

Psikometrik Ağ Analizi

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Sunum İçeriği

- Psikometride geleneksel ve ağ analizi yaklaşımları
- Gaussian grafik modeli (GGM)
- Ağ modellerinin kestirimi
- Psikometride ağ analizi uygulamaları

Psikometride Geleneksel Yaklaşımlar

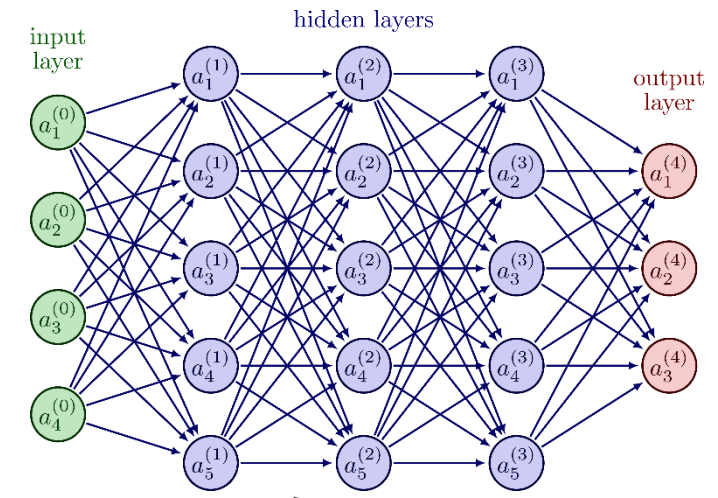
Eğitimsel ve psikolojik ölçmede:

1. **Yansıtıcı modelleme** (ör., faktör analitik modeller) Ölçülen özellik, gözlemlenen değişkenlere bağlı olarak kavramsallaştırılır.
2. **Biçimlendirici modelleme** (ör., temel bileşen analizi) Ölçülen özellik, gözlemlenen değişkenlerin ortak etkisi olarak görülür.
3. **Üçüncü bir modelleme yaklaşımı???**



Psikometride Ağ Analizi Yaklaşımı

- Ağ analizinde ölçülen özellikler, “gözlemlenen değişkenler arasındaki nedensel ilişkiler sistemi” olarak tanımlanır ([Schmittmann et al., 2013](#)).
- Örneğin,
 - Uyku eksikliği, yorgunluk ve konsantrasyon sorunlarına odaklanan bir depresyon envanterine sahip olduğumuzu varsayalım.
 - Geleneksel psikometrik bakış açısında, bu maddelerden elde edilen sonuçları birleştirir ve bir kişinin depresyon düzeyini temsil eden tek bir puan oluştururuz.
 - Ağ analizi perspektifinde ise öncelikli olarak belirtiler arasındaki olası doğrudan ilişkileri dikkate alırız.
 - Ör. uyku eksikliği → yorgunluk → konsantrasyon sorunları



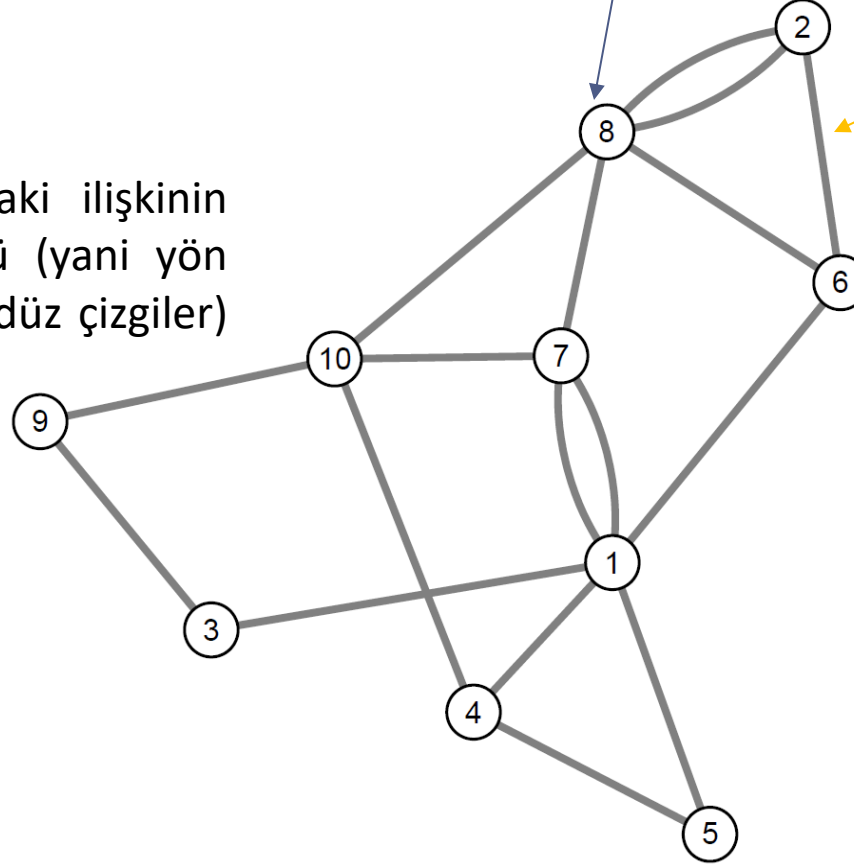
Psikometrik Ağlar \neq Sosyal Ağ ya da Yapay Sinir Ağları



Gaussian Grafik Modelleri (GGM)

- GGM ([Costantini et al., 2015](#) n sayıda **düğüm (node)** ve m sayıda **ayrıt (edge)** içeren bir ağ modelidir

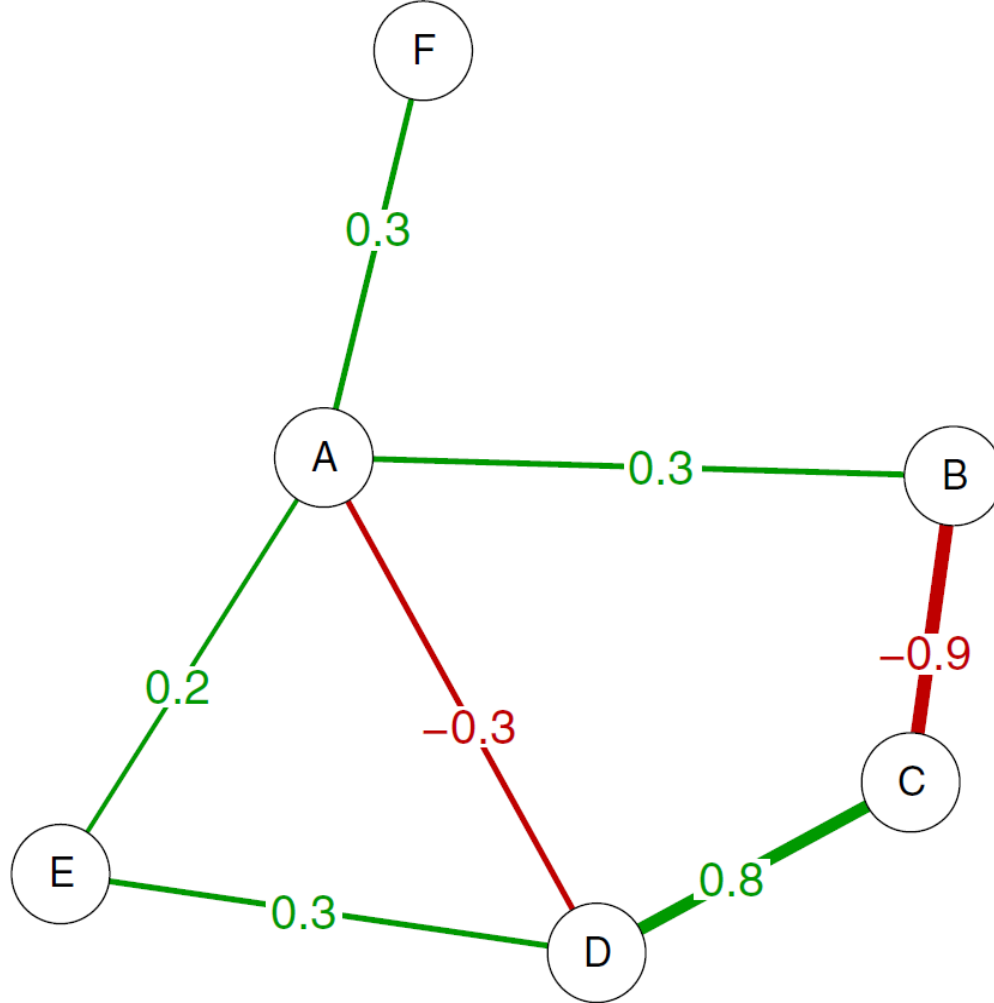
Her bir ayrıt değişkenler arasındaki ilişkinin gücünü temsil eder. Ayrıtlar yönlü (yani yön gösteren oklar) ya da yönsüz (yani düz çizgiler) olabilir.



