1. **Answer:**

   (a) The graph is connected as you can get from any one vertex to any other vertex on the graph.

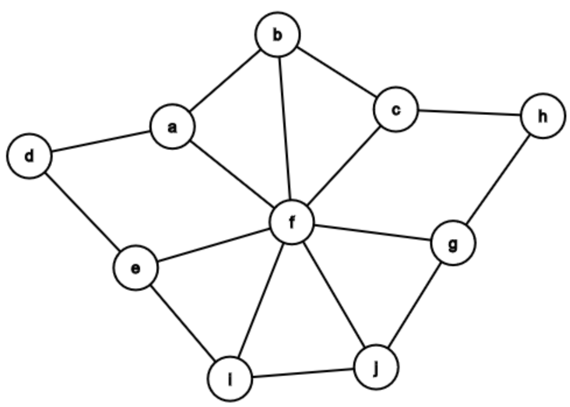
   (b) The graph does not have a Eulerian trail. In order to have a Eulerian trail, you must be able to visit each edge once and only once. However, going from some vertex v to the vertex f via any edge blocks you from visiting an edge next to v, which rules out at least one edge.

   (c) Yes. There exists a Eulerian circuit (note that vertex order in edges are indicative of vertex traversed) $fa \to af \to fb \to bf \to fc \to cf \to fg \to gf \to fj \to jf \to fi \to if \to fe \to ed \to da \to ab \to bc \to ch \to hg \to gj \to ji \to ie \to ed \to da \to af$.

   (d) Yes: $fabchgjied$.

   (e) No. A tree cannot contain any cycles, and the cycle $acbfa$ exists in the graph.

   (f) Graph:



   $d \to a \to b \to f \to c \to h \to g \to j \to i \to e \to d$.

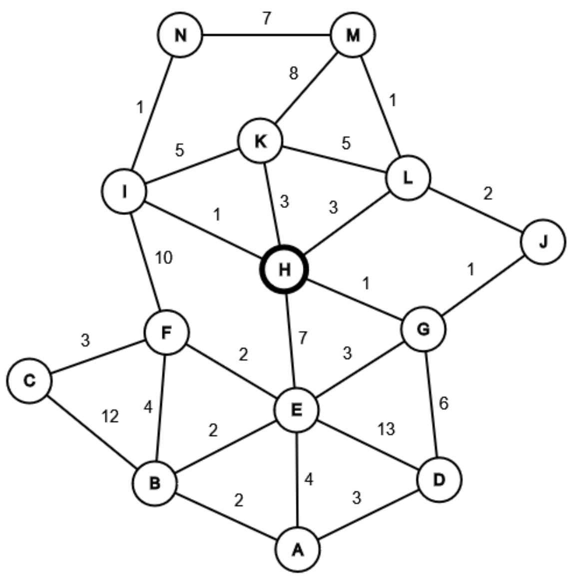
2. **Answer:**

   (a) Since the graph is connected, you must be able to get from any one vertex to any other vertex on the graph. If the two sets of edges E_1 and E_2 come together to form E, then there must be an edge that connects both subgraphs. Since an edge must contain two vertices, there must also be a vertex in both subgraphs due to it exist on the edge in both subgraphs. Therefore, a vertex exists in both subgraphs, and $V_1 \cap V_2 \neq \emptyset$.

   (b) A Hamiltonian path is a path that passes once through every vertex in a graph. Since you can get from any vertex to any other vertex in a connected graph, this path can be acyclical, as a tree as a result. Since we are only connecting vertices, and not forming a circuit of our own, it stands to reason that this path can become a spanning tree, due to its nature of being a connected acyclical path.

3. **Answer:**

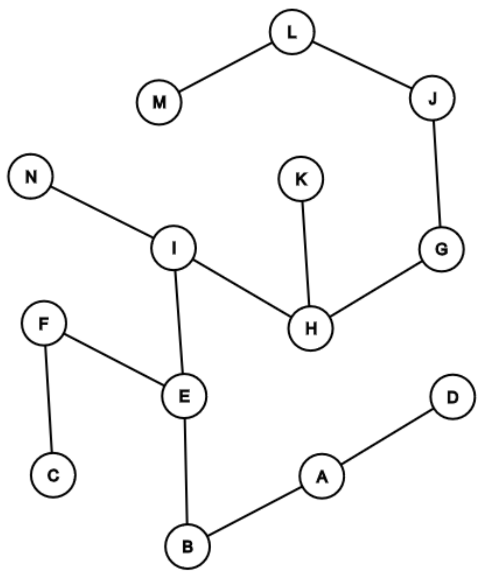
(a) Graph:



