STU22005 Applied Probability II

Professor Caroline Brophy

Continuous Assessment Sheet 2

Due date: submit before 4pm Friday 25th February 2022.

For this assignment, you must do and submit question 1. All other questions should also be done for practice and preparation for the live quiz during the week following submission, but are not handed up.

- Question 1 is to be done on your own and not in consultation with anyone else, do not discuss or show your answers to anyone else and do not look at answers belonging to anyone else. You are welcome to discuss your answers to the additional questions with other students.

To submit the assignment, you will submit one single pdf file:

- **File to be submitted**: Fill in the answer sheet that accompanies this assignment. Fill in the answers on the pdf document, save the file, and upload to Blackboard.

1. Lab results indicate that a drug for a disease is effective 80% of the time and ineffective in the remainder of cases. When effective, the drug increases the lifespan of a patient by 6 years. When ineffective, the drug causes a complication which decreases the lifespan of the patient by 1 year. As part of a trial study, the drug is administered to a large number of patients.

   (a) Using the lab results, calculate the expected value and variance of the lifespan increase for a single patient being treated with this drug.

   (b) What is the (approximate) probability that the average lifespan increase for 100 randomly selected patients will be 4 and a half years or more?

   Hint: Let L be the random variable in part (a) and start by considering the possible outcomes for L and with what probability each would occur, then find $E[L]$ and the $Var(L)$. In part (b), begin by defining the distribution of the random variable $\bar{L}$.

Below are additional questions that do not need to be handed up.

2. Consider $Y_1, \ldots, Y_9$ IID from a normal distribution with unknown mean μ and variance $\sigma^2 = 1$.

   (a) What is $P(|Y_1 - \mu| \leq \frac{1}{2})$?

   (b) What is $P(|\bar{Y} - \mu| \leq \frac{1}{2})$?

   Hint: Begin by defining the distribution of Y_1 and of $Y_1 - \mu$ in (a) and the distribution of $\bar{Y}$ and of $\bar{Y} - \mu$ in (b). Use sketches to understand the 'area under the curve' of interest to assist finding the probability in each case.

3. Experience has shown that the number of accidents that occur along a particular 10-mile stretch of a motorway is a Poisson random variable with a mean of 2 per week. What is the (approximate) probability that there will be less than 100 accidents on this stretch of motorway in a year (assuming 1 year = 52 weeks)?

   Hint: Begin by defnining the distribution of the random variable the number of accidents that occur in one week, call it X_i. Then define the distribution of the random variable X which is the sum of the X_i over 52 weeks.

4. The lifetime of a brand of light bulb in years is exponentially distributed with mean 10. What is the probability that the average lifetime of a random sample of 36 light bulbs is at least 10.5?

5. A sample of size 15 was collected from a normally distributed population. The standard deviation was computed as 4.362. Construct a 90% confidence interval for the variance.

6. Let $Y_1, \ldots, Y_n$ be an independent random sample from a $\mathrm{N}(\mu, \sigma^2)$. Define:

$$S^2 = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n-1}.$$

   Prove that

$$E(S^2) = \sigma^2$$

   Hint: start by expressing the of sum of squares as: $\sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2 = \sum_i \left[(Y_i - \mu) - (\bar{Y} - \mu)\right]^2$. *Then expand it and find the expectation.*