Gaussian Grafik Modelleri (GGM)

- GGM modelleri genellikle kısmi korelasyon değerleri üzerinden kestirilir.
- Ağ grafikleri oluşturulurken araştırmacılar genellikle negatif kısmi korelasyonlar için “**kırmızı**” çizgiler, pozitif kısmi korelasyonlar için ise “**yeşil**” çizgiler kullanır.
- **Ağırlıklı ayırtlar:** Daha geniş (yani daha kalın) ve daha doygun çizgiler (yani bağlantılar), daha güçlü kısmi korelasyonları (yani sıfırdan uzak korelasyonları) gösterir.
- **Ağırlıksız ayırtlar:** Ayırt, yalnızca bir ilişkinin varlığını veya yokluğunu temsil eder.
- Eğer iki değişken arasında kısmi korelasyon = 0 ise, değişkenlerin birbirinden bağımsız olduğunu göstermek için iki düğüm arasında bağlantı çizilmez.

Ağırlıklı ayrıtların olduğu bir ağ modeli örneği



Altı düğümden (yani değişkenden) ve yedi ayrıttan oluşan bir ağ.

Pozitif ayrıtlar yeşil, negatif ayrıtlar kırmızı renk ile gösterilmektedir.

Harfler farklı düğümleri tanımlar, sayılar ise ayrıtlarla ilişkilendirilen ağırlıkları temsil etmektedir.

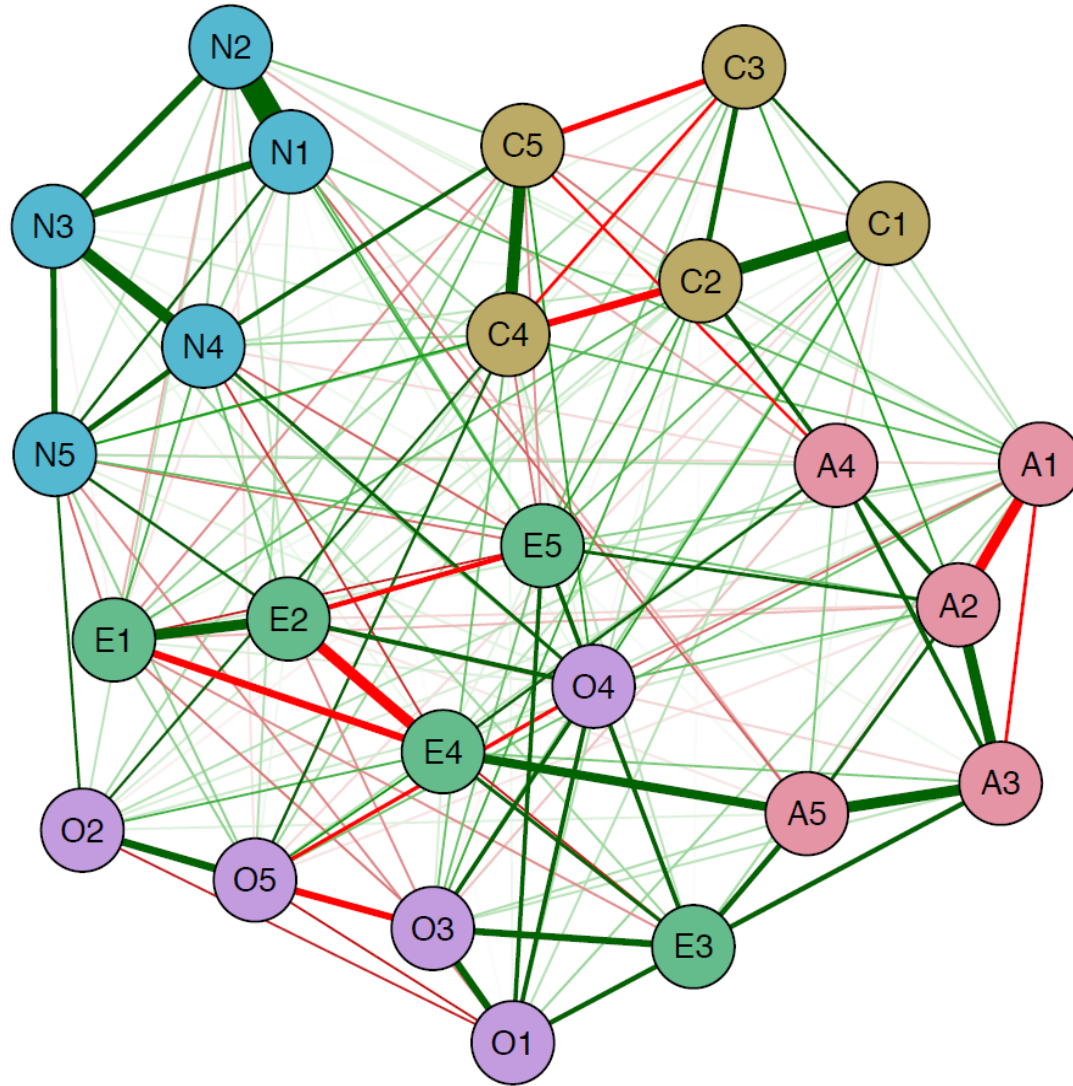
Kaynak: [Costantini et al. \(2015\)](#)



visualized
by Anna Vital

Source: J. M. Digman
Personality Structure: Emergence of the Five-Factor Model

Big Five Personality Test



Agreeableness

- A1: Am indifferent to the feelings of others.
- A2: Inquire about others' well-being.
- A3: Know how to comfort others.
- A4: Love children.
- A5: Make people feel at ease.

Conscientiousness

- C1: Am exacting in my work.
- C2: Continue until everything is perfect.
- C3: Do things according to a plan.
- C4: Do things in a half-way manner.
- C5: Waste my time.

Extraversion

- E1: Don't talk a lot.
- E2: Find it difficult to approach others.
- E3: Know how to captivate people.
- E4: Make friends easily.
- E5: Take charge.

Neuroticism

- N1: Get angry easily.
- N2: Get irritated easily.
- N3: Have frequent mood swings.
- N4: Often feel blue.
- N5: Panic easily.

Openness

- O1: Am full of ideas.
- O2: Avoid difficult reading material.
- O3: Carry the conversation to a higher level.
- O4: Spend time reflecting on things.
- O5: Will not probe deeply into a subject.

Kaynak: Epskamp, S. (2017). Network psychometrics (Doctoral dissertation). *Amsterdam: University of Amsterdam.*



Peki neden basit korelasyon varken kısmi korelasyon kullanıyoruz?

Aşağıdaki basit korelasyon matrislerine bakarak kaç faktörlü bir yapı olduğunu tahmin edelim...

