i],[r]]).
```

Determinization (eliminate choice)

A fsm `[Trans, Final, Q0]` such that

for all `[Q,X,Qn]` *and* `[Q,X,Qn']` *in* `Trans`, $\texttt{Qn} = \texttt{Qn}'$

is a *deterministic finite automaton* (DFA).

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Fact. Every fsm has a DFA accepting the same language.

Proof: Subset (powerset) construction

Apply to `arc`,`goal`, contra `Trans`,`Final`:

```
arcD(NodeList,NextList) :-
      setof(Next, arcLN(NodeList,Next), NextList).
arcLN(NodeList,Next) :- member(Node,NodeList),
                        arc(Node,Next).
```

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goalD(NodeList):-member(Node,NodeList),goal(Node).
```

Determinization (eliminate choice)

A fsm `[Trans, Final, Q0]` such that

for all `[Q,X,Qn]` *and* `[Q,X,Qn′]` *in* `Trans`, $\texttt{Qn} = \texttt{Qn}'$

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                        arc(Node,Next).

goalD(NodeList):-member(Node,NodeList),goal(Node).
```

```
searchD(NL) :- goalD(NL);
               (arcD(NL,NL2), searchD(NL2)).
```

Determinization (eliminate choice)

A fsm `[Trans, Final, Q0]` such that

for all `[Q,X,Qn]` *and* `[Q,X,Qn']` *in* `Trans`, $\texttt{Qn} = \texttt{Qn}'$

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```
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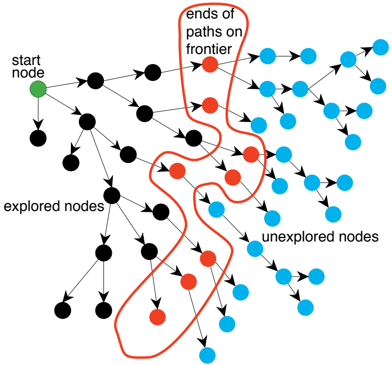
arcLN(NodeList,Next) :- member(Node,NodeList),
                        arc(Node,Next).

goalD(NodeList):-member(Node,NodeList),goal(Node).
```

```
searchD(NL) :- goalD(NL);
               (arcD(NL,NL2), searchD(NL2)).

search(Node) :- searchD([Node]).
```

Frontier search



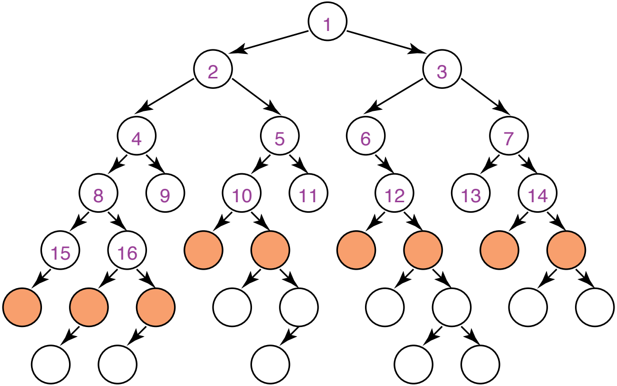
Poole & Mackworth

```
search(Node) :- frontierSearch([Node]).

frontierSearch([Node|_]) :- goal(Node).

frontierSearch([Node|Rest]) :-
            findall(Next, arc(Node,Next), Children),
            add2frontier(Children,Rest,NewFrontier),
            frontierSearch(NewFrontier).
```

Breadth-first: queue (FIFO)



$$[1] \rightsquigarrow [2,3] \rightsquigarrow [3,4,5] \rightsquigarrow [4,5,6,7] \rightsquigarrow \cdots$$

