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Example

$$KB = \begin{cases} \texttt{false :- a,b.} \\ \texttt{a :- c.} \\ \texttt{b :- c.} \end{cases}$$

$$KB \models \neg\texttt{c}$$

Satisfiability

Every set of definite clauses is satisfiable (i.e., has a model).
Not so with Horn clauses

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Horn-SAT is feasible, whereas 3-SAT is likely not.

Non-monotonicity

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Example: Assume a database of video segments is complete.

Rules

Encode *birds fly*

$$\texttt{fly}(X)\ \texttt{:-}\ \texttt{bird}(X).$$

to allow for exceptions.

Rules and defaults

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$$\frac{\text{prerequisite } p \; : \; \text{justification } j}{\text{conclusion } c}$$

applied to KB says:

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Birds and bees

$$\frac{\mathtt{bird}(X) \; : \; \mathtt{fly}(X)}{\mathtt{fly}(X)}$$

Let KB be

```
bird(robin).
bird(penguin).
false :- fly(penguin).
fly(bee).
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Conclude:

Birds and bees

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which we can block by adding to KB the rule

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Non-determinism

Conflicting defaults

$$\frac{\mathtt{quaker}(X) : \mathtt{pacifist}(X)}{\mathtt{pacifist}(X)} \qquad \frac{\mathtt{republican}(X) : \mathtt{hawk}(X)}{\mathtt{hawk}(X)}$$

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Applying one default to Nixon makes the other inapplicable.

KB has two incompatible extensions, breaking

least fixed point (provability model) for Horn clauses.

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N.B. Checking finite failure can be as hard as the Halting Problem.

3 modes of inference (C.S. Peirce)

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From $\models$ as inclusion $\subseteq$

$$\begin{aligned} KB \models g &\iff Mod(KB) \subseteq Mod(g) \\ KB \text{ satisfiable} &\iff Mod(KB) \not\subseteq Mod(\texttt{false}) \\ &\iff Mod(KB) \neq \emptyset \end{aligned}$$

to weighing alternatives $d \in D$ via probabilities given KB

$$prob(d|KB) \ = \ \text{conditional probability of } d \text{ given } KB$$

$\rightsquigarrow$ Bayesian networks ...