V1	1	0.22	0.22	0.22	0.17	0.16	0.12	0.13	0.16	0.15	0.16	0.15
V2	0.22	1	0.27	0.28	0.21	0.19	0.15	0.16	0.2	0.18	0.2	0.19
V3	0.22	0.27	1	0.27	0.21	0.19	0.15	0.16	0.19	0.18	0.2	0.19
V4	0.22	0.28	0.27	1	0.21	0.19	0.15	0.16	0.2	0.18	0.2	0.19
V5	0.17	0.21	0.21	0.21	1	0.3	0.23	0.26	0.21	0.2	0.22	0.2
V6	0.16	0.19	0.19	0.19	0.3	1	0.21	0.24	0.2	0.19	0.2	0.19
V7	0.12	0.15	0.15	0.15	0.23	0.21	1	0.18	0.15	0.14	0.15	0.14
V8	0.13	0.16	0.16	0.16	0.26	0.24	0.18	1	0.17	0.16	0.17	0.16
V9	0.16	0.2	0.19	0.2	0.21	0.2	0.15	0.17	1	0.27	0.29	0.27
V10	0.15	0.18	0.18	0.18	0.2	0.19	0.14	0.16	0.27	1	0.27	0.26
V11	0.16	0.2	0.2	0.2	0.22	0.2	0.15	0.17	0.29	0.27	1	0.27
V12	0.15	0.19	0.19	0.19	0.2	0.19	0.14	0.16	0.27	0.26	0.27	1
V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	

V1	1	0.45	0.45	0.32	0.34	0.32	0.26	0.28	0.32	0.31	0.32	0.31
V2	0.45	1	0.52	0.37	0.39	0.37	0.31	0.33	0.37	0.36	0.37	0.36
V3	0.45	0.52	1	0.37	0.39	0.37	0.3	0.33	0.37	0.36	0.37	0.36
V4	0.32	0.37	0.37	1	0.56	0.53	0.31	0.33	0.37	0.36	0.37	0.36
V5	0.34	0.39	0.39	0.56	1	0.56	0.33	0.35	0.4	0.38	0.4	0.38
V6	0.32	0.37	0.37	0.53	0.56	1	0.31	0.33	0.37	0.36	0.38	0.36
V7	0.26	0.31	0.3	0.31	0.33	0.31	1	0.39	0.44	0.3	0.31	0.3
V8	0.28	0.33	0.33	0.33	0.35	0.33	0.39	1	0.47	0.32	0.33	0.32
V9	0.32	0.37	0.37	0.37	0.4	0.37	0.44	0.47	1	0.36	0.38	0.36
V10	0.31	0.36	0.36	0.36	0.38	0.36	0.3	0.32	0.36	1	0.52	0.5
V11	0.32	0.37	0.37	0.37	0.4	0.38	0.31	0.33	0.38	0.52	1	0.52
V12	0.31	0.36	0.36	0.36	0.38	0.36	0.3	0.32	0.36	0.5	0.52	1
V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	

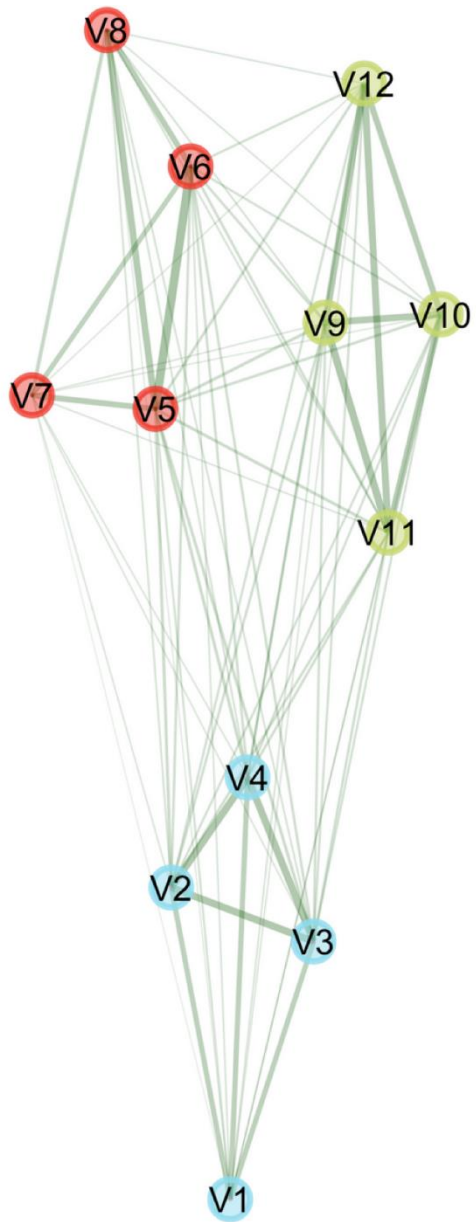
V1	1	0.45	0.45	0.45	0.48	0.45	0.26	0.28	0.32	0.31	0.32	0.31
V2	0.45	1	0.52	0.52	0.56	0.53	0.31	0.33	0.37	0.36	0.37	0.36
V3	0.45	0.52	1	0.52	0.56	0.53	0.3	0.33	0.37	0.36	0.37	0.36
V4	0.45	0.52	0.52	1	0.56	0.53	0.31	0.33	0.37	0.36	0.37	0.36
V5	0.48	0.56	0.56	0.56	1	0.56	0.33	0.35	0.4	0.38	0.4	0.38
V6	0.45	0.53	0.53	0.53	0.56	1	0.31	0.33	0.37	0.36	0.38	0.36
V7	0.26	0.31	0.3	0.31	0.33	0.31	1	0.39	0.44	0.42	0.44	0.43
V8	0.28	0.33	0.33	0.33	0.35	0.33	0.39	1	0.47	0.45	0.48	0.46
V9	0.32	0.37	0.37	0.37	0.4	0.37	0.44	0.47	1	0.52	0.54	0.52
V10	0.31	0.36	0.36	0.36	0.38	0.36	0.42	0.45	0.52	1	0.52	0.5
V11	0.32	0.37	0.37	0.37	0.4	0.38	0.44	0.48	0.54	0.52	1	0.52
V12	0.31	0.36	0.36	0.36	0.38	0.36	0.43	0.46	0.52	0.5	0.52	1
	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12

Şimdi de aynı verilerden elde edilen "kısmi" korelasyon matrislerine bakarak tahmin yapalım...

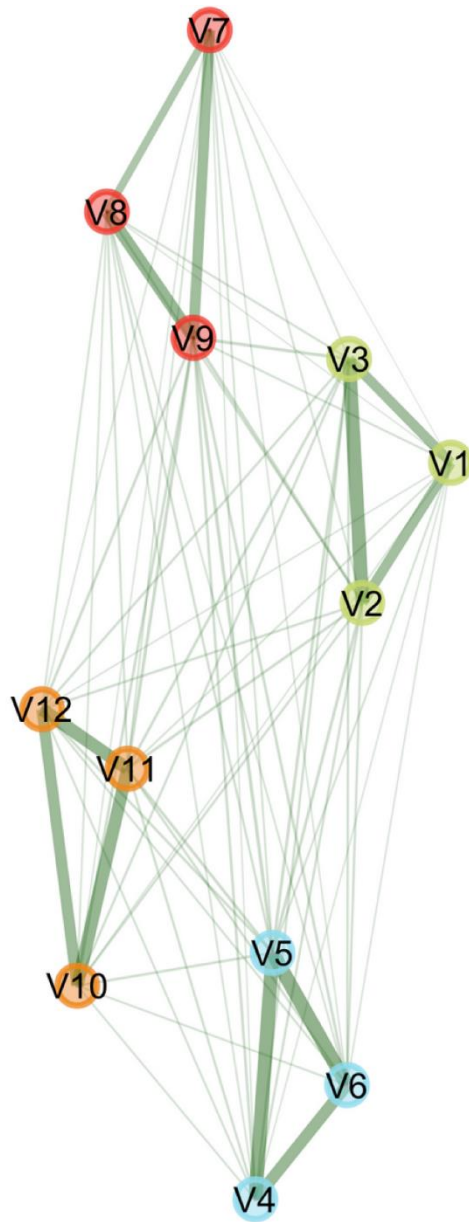
V1	0	0.1	0.1	0.1	0.04	0.03	0.02	0.02	0.03	0.03	0.03	0.03
V2	0.1	0	0.14	0.14	0.06	0.05	0.03	0.04	0.05	0.04	0.05	0.04
V3	0.1	0.14	0	0.14	0.06	0.05	0.03	0.04	0.05	0.04	0.05	0.04
V4	0.1	0.14	0.14	0	0.06	0.05	0.03	0.04	0.05	0.04	0.05	0.04
V5	0.04	0.06	0.06	0.06	0	0.16	0.11	0.13	0.06	0.05	0.06	0.05
V6	0.03	0.05	0.05	0.05	0.16	0	0.09	0.11	0.05	0.04	0.05	0.04
V7	0.02	0.03	0.03	0.03	0.11	0.09	0	0.07	0.03	0.03	0.03	0.03
V8	0.02	0.04	0.04	0.04	0.13	0.11	0.07	0	0.04	0.03	0.04	0.03
V9	0.03	0.05	0.05	0.05	0.06	0.05	0.03	0.04	0	0.13	0.14	0.13
V10	0.03	0.04	0.04	0.04	0.05	0.04	0.03	0.03	0.13	0	0.13	0.12
V11	0.03	0.05	0.05	0.05	0.06	0.05	0.03	0.04	0.14	0.13	0	0.13
V12	0.03	0.04	0.04	0.04	0.05	0.04	0.03	0.03	0.13	0.12	0.13	0
	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12