(b) Steps (Kruskal):

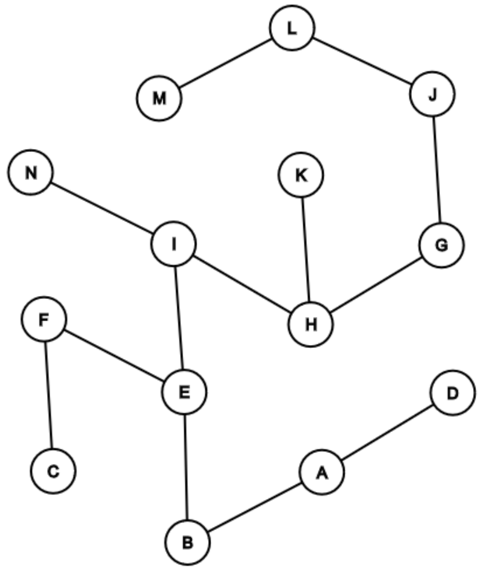
i. Add edge GJ
ii. Add edge LM
iii. Add edge HI
iv. Add edge GH
v. Add edge IN
vi. Add edge BE
vii. Add edge AB
viii. Add edge EF
ix. Add edge EI
x. Add edge JL
xi. Ignore edge EG
xii. Add edge CF
xiii. Add edge HK
xiv. Add edge AD
xv. Ignore edge HL
xvi. Ignore edge BF
xvii. Ignore edge AE
xviii. Ignore edge KI

xix. Ignore edge KL
xx. Ignore edge DG
xxi. Ignore edge EH
xxii. Ignore edge MN
xxiii. Ignore edge KM
xxiv. Ignore edge FI
xxv. Ignore edge BC
xxvi. Ignore edge DE



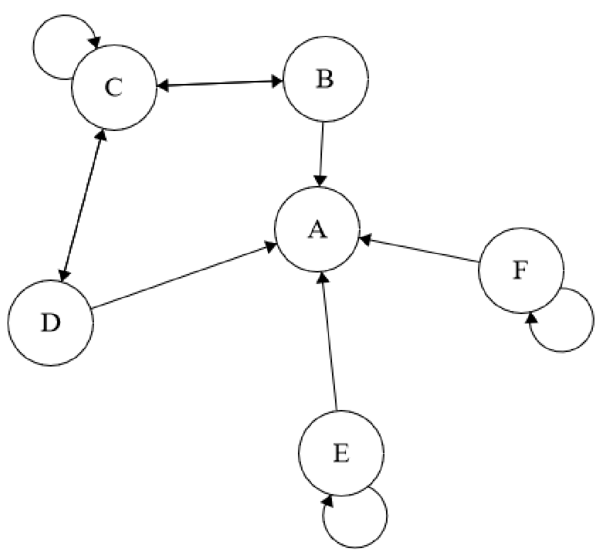
(c) Steps (Prim):

i. Add edge AD
ii. Add edge AB
iii. Add edge BE
iv. Add edge EF
v. Add edge EI
vi. Add edge HI
vii. Add edge GH
viii. Add edge GJ
ix. Add edge IN
x. Add edge JL
xi. Add edge LM
xii. Add edge CF
xiii. Add edge HK



4. **Answer:**

   (a) Graph:



(b) Adjacency matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

(c) $\varphi(B) = D$, $\varphi(D) = B$, $\varphi(C) = C$, $\varphi(A) = A$, $\varphi(E) = E$, $\varphi(F) = F$.

5. **Answer:** A directed walk between two points must respect edges between any two vertices v_{i-1}, v_i, whereas a semiwalk doesn't have to. In other words, each edge of a directed walk is an element of a Cartesian product of $V \times V$. Since a semiwalk can be in either order (e.g. v_{i-1}, v_i or v_i, v_{i-1}), it has to also exist where a directed walk exists. In other words, since a directed walk can exist between two vertices v_i and v_j in an ordered fashion, and a semiwalk can exist between the same two vertices potentially in an ordered fashion, if a directed walk exists, then there must exist a semiwalk.

   Conversely, if there exists a semiwalk between the two vertices, then a directed walk must exist, since the semiwalk can be written out in a way that respects the direction of edges, much like the directed walk.