Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

DECLARATION

I understand that this is an individual assessment and that collaboration is not permitted. I have not received any assistance with my work for this assessment. Where I have used the published work of others, I have indicated this with appropriate citation.

I have not and will not share any part of my work on this assessment, directly or indirectly, with any other student.

I have read and I understand the plagiarism provisions in the General Regulations of the University Calendar for the current year, found at http://www.tcd.ie/calendar.

I have also completed the Online Tutorial on avoiding plagiarism 'Ready Steady Write', located at http://tcd-ie.libguides.com/plagiarism/ready-steady-write."

I understand that by returning this declaration with my work, I am agreeing with the above statement.

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Date: 18/03/2022

1. **Answer:**

   (a) The graph is connected as you can get from any one vertex to any other vertex on the graph.

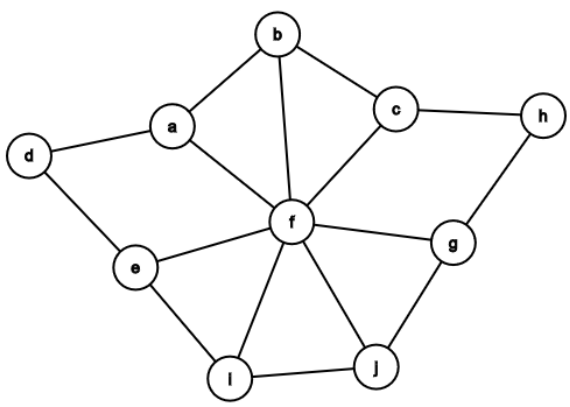
   (b) The graph does not have a Eulerian trail. In order to have a Eulerian trail, you must be able to visit each edge once and only once. However, going from some vertex v to the vertex f via any edge blocks you from visiting an edge next to v, which rules out at least one edge.

   (c) Yes. There exists a Eulerian circuit (note that vertex order in edges are indicative of vertex traversed) $fa \to af \to fb \to bf \to fc \to cf \to fg \to gf \to fj \to jf \to fi \to if \to fe \to ed \to da \to ab \to bc \to ch \to hg \to gj \to ji \to ie \to ed \to da \to af$.

   (d) Yes: $fabchgjied$.

   (e) No. A tree cannot contain any cycles, and the cycle $acbfa$ exists in the graph.

   (f) Graph:



   $d \to a \to b \to f \to c \to h \to g \to j \to i \to e \to d.$

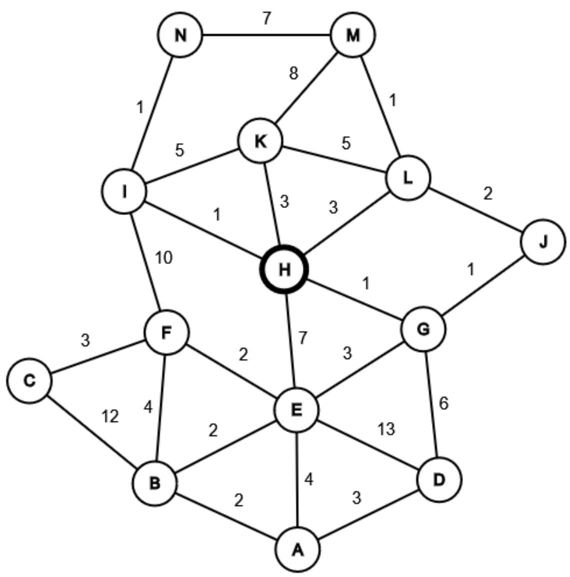
2. **Answer:**

   (a) Since the graph is connected, you must be able to get from any one vertex to any other vertex on the graph. If the two sets of edges E_1 and E_2 come together to form E, then there must be an edge that connects both subgraphs. Since an edge must contain two vertices, there must also be a vertex in both subgraphs due to it exist on the edge in both subgraphs. Therefore, a vertex exists in both subgraphs, and $V_1 \cap V_2 \neq \emptyset$.

   (b) A Hamiltonian path is a path that passes once through every vertex in a graph. Since you can get from any vertex to any other vertex in a connected graph, this path can be acyclical, as a tree as a result. Since we are only connecting vertices, and not forming a circuit of our own, it stands to reason that this path can become a spanning tree, due to its nature of being a connected acyclical path.

