CSU11001 Chike Okafor

Q1 Identity Number: 20332400

$\vec{x} = (4, 0, 0)$

$\vec{y} = (3, 3, 2)$

$\vec{z} = (4, 2, 3)$

Q2

i $x \cdot y = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} = 4(3) + 0(3) + 0(2)$

$= 12$

ii $y \times z = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3(3) - 2(2) \\ 2(4) - 3(3) \\ 3(2) - 3(4) \end{pmatrix}$

$= \begin{pmatrix} 9 - 4 \\ 8 - 9 \\ 6 - 12 \end{pmatrix}$

$= \begin{pmatrix} 5 \\ -1 \\ -6 \end{pmatrix}$

iii The vector projection of x onto y is defined as

$$\text{proj}_y x \quad \frac{x \cdot y^2}{\|y\|^2}$$

where x and y are vectors. These are both defined.

Solving for $\|y\|$ gives:

$\|y\| = \sqrt{3^2 + 3^2 + 2^2}$

$= \sqrt{9 + 9 + 4}$

$= \sqrt{22}$

$x \cdot y$ has already been solved in part (i):

$$x \cdot y = 12$$

Hence:

$$\text{proj}_y x = \frac{12}{(\sqrt{22})^2} \times \vec{y}$$

$$= \frac{6}{11} \times \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{6}{11} \times 3 \\ \frac{6}{11} \times 3 \\ \frac{6}{11} \times 2 \end{pmatrix}$$

$$= \begin{pmatrix} 18/11 \\ 18/11 \\ 12/11 \end{pmatrix}$$

iv Since the vectors $\vec{x}$, $\vec{y}$ and $\vec{z}$ all begin from $\vec{0}$ (the origin), the endpoints for each vector become:

$$x = \vec{0} + \vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} = (4, 0, 0).$$

$$y = \vec{0} + \vec{y} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} = (3, 3, 2)$$

$$z = \vec{0} + \vec{z} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} = (4, 2, 3)$$

The implicit equation of a plane is

$$n_1(x - p_1) + n_2(y - p_2) + n_3(z - p_3) = 0$$

where $\vec{n}$ is a vector in R^3 that is normal to the plane and P is a point in the plane with coordinates (p_1, p_2, p_3).

In order to get a vector normal (perpendicular) to the plane, we need to obtain the cross (vector) product of two vectors on the plane. This can be done using the three endpoints we have created.

$$\vec{xy} = \begin{pmatrix} y_1 - x_1 \\ y_2 - x_2 \\ y_3 - x_3 \end{pmatrix} = \begin{pmatrix} 3 - 4 \\ 3 - 0 \\ 2 - 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$$

$$\vec{xz} = \begin{pmatrix} 4 - 4 \\ 2 - 0 \\ 3 - 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$$

$$\vec{xy} \times \vec{xz} = \vec{n}$$

$$\vec{n} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3(3) - 2(2) \\ 2(0) - (-1)(3) \\ -1(2) - 3(0) \end{pmatrix} = \begin{pmatrix} 9 - 4 \\ 0 + 3 \\ -2 - 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$$

Now that we have the vector $\vec{n}$, we can calculate the equation of the plane. The point P used can be any point on the plane, so I used the point x. So:

$$5(x-4)+3(y-0)-2(z-0)=0$$
$$5x-20+3y-2z=0$$
$$\boxed{5x+3y-2z-20=0}$$

v. The distance between a point and a line is described as:

$$\text{distance} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

where the plane has the equation $ax+by+cz+d=0$, and the point has the coordinates (x_1, y_1, z_1). Using the point $(1,3,6)$ and the plane we got in part (iv), the distance can be calculated as follows:

$$\text{distance} = \frac{|5(1)+3(3)-2(6)-20|}{\sqrt{(5)^2+(3)^2+(-2)^2}}$$

$$= \frac{|5+9-12-20|}{\sqrt{38}}$$

$$= \frac{|-18|}{\sqrt{38}} = \frac{18}{\sqrt{38}}$$

$$= 2.91998\ldots$$
$$= \boxed{2.92 \text{ units (to 2 d.p.)}}$$

Q3 $A = \begin{pmatrix} 4 & 0 & 0 \\ 3 & 3 & 2 \\ 4 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 4 & 4 \\ 3 & 0 & 2 \\ 2 & 0 & 3 \end{pmatrix}$

