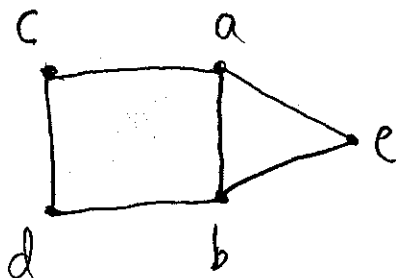


MAU22C00: TUTORIAL 10 SOLUTIONS
GRAPH THEORY

- 1) Let (V, E) be the graph with vertices a, b, c, d , and e and edges ab, bd, be, ac, cd , and ae .
- (a) Draw this graph. Write down its incidence table and its incidence matrix.
 - (b) Write down this graph's adjacency table and its adjacency matrix.
 - (c) Is this graph complete? Justify your answer.
 - (d) Is this graph bipartite? Justify your answer.
 - (e) Is this graph regular? Justify your answer.
 - (f) Does this graph have any regular subgraph? Justify your answer.
 - (g) Give an example of an isomorphism from the graph (V, E) specified at the beginning of this problem to the graph (V', E') with vertices p, q, r, s , and t , and edges pq, ps, rt, st, rs , and rq .

Solution: Let (V, E) be the graph with vertices a, b, c, d , and e and edges ab, bd, be, ac, cd , and ae .

- (a) Here is the graph:



If we keep the same order of the vertices and edges given in the statement of the problem, the incidence table is:

	ab	bd	be	ac	cd	ae
a	1	0	0	1	0	1
b	1	1	1	0	0	0
c	0	0	0	1	1	0
d	0	1	0	0	1	0
e	0	0	1	0	0	1

The corresponding incidence matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}.$$

- (b) If we keep the same order of the vertices given in the statement of the problem, the adjacency table is:

	a	b	c	d	e
a	0	1	1	0	1
b	1	0	0	1	1
c	1	0	0	1	0
d	0	1	1	0	0
e	1	1	0	0	0

The corresponding adjacency matrix is

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

- (c) No, as for example edge bc does not belong to the graph, so not every vertex is connected to every other vertex.
- (d) No, as the graph contains the complete subgraph $V' = \{a, b, e\}$ and $E' = \{ab, ae, be\}$, which cannot be partitioned.
- (e) No, as vertices a and b have degree 3, whereas the other vertices have degree 2.
- (f) Any two vertices that have an edge between them taken with that edge form a regular subgraph (1-regular) as do $\{a, b, e\}$ and the edges between them (2-regular) and $\{a, b, c, d\}$ and the edges between them (2-regular).

- (g) It does NOT suffice to show the two graphs have the same structure. An isomorphism is a MAP, so you must provide the map on vertices. Two possible isomorphisms are $\varphi(a) = s$, $\varphi(b) = r$, $\varphi(c) = p$, $\varphi(d) = q$, and $\varphi(e) = t$ or the following: $\varphi(a) = r$, $\varphi(b) = s$, $\varphi(c) = q$, $\varphi(d) = p$, and $\varphi(e) = t$.