V1	0	0.19	0.19	0.03	0.04	0.03	0.02	0.03	0.04	0.03	0.04	0.03
V2	0.19	0	0.26	0.05	0.06	0.05	0.04	0.04	0.06	0.05	0.05	0.05
V3	0.19	0.26	0	0.05	0.06	0.05	0.04	0.04	0.06	0.05	0.05	0.05
V4	0.03	0.05	0.05	0	0.27	0.22	0.03	0.04	0.05	0.04	0.05	0.04
V5	0.04	0.06	0.06	0.27	0	0.27	0.04	0.05	0.06	0.05	0.06	0.05
V6	0.03	0.05	0.05	0.22	0.27	0	0.03	0.04	0.05	0.04	0.05	0.04
V7	0.02	0.04	0.04	0.03	0.04	0.03	0	0.15	0.2	0.03	0.04	0.03
V8	0.03	0.04	0.04	0.04	0.05	0.04	0.15	0	0.23	0.04	0.04	0.04
V9	0.04	0.06	0.06	0.05	0.06	0.05	0.2	0.23	0	0.05	0.06	0.05
V10	0.03	0.05	0.05	0.04	0.05	0.04	0.03	0.04	0.05	0	0.24	0.22
V11	0.04	0.05	0.05	0.05	0.06	0.05	0.04	0.04	0.06	0.24	0	0.25
V12	0.03	0.05	0.05	0.04	0.05	0.04	0.03	0.04	0.05	0.22	0.25	0
	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12

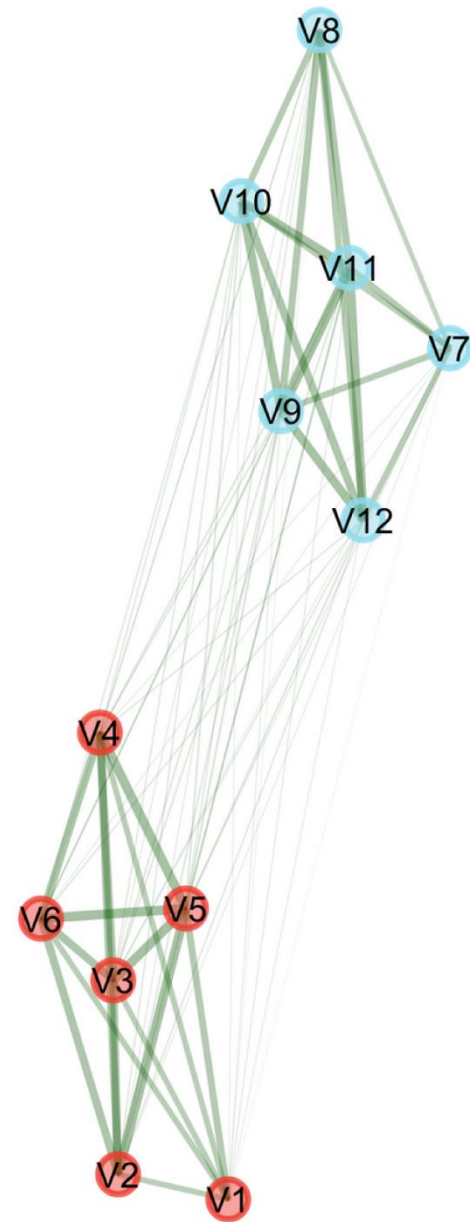
V1	0	0.11	0.11	0.11	0.13	0.11	0.01	0.01	0.02	0.02	0.02	0.02
V2	0.11	0	0.15	0.15	0.18	0.15	0.02	0.02	0.03	0.03	0.03	0.03
V3	0.11	0.15	0	0.15	0.18	0.15	0.02	0.02	0.03	0.03	0.03	0.03
V4	0.11	0.15	0.15	0	0.18	0.15	0.02	0.02	0.03	0.03	0.03	0.03
V5	0.13	0.18	0.18	0.18	0	0.18	0.02	0.03	0.04	0.03	0.04	0.03
V6	0.11	0.15	0.15	0.15	0.18	0	0.02	0.02	0.03	0.03	0.03	0.03
V7	0.01	0.02	0.02	0.02	0.02	0.02	0	0.09	0.12	0.11	0.12	0.11
V8	0.01	0.02	0.02	0.02	0.03	0.02	0.09	0	0.14	0.12	0.14	0.13
V9	0.02	0.03	0.03	0.03	0.04	0.03	0.12	0.14	0	0.16	0.18	0.17
V10	0.02	0.03	0.03	0.03	0.03	0.03	0.11	0.12	0.16	0	0.17	0.15
V11	0.02	0.03	0.03	0.03	0.04	0.03	0.12	0.14	0.18	0.17	0	0.17
V12	0.02	0.03	0.03	0.03	0.03	0.03	0.11	0.13	0.17	0.15	0.17	0
	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12