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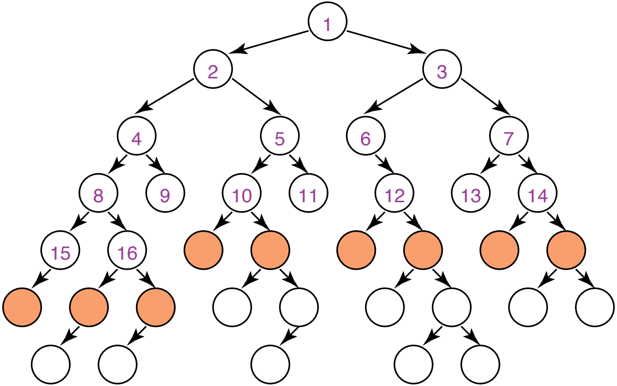
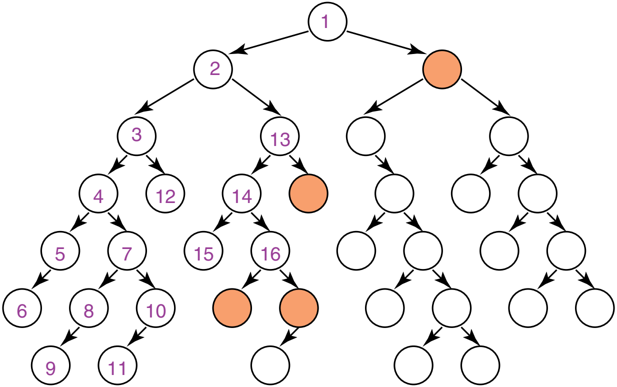


[1] $\rightsquigarrow$ [2,3] $\rightsquigarrow$ [3,4,5] $\rightsquigarrow$ [4,5,6,7] $\rightsquigarrow \cdots$

```
add2frontier(Children,[],Children).

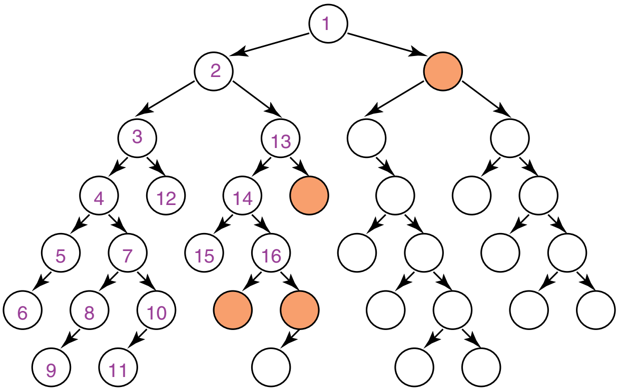
add2frontier(Children,[H|T],[H|More]) :-
                        add2frontier(Children,T,More).
```

Depth-first: stack (LIFO)



$[1] \rightsquigarrow [2, \bullet] \rightsquigarrow [3, 13, \bullet] \rightsquigarrow [4, 12, 13, \bullet] \rightsquigarrow \cdots$

Depth-first: stack (LIFO)



[1] ⇝ [2,•] ⇝ [3,13,•] ⇝ [4,12,13,•] ⇝ ···

```
add2frontier([],Rest,Rest).

add2frontier([H|T],Rest,[H|TRest]) :-
                        add2frontier(T,Rest,TRest).
```

If-then-else and cut !

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

If-then-else and cut !

```
i :- p,!,q.                    [i]

i :- r.

p.

r.
```

```
| ?- i.
```

If-then-else and cut !

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

$$
\begin{tikzcd}
 & \texttt{[i]} \arrow[dl] \arrow[dr] & \\
\texttt{[p,!,q]} & & \texttt{[r]}
\end{tikzcd}
$$

If-then-else and cut !

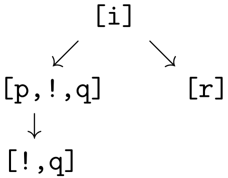
```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```



Cut ! is true but destroys backtracking.

If-then-else and cut !

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

```
      [i]
     ↙
[p,!,q]
   ↓
 [!,q]
   ↓
  [q]
```

Cut ! is true but destroys backtracking.

If-then-else and cut !

```
i :- p,!,q.

i :- r.

p.

r.
```

```
        [i]
       ↙
   [p,!,q]
     ↓
   [!,q]
     ↓
    [q]
```

```
| ?- i.

no
```

Cut ! is true but destroys backtracking.

Review: Depth-first as frontier search

```
prove([],_).       % goal([]).
prove(Node,KB) :- arc(Node,Next,KB), prove(Next,KB).
```

Review: Depth-first as frontier search

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prove([],_).        % goal([]).
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```

```
fs([[]|_],_).

fs([Node|More],KB) :- findall(X,arc(Node,X),L),
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```

Review: Depth-first as frontier search

```
prove([],_).      % goal([]).
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                      fs(NewFrontier,KB).
```

Cut?

