Discrete Fourier Transform

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Contents

1 Summary

In this report, calculation of Discrete Fourier Transform (DFT) and it's implementation on C will be explained. The role of DFT is very important in many areas and spectrum analysis is one of them. Source code of the C program can be found in the Git Repository.

2 Introduction

DFT is designed to give information about frequency and phase information related to the desired signal. DFT deals with the signal which is discrete. Discrete–Time Fourier Transform (DTFT) should not be confused with DFT. DTFT works on sampled continuous time signals while DFT works on discrete signals. Also it should be mentioned that, in order to stabilize the speed of algorithm, size of the DFT must be a power of 2.

3 Calculation Step by Step

Before implementing the C Program for the DFT, whole process should be done by hand. DFT of any signal can be calculated as indicate in the equation 1 below;

$$X[k] = \sum_{n=0}^{N-1} X[n]e^{-j2\pi\frac{k}{N}n} \tag{1}$$

For example, $X[n]$ can be assumed as an array which is given below;

$$X[n] = [1, 2, 3, 4].$$

Each iteration will be explained and DFT calculation will be made for the $X[n]$ array. For $k = 0$;

$$\begin{aligned} X[0] &= 1e^{-j2\pi\frac{0}{4}0} + 2e^{-j2\pi\frac{0}{4}1} + 3e^{-j2\pi\frac{0}{4}2} + 4e^{-j2\pi\frac{0}{4}3} \\ &= 1 + 2 + 3 + 4 \\ &= 10 \end{aligned}$$

For $k = 1$;

$$\begin{aligned}
X[1] &= 1e^{-j2\pi\frac{1}{4}0} + 2e^{-j2\pi\frac{1}{4}1} + 3e^{-j2\pi\frac{1}{4}2} + 4e^{-j2\pi\frac{1}{4}3} \\
&= 1 + 2e^{-j\frac{\pi}{2}} + 3e^{-j\pi} + 4e^{-j\frac{3\pi}{2}} \\
&= 1 + 2(-j) + 3(-1) + 4(j) \\
&= -2 + 2j
\end{aligned}$$

For $k = 2$;

$$\begin{aligned}
X[2] &= 1e^{-j2\pi\frac{2}{4}0} + 2e^{-j2\pi\frac{2}{4}1} + 3e^{-j2\pi\frac{2}{4}2} + 4e^{-j2\pi\frac{2}{4}3} \\
&= 1e^{0} + 2e^{-j\pi} + 3e^{-j2\pi} + 4e^{-j3\pi} \\
&= 1 - 2 + 3 - 4 \\
&= -2
\end{aligned}$$

For $k = 3$;

$$\begin{aligned}
X[3] &= 1e^{-j2\pi\frac{3}{4}0} + 2e^{-j2\pi\frac{3}{4}1} + 3e^{-j2\pi\frac{3}{4}2} + 4e^{-j2\pi\frac{3}{4}3} \\
&= e^{0} + 2e^{-j\frac{3\pi}{2}} + 3e^{-j3\pi} + 4e^{-j\frac{9\pi}{2}} \\
&= 1 + 2j + 3(-1) + 4(-j) \\
&= -2 - 2j
\end{aligned}$$