Slides mainly from Poole & Mackworth, chap 13

Datalog based on the following assumptions

- An agent's knowledge can be usefully described in terms of *individuals* and *relations* among individuals.
- An agent's knowledge base consists of *definite* and *positive* statements.
- The environment is *static*.
- There are only a finite number of individuals of interest in the domain.
  An individual can be named.

Datalog syntax

- A variable starts with upper-case letter.
- A constant starts with lower-case letter or is a sequence of digits (numeral).
- A predicate symbol starts with lower-case letter.
- A term is either a variable or a constant.
- An atomic symbol (atom) is of the form p or $p(t_1, \ldots, t_n)$ where p is a predicate symbol and t_i are terms.

Datalog syntax (ctd)

- A definite clause is either an atomic symbol (a fact) or of the form:

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1 \wedge \cdots \wedge b_m}_{\text{body}}$$

  where a and b_i are atomic symbols.
- query is of the form $?b_1 \wedge \cdots \wedge b_m$.
- knowledge base is a set of definite clauses.

Semantics: General Idea

A semantics specifies the meaning of sentences in the language.

An interpretation specifies:

- what objects (individuals) are in the world
- the correspondence between symbols in the computer and objects & relations in world
  - constants denote individuals
  - predicate symbols denote relations

Formal Semantics

An interpretation is a triple $I = \langle D, \phi, \pi \rangle$, where

- D, the domain, is a nonempty set. Elements of D are individuals.
- ϕ is a mapping that assigns to each constant an element of D. Constant c denotes individual $\phi(c)$.
- π is a mapping that assigns to each n-ary predicate symbol a relation: a function from D^n into $\{\textit{TRUE}, \textit{FALSE}\}$.

Example Interpretation

Constants: *phone*, *pencil*, *telephone*.
Predicate Symbol: *noisy* (unary), *left_of* (binary).

- $D = \{$✂, ☎, ✎$\}$.
- $\phi(phone) =$ ☎, $\phi(pencil) =$ ✎, $\phi(telephone) =$ ☎.
- $\pi(noisy)$:

⟨✂⟩	*FALSE*	⟨☎⟩	*TRUE*	⟨✎⟩	*FALSE*

  $\pi(left\_of)$:

⟨✂, ✂⟩	*FALSE*	⟨✂, ☎⟩	*TRUE*	⟨✂, ✎⟩	*TRUE*
⟨☎, ✂⟩	*FALSE*	⟨☎, ☎⟩	*FALSE*	⟨☎, ✎⟩	*TRUE*
⟨✎, ✂⟩	*FALSE*	⟨✎, ☎⟩	*FALSE*	⟨✎, ✎⟩	*FALSE*

Important points to note

- The domain D can contain real objects. (e.g., a person, a room, a course). D can't necessarily be stored in a computer.
- $\pi(p)$ specifies whether the relation denoted by the n-ary predicate symbol p is true or false for each n-tuple of individuals.
- If predicate symbol p has no arguments, then $\pi(p)$ is either *TRUE* or *FALSE*.

Truth in an interpretation

A constant c denotes in I the individual $\phi(c)$.
Ground (variable-free) atom $p(t_1, \ldots, t_n)$ is

- true in interpretation I if $\pi(p)(\langle\phi(t_1), \ldots, \phi(t_n)\rangle) = \textit{TRUE}$ in interpretation I and
- false otherwise.

Ground clause $h \leftarrow b_1 \land \ldots \land b_m$ is false in interpretation I if h is false in I and each b_i is true in I, and is true in interpretation I otherwise.

Example Truths

In the interpretation given before, which of following are true?

$$noisy(phone)$$
$$noisy(telephone)$$
$$noisy(pencil)$$
$$left\_of(phone, pencil)$$
$$left\_of(phone, telephone)$$
$$noisy(phone) \leftarrow left\_of(phone, telephone)$$
$$noisy(pencil) \leftarrow left\_of(phone, telephone)$$
$$noisy(pencil) \leftarrow left\_of(phone, pencil)$$
$$noisy(phone) \leftarrow noisy(telephone) \wedge noisy(pencil)$$

Example Truths

In the interpretation given before, which of following are true?

$noisy(phone)$	true
$noisy(telephone)$	true
$noisy(pencil)$	false
$left\_of(phone, pencil)$	true
$left\_of(phone, telephone)$	false
$noisy(phone) \leftarrow left\_of(phone, telephone)$	true
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	true
$noisy(pencil) \leftarrow left\_of(phone, pencil)$	false
$noisy(phone) \leftarrow noisy(telephone) \wedge noisy(pencil)$	true

Models and logical consequences

- A knowledge base, KB, is true in interpretation I if and only if every clause in KB is true in I.
- A model of a set of clauses is an interpretation in which all the clauses are true.
- If KB is a set of clauses and g is a conjunction of atoms, g is a logical consequence of KB, written $KB \models g$, if g is true in every model of KB.
- That is, $KB \models g$ if there is no interpretation in which KB is true and g is false.