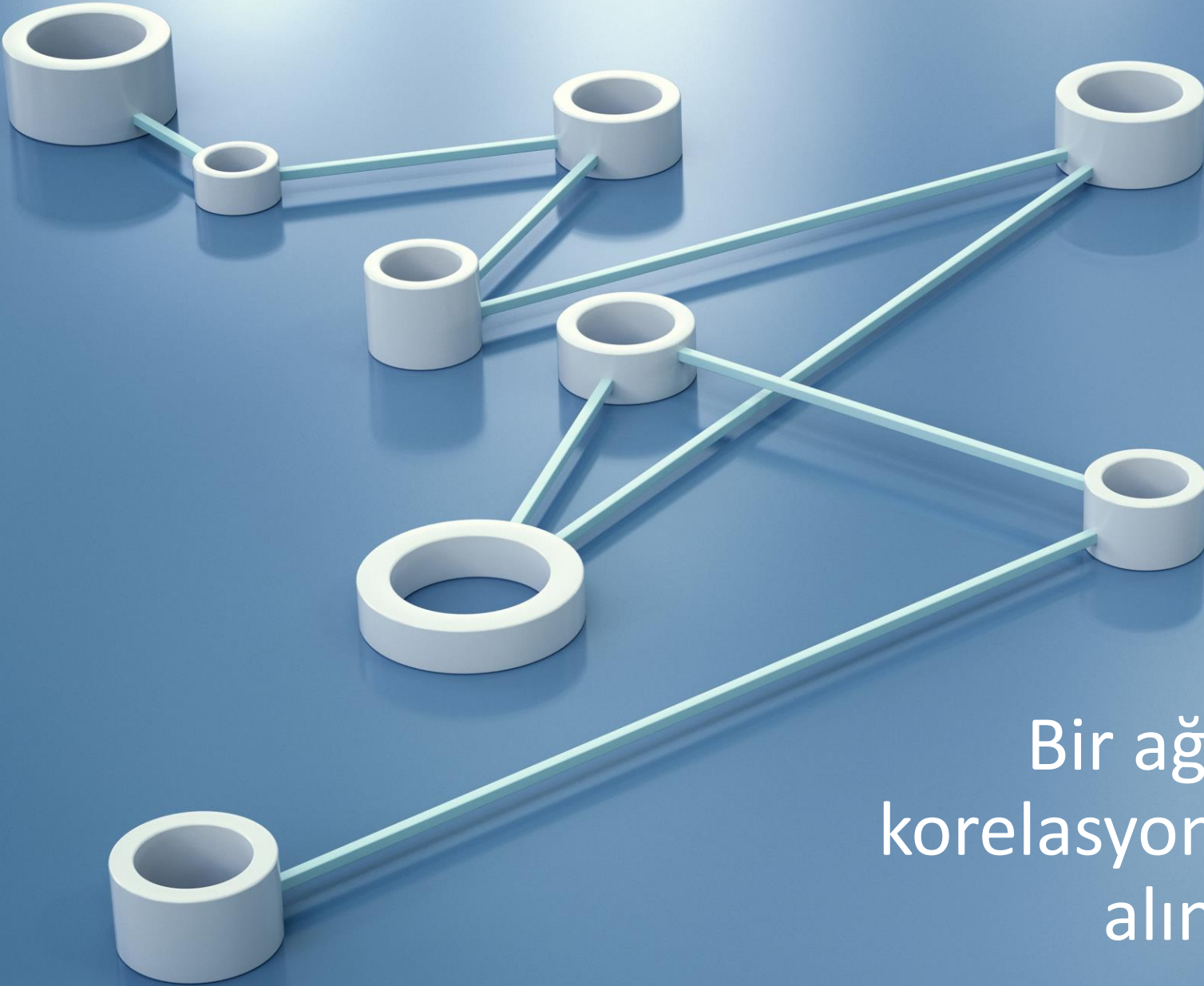
- 1
- 2
- 3



- 1
- 2
- 3
- 4



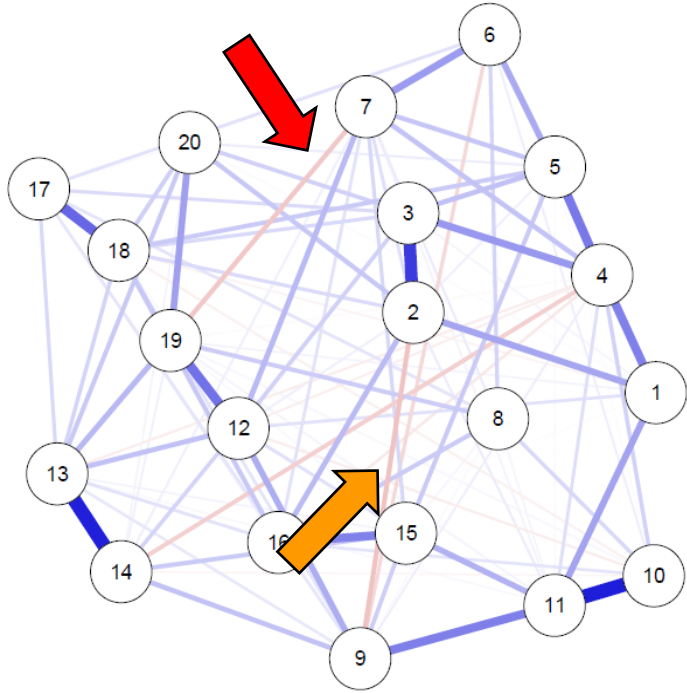
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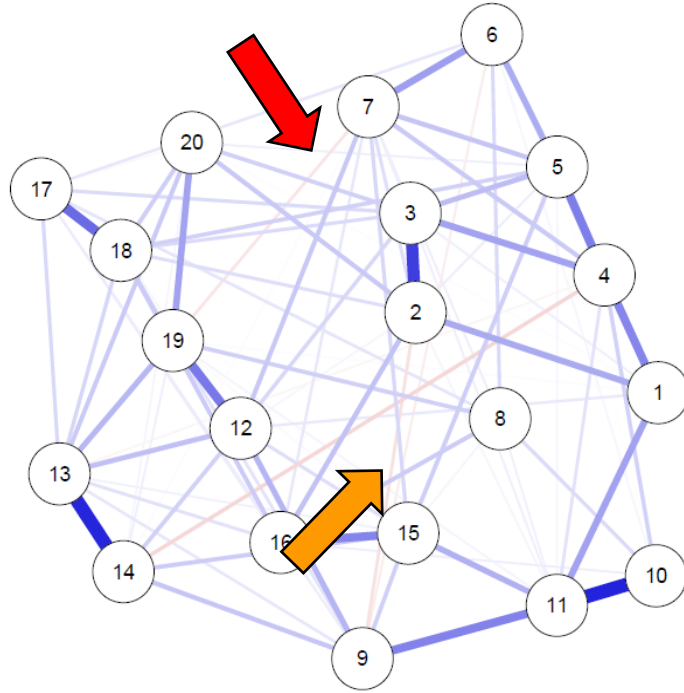
Bir ađ içindeki kısmi
korelasyonların hepsi dikkate
alınmalı mıdır?

Ağ Modellerinin Kestirimi: Lasso ve Grafiksel Lasso

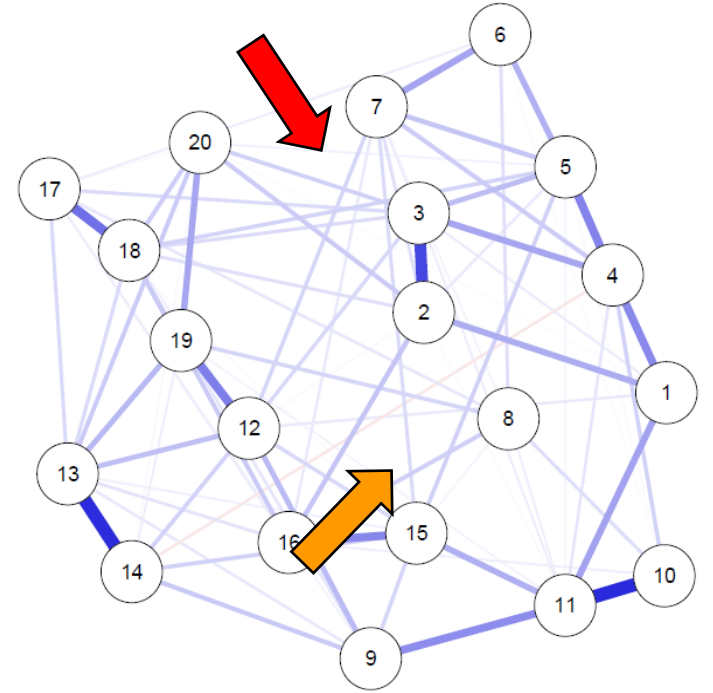
- Ağ yapılarında kompleks modelleri önlemek için zayıf bağlantıların sayısını sınırlamak gereklidir.
- Least absolute shrinkage and selection operator (Lasso; [Tibshirani, 1996](#)) gibi istatistiksel düzenleme tekniklerini kullanarak zayıf bağlantıları ortadan kaldırmak mümkündür.
- Lasso (L1) düzenlemesi, bir ağ modeli tahmin edilirken kısmi korelasyon katsayılarını daraltır, bu da küçük korelasyon katsayıların sıfır olarak tahmin edildiği anlamına gelir.
- Lasso GGM modellerinin kestiriminde kullanıldığında grafiksel lasso ya da kısaca Glasso ([Friedman et al., 2008](#)) ismini alır.