$$AB = \begin{pmatrix} 4 & 0 & 0 \\ 3 & 3 & 2 \\ 4 & 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 4 & 4 \\ 3 & 0 & 2 \\ 2 & 0 & 3 \end{pmatrix}$$

$$\begin{pmatrix} (4\times3)+(0\times3)+(0\times2) & (4\times4)+(0\times0)+(0\times0) & (4\times4)+(0\times2)+(0\times3) \\ (3\times3)+(3\times3)+(2\times2) & (3\times4)+(3\times0)+(2\times0) & (3\times4)+(3\times2)+(2\times3) \\ (4\times3)+(2\times3)+(3\times2) & (4\times4)+(2\times0)+(3\times0) & (4\times4)+(2\times2)+(3\times3) \end{pmatrix}$$

$$= \begin{pmatrix} 12+0+0 & 16+0+0 & 16+0+0 \\ 9+9+4 & 12+0+0 & 12+6+6 \\ 12+6+6 & 16+0+0 & 16+4+9 \end{pmatrix}$$

$$= \begin{pmatrix} 12 & 16 & 16 \\ 22 & 12 & 24 \\ 24 & 16 & 29 \end{pmatrix}$$

$$BA = \begin{pmatrix} 3 & 4 & 4 \\ 3 & 0 & 2 \\ 2 & 0 & 3 \end{pmatrix}\begin{pmatrix} 4 & 0 & 0 \\ 3 & 3 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 12+12+16 & 0+12+8 & 0+8+12 \\ 12+0+8 & 0+0+4 & 0+0+6 \\ 8+0+12 & 0+0+6 & 0+0+9 \end{pmatrix}$$

$$= \begin{pmatrix} 40 & 20 & 20 \\ 20 & 4 & 6 \\ 20 & 6 & 9 \end{pmatrix}$$

$$\therefore AB = \begin{pmatrix} 12 & 16 & 16 \\ 22 & 12 & 24 \\ 24 & 16 & 29 \end{pmatrix}, \quad BA = \begin{pmatrix} 40 & 20 & 20 \\ 20 & 4 & 6 \\ 20 & 6 & 9 \end{pmatrix}$$

Hence $AB \neq BA$ as for each value in AB, the value in that position in BA is not equal.

Q 4 $2x - 2y - 4z = 4$

$x + y + z = 3$

$4x + 6y + 2z = 8$

These equations are used to form an augmented matrix. Solving using the Gaussian elimination method gives:

$$\begin{pmatrix} 2 & -2 & -4 & 4 \\ 1 & 1 & 1 & 3 \\ 4 & 6 & 2 & 8 \end{pmatrix}$$

$$\begin{matrix} R1 \to \frac{1}{2} \times R1 \\ \longrightarrow \\ R3 \to \frac{1}{2} \times R3 \end{matrix} \quad \begin{pmatrix} 1 & -1 & -2 & 2 \\ 1 & 1 & 1 & 3 \\ 2 & 3 & 1 & 4 \end{pmatrix}$$

$$\begin{matrix} R2 \to R2 - R1 \\ \longrightarrow \\ R3 \to R3 - 2R1 \end{matrix} \quad \begin{pmatrix} 1 & -1 & -2 & 2 \\ 0 & -2 & 3 & 1 \\ 0 & 5 & 5 & 0 \end{pmatrix}$$

$$\begin{matrix} R2 \leftrightarrow R3 \\ \longrightarrow \end{matrix} \quad \begin{pmatrix} 1 & -1 & -2 & 2 \\ 0 & 5 & 5 & 0 \\ 0 & -2 & 3 & 1 \end{pmatrix}$$

$$\begin{matrix} R2 \to \frac{1}{5} \times R2 \\ \longrightarrow \end{matrix} \quad \begin{pmatrix} 1 & -1 & -2 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 3 & 1 \end{pmatrix}$$

$$\begin{matrix} R1 \to R1 + R2 \\ \longrightarrow \\ R3 \to R3 - 2R2 \end{matrix} \quad \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

$R1 \to R1 + R3$
$R2 \to R2 - R3$
$$\longrightarrow \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

The matrix is now in reduced row echelon form and we can now see our original linear system row is equal to

$$x = 3$$
$$y = -1$$
$$z = 1$$

So our solution is $x = 3$, $y = -1$ and $z = 1$.