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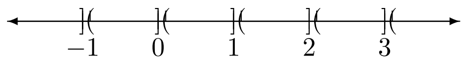
## 4.2 Equivalence Relations and Partitions

**Task:** Understand how equivalence relations divide sets.

**Definition:** Let $A$ be a set. A partition of $A$ is a collection of non-empty sets, any two of which are disjoint such that their union is $A$, **i.e.** $\lambda = \{A_\alpha \mid \alpha \in I\}$ s.t. $\forall \alpha, \alpha' \in I$ satisfying $\alpha \neq \alpha'$, $A_\alpha \cap A_{\alpha'} = \emptyset$ and $\bigcup\limits_{\alpha \in I} A_\alpha = A$

Here $I$ is an indexing act (may be infinite). $\bigcup\limits_{\alpha \in I} A_\alpha$ is the union of all the $A_\alpha$'s (possibly an infinite union)

**Example** $\{(n, n+1] \mid n \in \mathbb{Z}\}$ is a partition of $\mathbb{R}$




$\bigcup\limits_{n \in \mathbb{Z}} (n, n+1] = \mathbb{R}$

$(n, n+1] \cap (m, m+1] = \emptyset$ if $n \neq m$

**Definition:** If $R$ is an equivalence relation on a set $A$ and $x \in A$, the equivalence class of $x$ denoted $[x]_R$ is the set $\{y \mid xRy\}$. The collection of all equivalence classes is called $A$ modulo $R$ and denoted $A/R$.

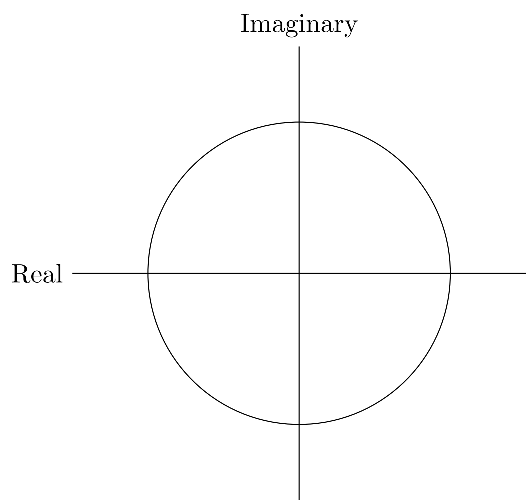
**Examples:**

1. $A = \mathbb{N}$ $\qquad x \equiv y \bmod 3$
   We have the equivalence classes $[0]_R, [1]_R$ and $[2]_R$ given by the three possible remainders under division by 3.
   $[0]_R = \{0, 3, 6, 9, ...\}$
   $[1]_R = \{1, 4, 7, 10, ...\}$
   $[2]_R = \{2, 5, 8, 11, ...\}$
   Clearly $[0]_R \cup [1]_R \cup [2]_R = \mathbb{N}$ and they are mutually disjoint $\Rightarrow R$ gives a partition of $\mathbb{N}$.

2. $ABC \sim A'B'C'$
   $[ABC] = \{$The set of all triangles with angles of magnitude $\measuredangle ABC, \measuredangle BAC, \measuredangle ACB\}$
   The union over the set of all $[ABC]$ is the set of all triangles and $[ABC] \cap [A'B'C'] = \emptyset$ if $ABC \nsim A'B'C'$ since it means these triangles have at least one angle that is different.

3. $A = \mathbb{C}$ $\qquad x \sim y$ if $|x| = |y|$ $\qquad$ equivalence relation
   $[x] = \{y \in \mathbb{C} \mid |x| = |y|\} = [r]$ for $r \in [0, +\infty)$ (meaning $r \geq 0$)

   circle of radius $|x|$

   $\bigcup_{r \in [0,+\infty)} [r] = \mathbb{C}$
   $[r_1] \cap [r_2] \neq \emptyset$ if $r_1 \neq r_2$ since two distinct circles in $\mathbb{C} \simeq \mathbb{R}^2$ with empty intersection.

circles $r_1 \wedge r_2$