$\lambda = 0$



$\lambda = 0.25$



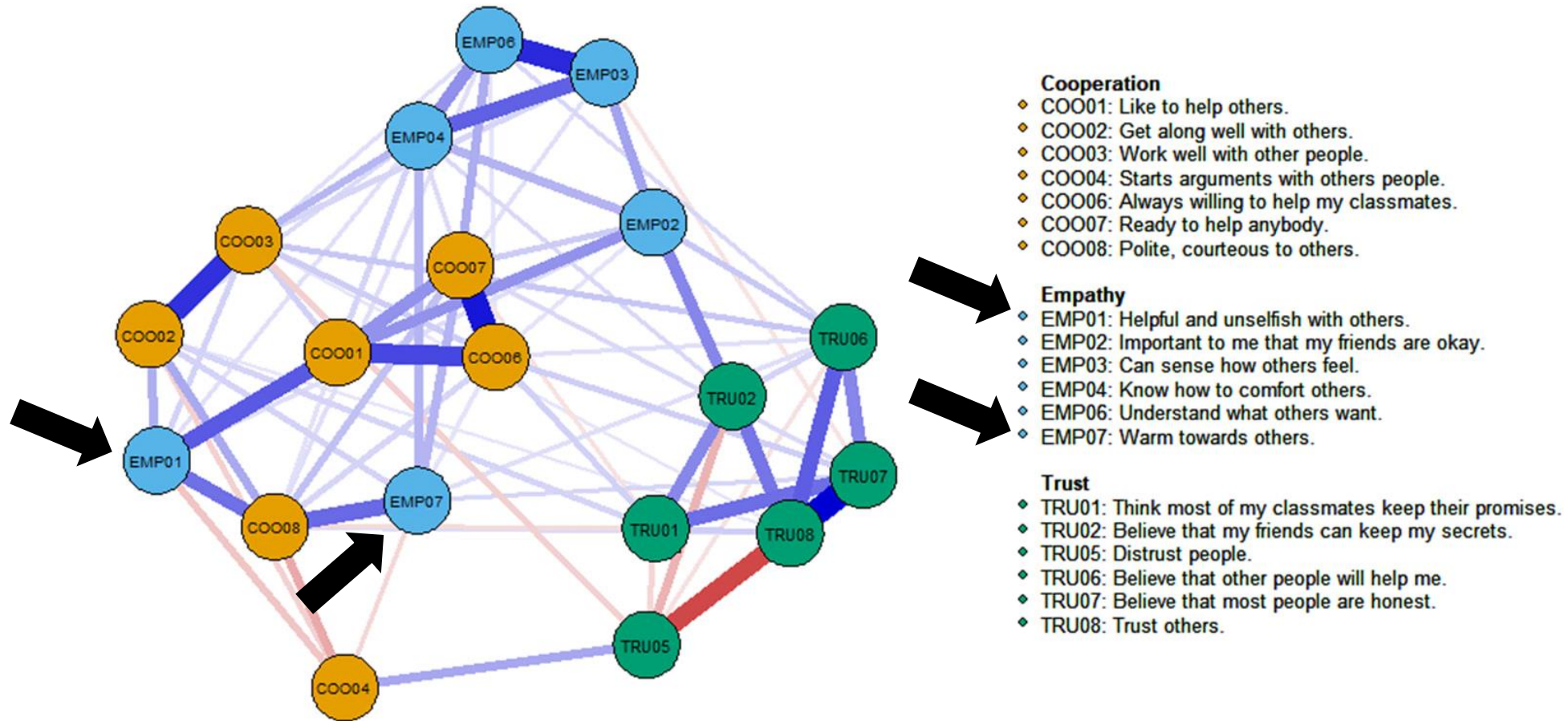
$\lambda = 0.5$

Kaynak: [Epskamp and Fried \(2017\)](#)

Psikometride Ağ Analizi Uygulamaları



Yapı ve Kapsam Geçerliliğini Test Etme



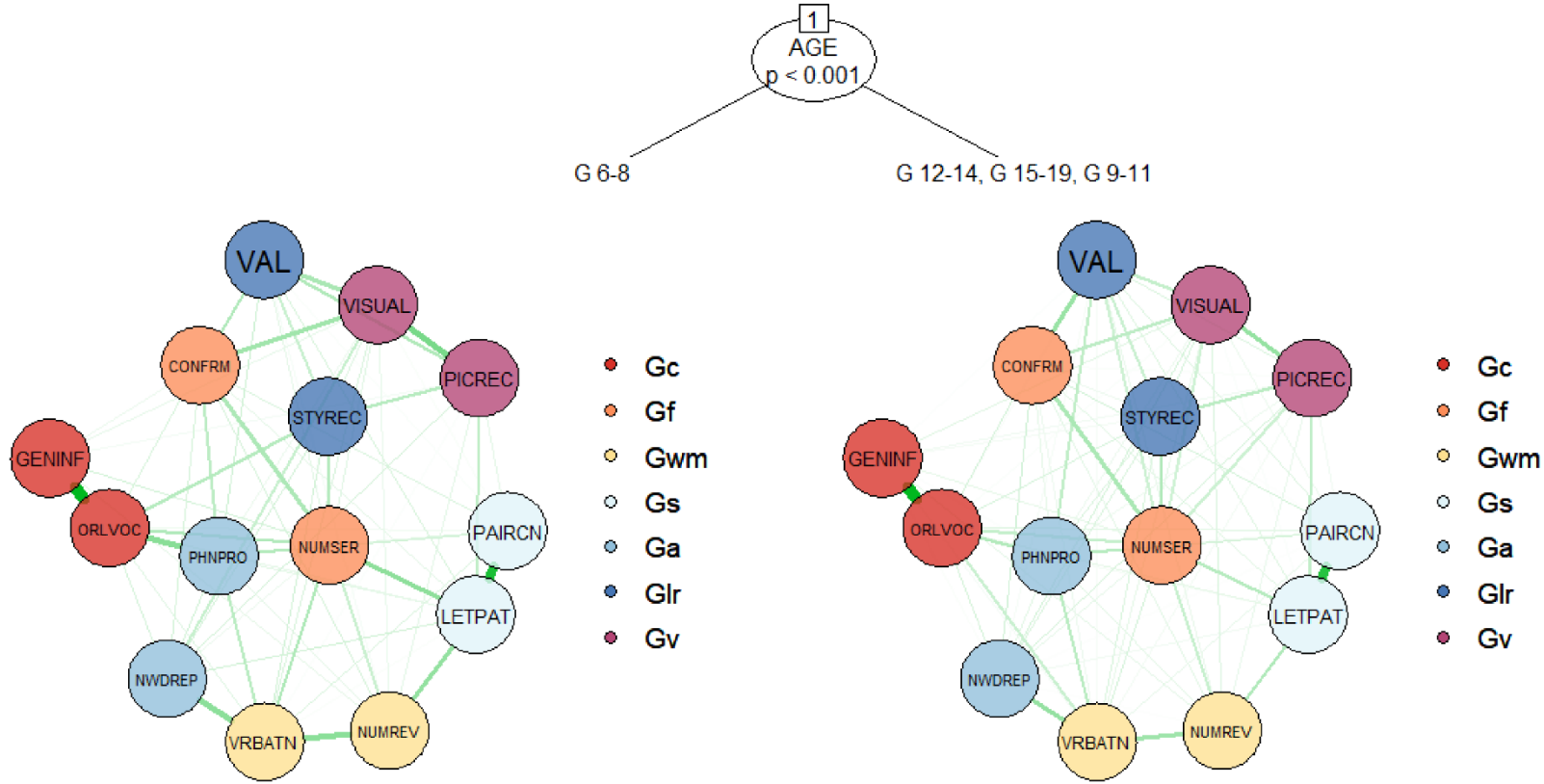
Clelland, A.C., & Bulut, O. (2024, April). *A network psychometrics approach to explore Canadian students' collaborative skills*. Poster presented at the annual meeting of the American Educational Research Association, Philadelphia, PA.

Test Boyutluluğu Analizi

- Exploratory Graph Analysis (EGA; [Golino & Epskamp, 2017](#))

V1	0	0.11	0.11	0.11	0.13	0.11	0.01	0.01	0.02	0.02	0.02	0.02
V2	0.11	0	0.15	0.15	0.18	0.15	0.02	0.02	0.03	0.03	0.03	0.03
V3	0.11	0.15	0	0.15	0.18	0.15	0.02	0.02	0.03	0.03	0.03	0.03
V4	0.11	0.15	0.15	0	0.18	0.15	0.02	0.02	0.03	0.03	0.03	0.03
V5	0.13	0.18	0.18	0.18	0	0.18	0.02	0.03	0.04	0.03	0.04	0.03
V6	0.11	0.15	0.15	0.15	0.18	0	0.02	0.02	0.03	0.03	0.03	0.03
V7	0.01	0.02	0.02	0.02	0.02	0.02	0	0.09	0.12	0.11	0.12	0.11
V8	0.01	0.02	0.02	0.02	0.03	0.02	0.09	0	0.14	0.12	0.14	0.13
V9	0.02	0.03	0.03	0.03	0.04	0.03	0.12	0.14	0	0.16	0.18	0.17
V10	0.02	0.03	0.03	0.03	0.03	0.03	0.11	0.12	0.16	0	0.17	0.15
V11	0.02	0.03	0.03	0.03	0.04	0.03	0.12	0.14	0.18	0.17	0	0.17
V12	0.02	0.03	0.03	0.03	0.03	0.03	0.11	0.13	0.17	0.15	0.17	0
	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12

