

MAU22C00: TUTORIAL 13 PROBLEMS
GRAPH THEORY

- 1) For what type of p and q does the complete bipartite graph $K_{p,q}$ have a Hamiltonian circuit? Justify your answer.
- 2) Let (V, E) be the graph with vertices a, b, c, d, e , and f and edges $ab, ac, bc, bd, cd, de, df$, and ef .
 - (a) Does this graph have a Hamiltonian circuit? Justify your answer.
 - (b) Is this graph a tree? Justify your answer.
 - (c) If it is not a tree, how many distinct spanning trees does it have?
- 3) Consider the statement “A graph (V, E) is a tree $\iff \#(E) = \#(V) - 1$. ” What hypothesis is needed for this equivalence to be true? Give an example to show why this hypothesis is necessary.

Recall that

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

read as “ n choose k ” gives the number of distinct combinations of k objects taken out of a possible n objects for $n \geq k \geq 0$ with the convention $0! = 1$.

- 4) Consider the complete graph K_n for $n = 2, 3, 4$. In each of the three cases
 - (a) Is this graph a tree? Justify your answer.
 - (b) If it is not a tree, how many distinct spanning trees does it have?(Hint: How many edges does K_n have?)