3. **Answer:**

(a) Graph:



(b) Steps (Kruskal):

i. Add edge GJ

ii. Add edge LM

iii. Add edge HI

iv. Add edge GH

v. Add edge IN

vi. Add edge BE

vii. Add edge AB

viii. Add edge EF

ix. Add edge EI

x. Add edge JL

xi. Ignore edge EG

xii. Add edge CF

xiii. Add edge HK

xiv. Add edge AD

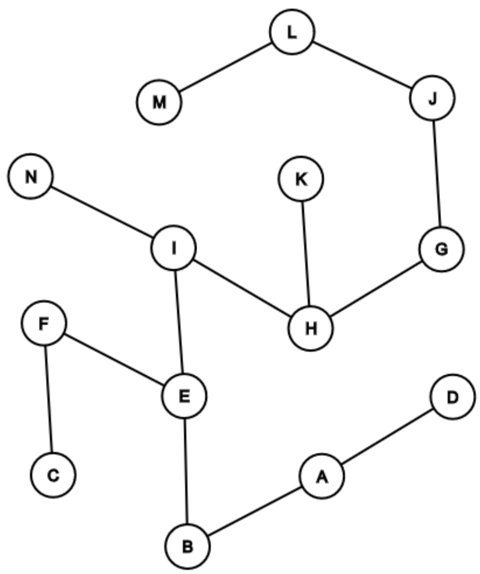
xv. Ignore edge HL

xvi. Ignore edge BF

xvii. Ignore edge AE

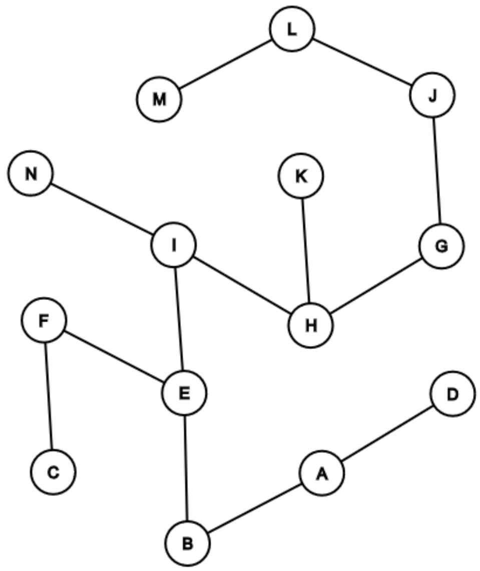
xviii. Ignore edge KI

xix. Ignore edge KL
xx. Ignore edge DG
xxi. Ignore edge EH
xxii. Ignore edge MN
xxiii. Ignore edge KM
xxiv. Ignore edge FI
xxv. Ignore edge BC
xxvi. Ignore edge DE



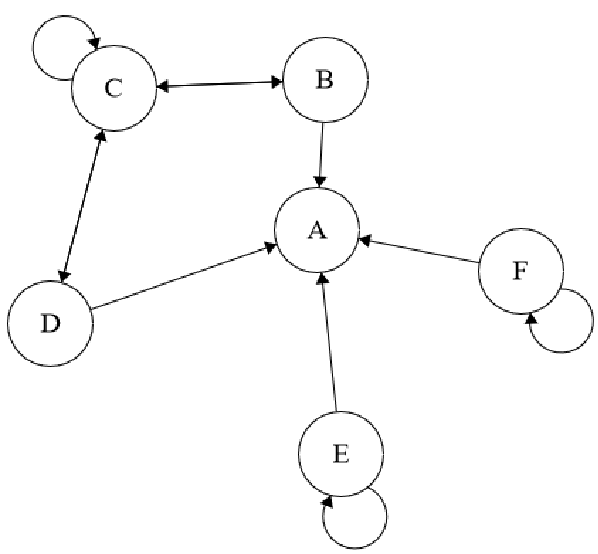
(c) Steps (Prim):

i. Add edge AD
ii. Add edge AB
iii. Add edge BE
iv. Add edge EF
v. Add edge EI
vi. Add edge HI
vii. Add edge GH
viii. Add edge GJ
ix. Add edge IN
x. Add edge JL
xi. Add edge LM
xii. Add edge CF
xiii. Add edge HK



4. **Answer:**

   (a) Graph:



(b) Adjacency matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(c) $\varphi(B) = D$, $\varphi(D) = B$, $\varphi(C) = C$, $\varphi(A) = A$, $\varphi(E) = E$, $\varphi(F) = F$.

5. **Answer:** A directed walk between two points must respect edges between any two vertices v_{i-1}, v_i, whereas a semiwalk doesn't have to. In other words, each edge of a directed walk is an element of a Cartesian product of $V \times V$. Since a semiwalk can be in either order (e.g. v_{i-1}, v_i or v_i, v_{i-1}), it has to also exist where a directed walk exists. In other words, since a directed walk can exist between two vertices v_i and v_j in an ordered fashion, and a semiwalk can exist between the same two vertices potentially in an ordered fashion, if a directed walk exists, then there must exist a semiwalk.

   Conversely, if there exists a semiwalk between the two vertices, then a directed walk must exist, since the semiwalk can be written out in a way that respects the direction of edges, much like the directed walk.