

MAU22C00 - TUTORIAL 1

1) Give an example of sentences P and Q such that the implication $P \rightarrow Q$ is true, but its converse $Q \rightarrow P$ is false. Give another example of sentences P and Q such that the implication $P \rightarrow Q$ is true and its converse $Q \rightarrow P$ is also true. Given an implication $P \rightarrow Q$, what can you deduce about the truth value of its converse?

2) Suppose we are told the following:

If turtles can sing then artichokes can fly. Artichokes can fly implies turtles can sing and dogs can't play chess. Dogs can play chess if and only if turtles can sing.

Deduce that turtles can't fly.

3) Prove that the order of quantifiers cannot be reversed without potentially modifying the truth value by providing a counterexample. The easiest counterexample can be given for two quantifiers. Find a sentence in predicate logic $P(x, y)$ such that $\forall x \exists y P(x, y)$ is true, but $\exists y \forall x P(x, y)$ is false. Do not forget to specify to which set x and y belong!