Ölçme Değişmezliği

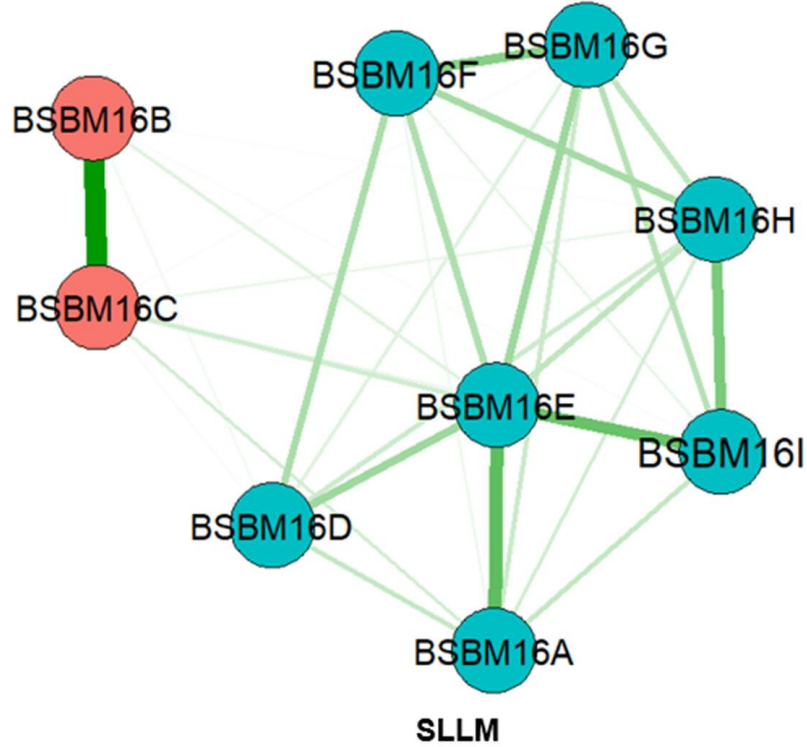


Kaynak: Bulut, O., Cormier, D. C., Aquilina, A. M., & Bulut, H. C. (2021). Age and sex invariance of the Woodcock-Johnson IV Tests of Cognitive Abilities: Evidence from psychometric network modeling. *Journal of Intelligence*, 9(3), 1-16.

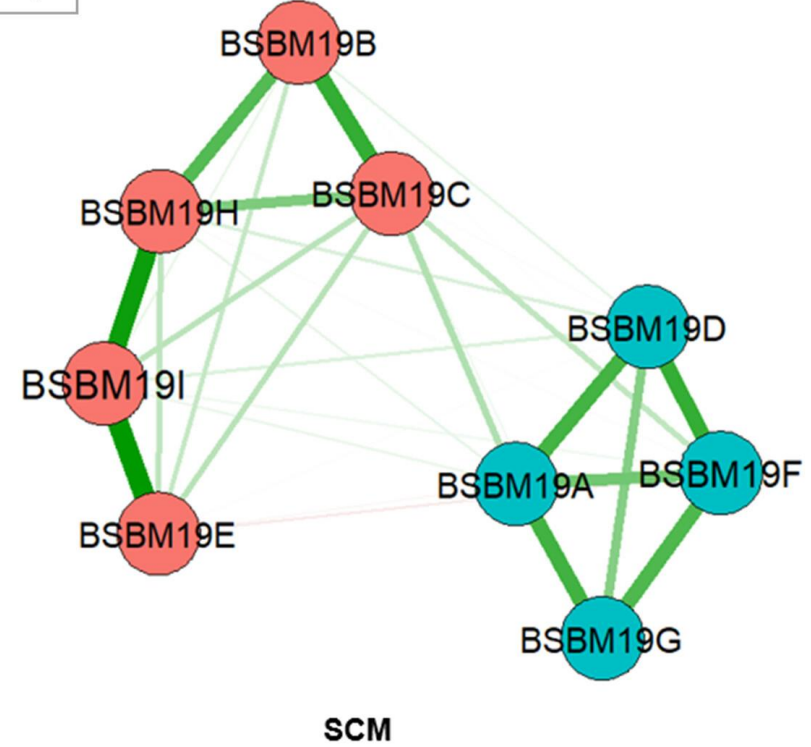
<https://doi.org/10.3390/jintelligence9030035>

Madde İfade Etkisi

a)



b)



- Negatively worded items
- Positively worded items

Kaynak: Bulut, H. C., Bulut, O., & Clelland, A. (2024). A psychometric network analysis approach for detecting item wording effects in self-report measures across subgroups. *Field Methods*. <https://doi.org/10.1177/1525822X241247444>

“Students Like Learning Mathematics” in TIMSS 2019

BSBM16A	I enjoy learning mathematics
BSBM16B_R	I wish I did not have to study mathematics
BSBM16C_R	Mathematics is boring
BSBM16D	I learn many interesting things in mathematics
BSBM16E	I like mathematics
BSBM16F	I like any schoolwork that involves numbers
BSBM16G	I like to solve mathematics problems
BSBM16H	I look forward to mathematics lessons
BSBM16I	Mathematics is one of my favorite subjects.

Olumsuzluk

Olumsuz çağrışım

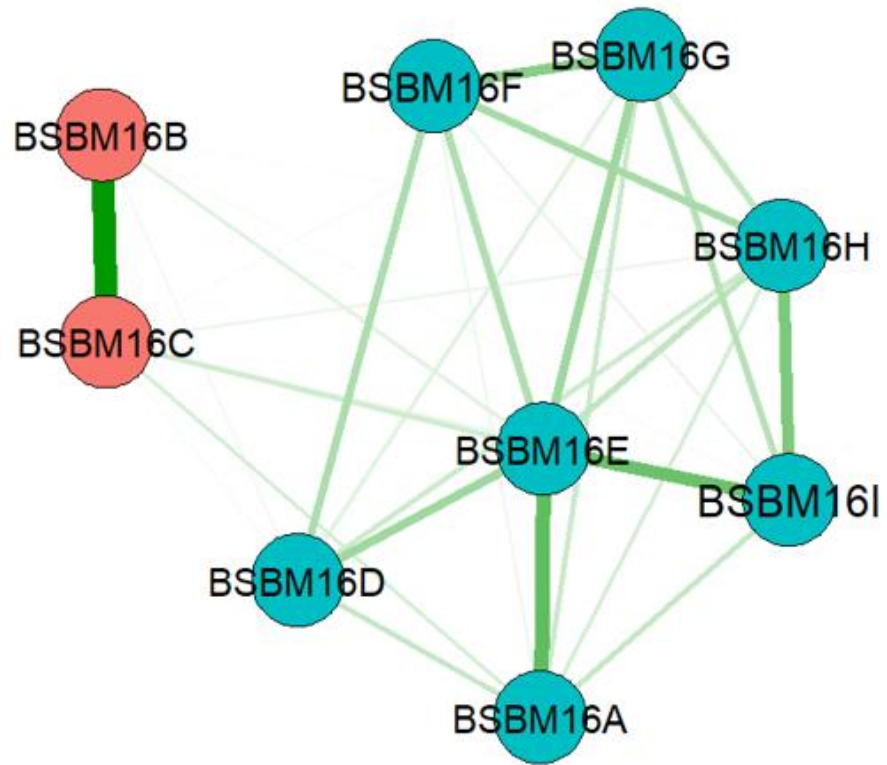
1 = Disagree a lot 2 = Disagree 3 = Agree 4 = Agree a lot