Tracking the frontier

[[i]]

```
i :- p,!,q.          [i]

i :- r.

p.

r.
```

```
| ?- i.
```

Tracking the frontier

$[[\mathtt{i}]] \rightsquigarrow [[\mathtt{p},!,\mathtt{q}],[\mathtt{r}]]$

```
i :- p,!,q.

i :- r.

p.

r.
```

$$\begin{tikzcd} & {[\mathtt{i}]} \arrow[dl] \arrow[dr] & \\ {[\mathtt{p},!,\mathtt{q}]} & & {[\mathtt{r}]} \end{tikzcd}$$

```
| ?- i.
```

Tracking the frontier

`[[i]]` $\rightsquigarrow$ `[[p,!,q],[r]]` $\rightsquigarrow$ `[[!,q],[r]]`

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

$$
\begin{tikzcd}
 & \texttt{[i]} \arrow[dl] \arrow[dr] & \\
\texttt{[p,!,q]} \arrow[d] & & \texttt{[r]} \\
\texttt{[!,q]} & &
\end{tikzcd}
$$

Tracking the frontier

`[[i]]` $\rightsquigarrow$ `[[p,!,q],[r]]` $\rightsquigarrow$ `[[!,q],[r]]` $\rightsquigarrow$ `[[q]]`

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.
```

$$
\begin{array}{c}
\texttt{[i]} \\
\swarrow \\
\texttt{[p,!,q]} \\
\downarrow \\
\texttt{[!,q]} \\
\downarrow \\
\texttt{[q]}
\end{array}
$$

Tracking the frontier

$$[[i]] \rightsquigarrow [[p,!,q],[r]] \rightsquigarrow [[!,q],[r]] \rightsquigarrow [[q]] \rightsquigarrow []$$

```
i :- p,!,q.

i :- r.

p.

r.
```

$$\begin{array}{cc} & [i] \\ & \swarrow \\ [p,!,q] & \\ \downarrow & \\ [!,q] & \\ \downarrow & \\ [q] & \end{array}$$

```
| ?- i.
```

Tracking the frontier

[[i]] $\rightsquigarrow$ [[p,!,q],[r]] $\rightsquigarrow$ [[!,q],[r]] $\rightsquigarrow$ [[q]] $\rightsquigarrow$ []

```
i :- p,!,q.

i :- r.

p.

r.
```

```
| ?- i.

no
```

```
        [i]
       ↙
[p,!,q]
   ↓
 [!,q]
   ↓
  [q]
```

Cut via frontier depth-first search

```
fs([[]|_],_).


fs([Node|More],KB) :-
                      findall(X,arc(Node,X),L),
                      append(L,More,NewFrontier),
                      fs(NewFrontier,KB).
```

Cut via frontier depth-first search

```
fs([[]|_],_).

fs([[cut|T]|_],KB)) :- fs([T],KB).

fs([Node|More],KB) :-
                        findall(X,arc(Node,X),L),
                        append(L,More,NewFrontier),
                        fs(NewFrontier,KB).
```

Cut via frontier depth-first search

```
fs([[]|_],_).

fs([[cut|T]|_],KB)) :- fs([T],KB).

fs([Node|More],KB) :-  Node = [H|_], H\== cut,
                       findall(X,arc(Node,X),L),
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                       fs(NewFrontier,KB).
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Cut via frontier depth-first search

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if(p,q,r) :- (p,!,q); r.        % contra (p,q);r
```

Cut via frontier depth-first search

```
fs([[]|_],_).

fs([[cut|T]|_],KB)) :- fs([T],KB).

fs([Node|More],KB) :-  Node = [H|_], H\== cut,
                       findall(X,arc(Node,X),L),
                       append(L,More,NewFrontier),
                       fs(NewFrontier,KB).


if(p,q,r) :- (p,!,q); r.        % contra (p,q);r

negation-as-failure(p) :- (p,!,fail); true.
```