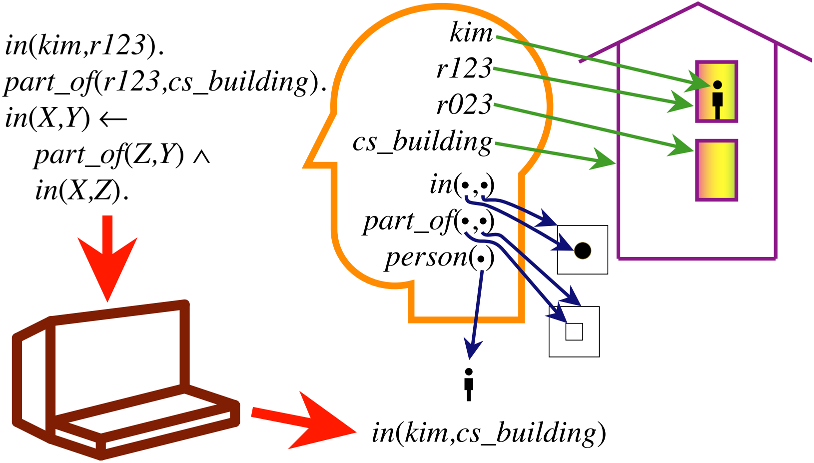
User's view of Semantics

1. Choose a task domain: intended interpretation.
2. Associate constants with individuals you want to name.
3. For each relation you want to represent, associate a predicate symbol in the language.
4. Tell the system clauses that are true in the intended interpretation: axiomatizing the domain.
5. Ask questions about the intended interpretation.
6. If $KB \models g$, then g must be true in the intended interpretation.

Computer's view of semantics

- The computer doesn't have access to the intended interpretation.
- All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
- If $KB \models g$ then g must be true in the intended interpretation.
- If $KB \not\models g$ then there is a model of KB in which g is false. This could be the intended interpretation.

Semantics in the mind



Soundness and completeness

Recall that g is a *logical consequence of* KB, $KB \models g$, precisely if g is true in all models of KB.

Let $\vdash$ be a mechanical procedure for deriving a formula g from a knowledge base KB, written $KB \vdash g$.

> $\vdash$ is sound if $KB \models g$ whenever $KB \vdash g$.

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Two extreme examples:

(1) $KB \vdash g$ for no g
sound

(2) $KB \vdash g$ for all g
complete

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Two extreme examples:

(1) $KB \vdash g$ for no g 'say nothing' undergenerates
sound

(2) $KB \vdash g$ for all g 'say everything' overgenerates
complete

Propositional KBs

Recall

```
i :- p,q.
i :- r.
p.
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KB = [[i,p,q],[i,r],[p],[r]]
arc([H|T],N,KB) :- member([H|B],KB),
                   append(B,T,N).
prove([],KB).
prove(Node,KB) :- arc(Node,Next,KB),
                  prove(Next,KB).
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Let

$$KB \vdash G \iff \texttt{prove([G],KB)}$$

Theorem.

(1) $\vdash$ is sound (proved by induction)

(2) $\vdash$ is *not* complete (why?)

Logical consequences bottom-up

$$
\begin{aligned}
C_0 &:= \emptyset \\
C_{n+1} &:= \{H \mid (\text{for some } B \subseteq C_n) \ \text{member}([H|B], KB)\} \\
C &:= \bigcup_{n \geq 0} C_n \\
&= \bigcup_{n \leq k} C_n \quad \text{where } k = \text{number of clauses in } KB
\end{aligned}
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$C_1 = \{\texttt{p,r}\}$

$C_2 = \{\texttt{p,r,i}\} = C_n$ for $n \geq 2$

A 0-ary predicate p is interpreted by $I = \langle D, \phi, \pi \rangle$ as

$$\pi(p) : D^0 \to \{\text{true,false}\}.$$

Substitutions and instances

A 0-ary predicate p is interpreted by $I = \langle D, \phi, \pi \rangle$ as

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Let K be a set of constants.

A K-substitution is a function from a finite set of variables to K — i.e. a set $\{V_1/c_1, \ldots, V_n/c_n\}$ of $c_i \in K$ and distinct variables V_i.

The application $e\theta$ of a K-substitution $\theta = \{V_1/c_1, \ldots, V_n/c_n\}$ to a clause e is e with each V_i replaced by c_i

e.g. $p(Z, U, Y, a, X)\{X/b, U/a, Z/b\} \ = \ p(b, a, Y, a, b)$.

A K-instance of e is $e\theta$ for some K-substitution θ.

Given a set B of clauses and a K-substitution θ, let

$$B\theta \ := \ \{e\theta \mid e \in B\}.$$

Bottom-up with substitutions

If KB has constants from some non-empty finite set K, let

$$
\begin{aligned}
C_0^K &:= \emptyset \\
C_{n+1}^K &:= \{H\theta \mid \theta \text{ is a } K\text{-substitution s.t. } B\theta \subseteq C_n^K \\
&\qquad\qquad\qquad \text{for some } B \text{ s.t. } \mathrm{member}([H|B], KB)\} \\
C^K &:= \bigcup_{n \geq 0} C_n^K
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E.g. for $KB =$`[[p(a,b)],[q(X),p(X,Y)]]` and $K = \{a, b\}$,

$$\begin{aligned}
C_1^K &= \{\mathtt{p(a,b)}\} \\
C_2^K &= \{\mathtt{p(a,b)}, \mathtt{q(a)}\} = C^K
\end{aligned}$$

Soundness & completeness via Herbrand

The Herbrand interpretation of a set KB of clauses with constants from a non-empty set K is the triple $I = \langle D, \phi, \pi \rangle$ where

- the domain D is the set K of constants
- ϕ is the identity function on K (each constant in K refers to itself)
- for each n-ary p and n-tuple $c_1 \ldots c_n$ from K,

$$\pi(p)(c_1 \ldots c_n) = \text{ true } \iff p(c_1 \ldots c_n) \in C^K$$

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Fact. I is a model of KB, and every clause true in I is true in every model of KB (interpreting constants in K).

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Corollary. The bottom-up procedure with substitutions is sound and complete (for Datalog).