Step1

Converting the transfer function to the state-space;

$$H(s) = \frac{s^2 + 2s + 50}{s^3 + 10s^2 + 24s}$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

Depending up on the original equation, b0, b1, b2, b3 and a1, a2, a3 can be found. Then using these values, firstly β1, β2 and β3 for B matrix after that state-space model is produced.

A =[0 1 0	B =[1	C =[1 0 0]
0 0 1	-8	
0 -24 -10]	106]	D= 0

Step2

To constitute desired polynomial using own roots; $s^3 + 7s^2 + 15s + 25$. (Actually this step is not require for method 3-acker, but when using method 1 or 2, it requires.)

Step3

To find state-feedback gain K;

K=[0.5000 -0.0904 -0.0398];

Step4

To find full-state observedr gain Ke;

Ke=[20; 76; -240];

Step5

$$\frac{U(s)}{-Y(s)} = K\left(sI - A + K_eC + BK\right)^{-1} K_e$$

$$\frac{12.69 * s^2 + \ 163.66 * s^1 + \ \ 500.0000}{s^3 \ \ 27 * s^2 \ \ 218.31 * s^1 \ \ 554.87}$$

Step6 (minumum order observer)

Minumum order observer desired poles are -10,-10 so we can say that; re_min=[-10,-10]

A =[0 1 0

0 0 1 C =[1 0 0] = Aab

0 -24 -10] = Abb

To determine the Ke for minumum order observer using acker(Abb’,Aab’,re_min);

Ke_min=[10; -24];

Step7

Using these notations: (calculations are made in MATLAB)

$$\hat{\mathbf{A}} = \mathbf{A}_{bb} - \mathbf{K}_e \mathbf{A}_{ab}$$
$$\hat{\mathbf{B}} = \hat{\mathbf{A}}\mathbf{K}_e + \mathbf{A}_{ba} - \mathbf{K}_e A_{aa}$$
$$\hat{\mathbf{F}} = \mathbf{B}_b - \mathbf{K}_e B_a$$

$$\tilde{\mathbf{A}} = (\hat{\mathbf{A}} - \hat{\mathbf{F}}\mathbf{K}_b) \qquad \tilde{\mathbf{B}} = \hat{\mathbf{B}} - \hat{\mathbf{F}}(K_a + \mathbf{K}_b\mathbf{K}_e)$$
$$\tilde{\mathbf{C}} = -\mathbf{K}_b \qquad \tilde{D} = -(K_a + \mathbf{K}_b\mathbf{K}_e)$$

When we take apart K, we get in this form [Ka(1. row) , Kb (2. and 3. rows)]

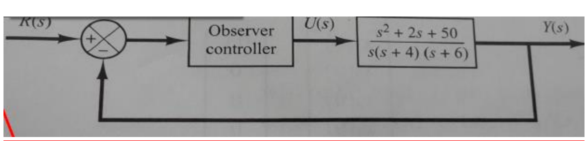
Using above terms transfer function of observer is;

0.5512 s^2 + 12.68 s + 50

----

s^2 + 16.45 s + 52.78

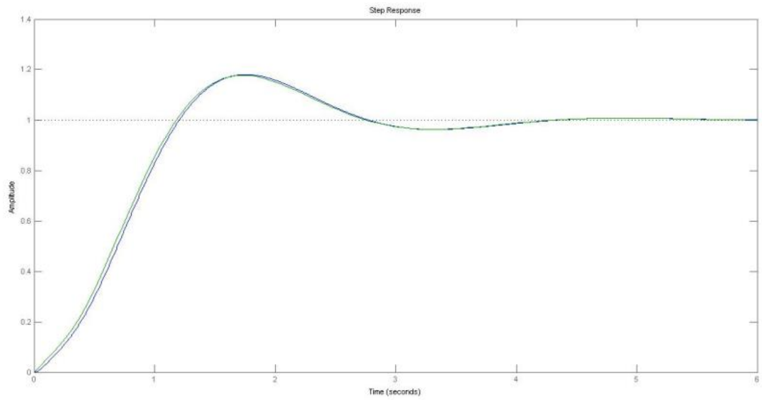
Step8



Plant is series to the observer and a negative feedback. Using the proper command in MATLAB, closed loop transfer function is obtained.

Step9

Step response of the closed loop transfer function like that;



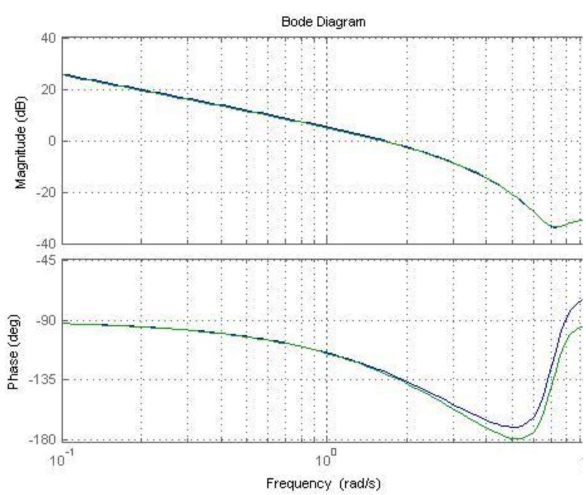
Green curve represents closed loop system with minumum-order observer

Blue curve represents closed loop system with full-state observer

Step response of the both design are nearly.

Step10

To reach the bandwidth information both system, bode plot is required.

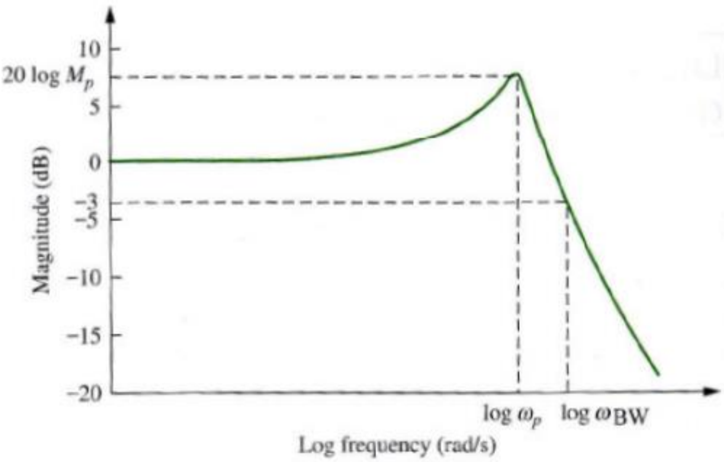


*The logspace is restricted in (-1,1,100) because of more readability.

Green curve represents closed loop system with minumum-order observer

Blue curve represents closed loop system with full-state observer

To get bandwidth from bode plot:



“The bandwidth of the system with **minimum-order observer** is **higher than** that of the system with the **full-order observer**, provided that multiple observer poles are placed at the same place for both observers.” (this information is taken from slide_10)

BW_moo = 2.6671 % minumum order observer bandwitdh

BW_fso = 2.6839 % full-state observer bandwitdh

The result of bandwitdh and our expectation are not fit. We expect more difference between bandwitdh. But result are nearly identical.

MATLAB code is available from [here.]()