Prob extra qs

cgregg

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Question 3

$X_1, X_2, ..., X_n$ are n independent discrete random variables with $\mathbb{P}(X_i = c) = 1$ for $i = 1, 2, ..., n$.

$$X = \sum_{i=1}^{n} X_i$$

X is also a random variable by the linearity property (basically being that if you sum two random variables together you get another random variable), with its expected value given by

$$\mathbb{E}(X) = \sum_{i=1}^{n} \mathbb{E}(X_i)$$

Now we calculate $\mathbb{E}(X_i)$. As the probability of any x_i being = c is 1, each X_i's expected value is c. This is done out here, starting with the formula for expected value.

$$\mathbb{E}(X) = \sum_{x} xp(x)$$

$$\mathbb{E}(X_i) = \sum_{x_i} x_i p(x_i)$$

$$\mathbb{E}(X_i) = \sum_{x_i} (c \times 1) + (< anyothervalueforx > \times 0)$$

$$\mathbb{E}(X_i) = c$$

Then, the expected value for X can be found

$$\mathbb{E}(X) = \sum_{i=1}^{n} \mathbb{E}(X)_i$$

$$\mathbb{E}(X) = \sum_{i=1}^{n} c$$

$$\mathbb{E}(X) = cn$$

The variance of X can be stated as

$$Var(X) = Var(\sum_{i=1}^{n} X_i)$$

From the properties of variance (that any constants can be removed before calculating variance), this can be written as

$$Var(X) = \sum_{i=1}^{n} Var(X_i)$$

Then the variance of X_i can be calculated.

$$Var(X_i) = \mathbb{E}(X^2) - \mathbb{E}(X)^2 = 0 - 0 = 0$$

Then this value is substituted into the initial calculation

$$Var(X) = \sum_{i=1}^{n} 0$$

$$Var(X) = n \times 0 = 0$$

Question 4

We have to start by finding the CDF of f(y), as we need to work with actual probabilities rather than densities. (this is found by the same calculations as in tutorial sheet 4 q4 lmk if those calculations are needed) Not bothered to do that out here but it's his first step in solution 2.
Now, we work with U.

$$U = 10Y - 4$$

(CDF)

$$F_U(u) = \mathbb{P}(U \leq u)$$

But we know another way to express U, so we sub this in.

$$F_U(u) = \mathbb{P}(10Y - 4 \leq u)$$

..a bit of manipulation...

$$F_U(u) = \mathbb{P}(Y \leq \frac{u+4}{10})$$

However, the CDF $F_Y(y)$ can also be written as $\mathbb{P}(Y \leq y)$. So we rephrase the above statement to

$$F_U(u) = F_Y(\frac{u+4}{10}$$

Then here you start writing the relevant values to y (seen in the CDF) in terms of u, not sure how to phrase it but it's kind of like normalisation/z-values.
So, for y=0

$$\frac{u+4}{10} = 0$$

$$u+4 = 0$$

$$u = -4$$

For y = 1

$$\frac{u+4}{10} = 1$$

$$u+4 = 10$$

$$u = 6$$

For y = 1

$$\frac{u+4}{10} = 1$$

$$u+4 = 10$$

$$u = 6$$

For y = 1.5

$$\frac{u+4}{10} = 1.5$$

$$u+4 = 15$$

$$u = 11$$

For $\frac{y^2}{2}$

$$\frac{(\frac{u+4}{10})^2}{2}$$

$$\frac{(u+4)^2}{200}$$

For y-$\frac{1}{2}$

$$\frac{u+4}{10} - \frac{1}{2}$$

$$\frac{u+4-5}{10}$$

$$\frac{u-1}{10}$$

Then, we can write out the CDF of U

$F_U(u) =$

$$\begin{cases} 0, & u < -4 \\ \dfrac{(u+4)^2}{200}, & -4 \le u < 6 \\ \dfrac{u-1}{10}, & 6 \le u < 11 \\ 1, & u \ge 11 \end{cases}$$

To finally find the pdf, we differentiate the CDF $f_U(u) =$

$$\begin{cases} \dfrac{u+4}{100}, & -4 \le u < 6 \\ \dfrac{1}{10}, & 6 \le u < 11 \\ 0, & otherwise \end{cases}$$