4 = Disagree a lot 3 = Disagree 2 = Agree 1 = Agree a lot

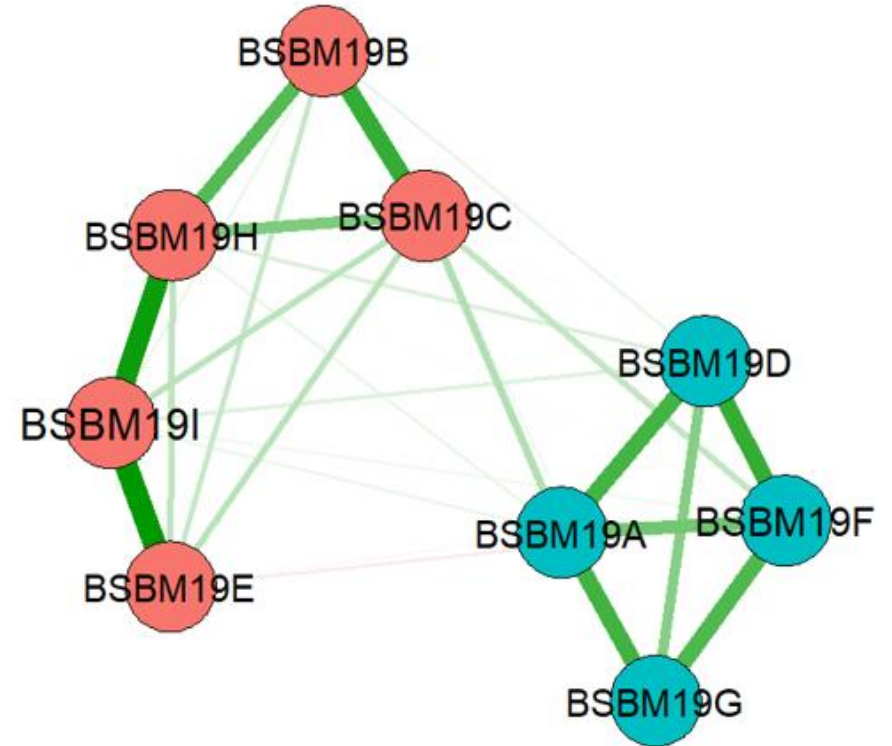
“Students Confident in Mathematics” in TIMSS 2019



BSBM19A	I usually do well in mathematics
BSBM19B_R*	Mathematics is more difficult for me than for many of my classmates
BSBM19C_R**	Mathematics is not one of my strengths
BSBM19D	I learn things quickly in mathematics
BSBM19E_R	Mathematics makes me nervous
BSBM19F	I am good at working out difficult mathematics problems
BSBM19G	My teacher tells me I am good at mathematics
BSBM19H_R	Mathematics is harder for me than any other subject
BSBM19I_R	Mathematics makes me confused

Students Like Learning Mathematics (SLLM)



Students Confident in Mathematics (SCM)



 Olumlu maddeler
 Olumsuz maddeler

Yapı ve Madde Tutarlılığı

Yapı Tutarlılığı

Tespit edilen ağ yapısının tekrarlı bootstrap örneklemeler alındığında yine ortaya çıkıyor mu?

Madde Tutarlılığı

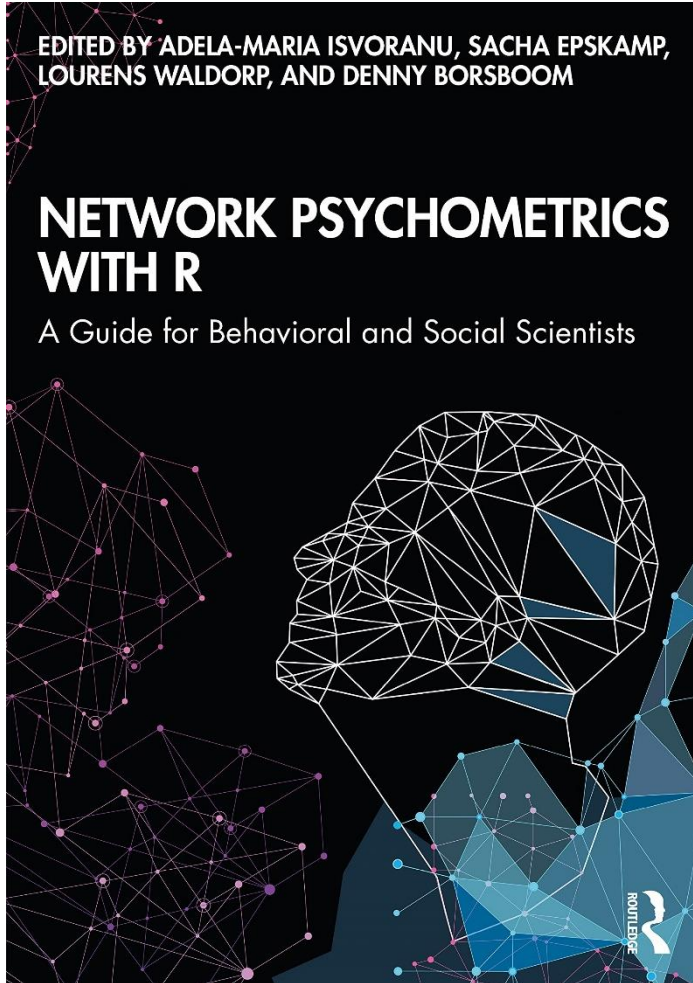
Analiz edilen maddeler tekrarlı bootstrap örneklemeler alındığında yine yine aynı madde grubu ya da komünitesine ait çıkıyor mu?

Kaynak: Christensen, A. P., & Golino, H. (2021). Estimating the stability of psychological dimensions via bootstrap exploratory graph analysis: A Monte Carlo simulation and tutorial. *Psych*, 3(3), 479-500. <https://doi.org/10.3390/psych3030032>

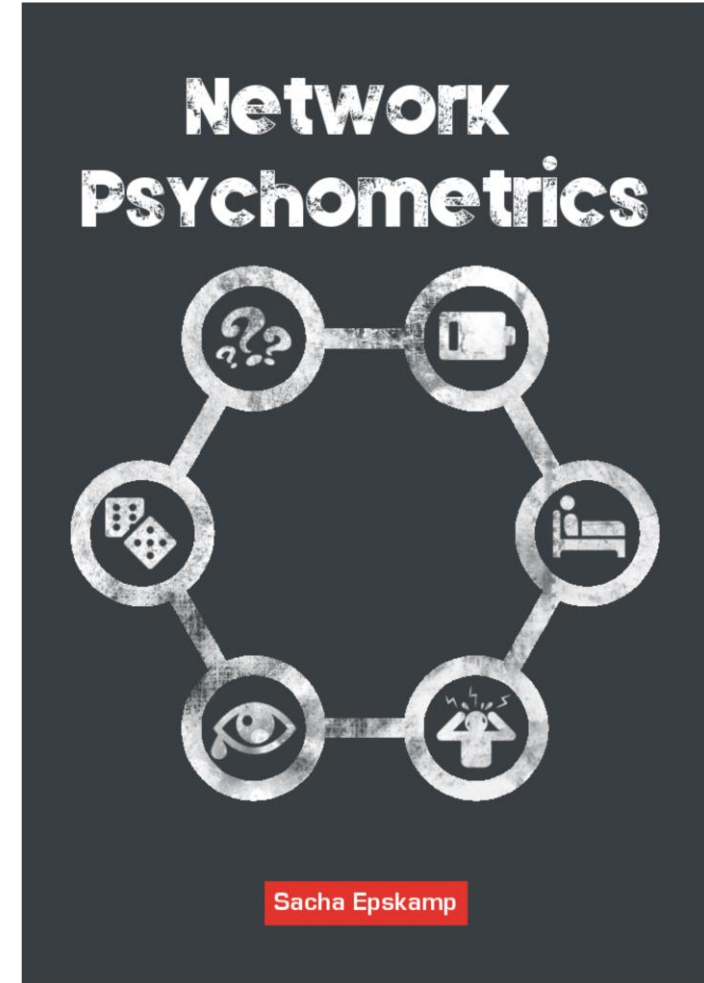
Yerel Bağımsızlık Varsayımını Test Etme

- Ölçülen ortak bir beceri olması dışında maddeleri birbirlerine bağlayan başka etken var mı? (Madde Tepki Kuramı)
- **Unique Variable Analysis** ([Christensen et al., 2022](#)) birbirlerine bağlı madde çiftlerini tespit etmekte kullanılır → weighted topological overlap (wTO)
- wTO ağ içindeki düğümlerin ne ölçüde benzer ayırıt ya da kenarlara sahip olduğunu ölçer.
- $wTO \geq 0.25$ → maddeler arası gereğinden fazla bağımlılık olduğunu, yani bazı soruların gereksiz olduğunu gösterir.

Ek Kaynaklar



<https://doi.org/10.4324/9781003111238>



https://www.researchgate.net/publication/313344229_Network_Psychometrics