

**MAU22C00: TUTORIAL 14 PROBLEMS**  
**MINIMAL SPANNING TREES**

1) (Annual Exam Trinity Term 2018) Consider the connected undirected graph with vertices  $A, B, C, D, E, F, G, H, I, J, K$ , and  $L$ , and with edges listed with associated costs in the following table:

$CF$	$JK$	$IJ$	$AD$	$CH$	$EI$	$BL$	$CE$	$HG$	$BH$
2	2	3	3	3	4	5	6	6	7
$EF$	$FJ$	$GK$	$CD$	$DE$	$HL$	$AC$	$FH$	$EJ$	$AB$
8	8	9	9	10	10	10	11	12	14

- (a) Draw the graph and label each edge with its cost.
  - (b) Determine the minimum spanning tree generated by Kruskal's Algorithm, where that algorithm is applied with the queue specified in the table above. For each step of the algorithm, write down the edge that is added.
- 2) In the previous problem, how many distinct ways can the edges of the graph be ordered in non-decreasing order of cost, i.e. how many different non-decreasing queues are there for the edges of the graph? Justify your answer.
- 3) Would every non-decreasing queue from problem 2 give a different minimal spanning tree when Kruskal's Algorithm is applied? Justify your answer by either a proof or a counterexample.
- 4) Prove that every nontrivial tree is a bipartite graph.