1)

a) step 1: $a_n = a_n^{(h)} + a_n^{(p)}$ → homo → particular

step 2: $a_n^{(h)} \Rightarrow \underbrace{a_n}_{r} = 3\underbrace{a_{n-1}}_{1}$

$\boxed{r=3}$
(characteristic root)

step 3: $a_n^{(h)} = c_1 \cdot (3)^n$ → homo equation

step 4:

$a_n^{(p)} = A.2^n$

$a_{n-1} = A.2^{n-1}$

$\Rightarrow A.2^n = \frac{3A}{2}.2^n + 2^n$

$\frac{-A}{2}.2^n = 2^n \Rightarrow \frac{-A}{2} = 1$

$\boxed{A=-2}$

$a_n^{(p)} = -2.2^n$

$\boxed{a_n^{(p)} = -2^{n+1}}$

b) $a_n = a_n^{(h)} + a_n^{(p)}$

$a_n = c_1 \cdot (3)^n - 2^{n+1}$

step 5:

$a_0 = 1 \Rightarrow c_1 - 2 = 1 \Rightarrow \boxed{c_1 = 3}$

$a_n = 3.3^n - 2^{n+1}$

$\boxed{a_n = 3^{n+1} - 2^{n+1}}$

2)

Step 1 : $f(n) = f(n)^{(h)} + f(n)^{(p)}$

Step 2 : $f(n)^{(h)} \Rightarrow \underbrace{f(n)}_{r^2} = 4\underbrace{f(n-1)}_{r} - 4\underbrace{f(n-2)}_{1}$

$r^2 = 4r - 4$

$r^2 - 4r + 4 = 0$

$\boxed{r_1 = r_2 = 2} \Rightarrow \boxed{f(n)^{(h)} = c_1 \cdot (2)^n + c_2 \cdot (2)^n \cdot n}$

Because roots are equal.

Step 3 :

Since the particular part is of 2nd degree, it is second order in the equation.

$f(n)^{(p)} = A \cdot n^2 + Bn + C$

$f(n-1) = A \cdot (n-1)^2 + B(n-1) + C$

$f(n-2) = A \cdot (n-2)^2 + B(n-2) + C$

Step 4 :

$An^2 + Bn + C = A(n-1)^2 + B(n-1) + C - A(n-2)^2 - B(n-2) - C + n^2$

Simplified $\Rightarrow An^2 + Bn + C = 8An - 12A + 4B + n^2$

$\boxed{A = 1}$

$n^2 + Bn + C = 8n - 12 + 4B$

$\boxed{B = 8}$

$C = -12 + 4 \cdot 8$

$\boxed{C = 20}$

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2- Continue)

$$\boxed{f(n)^{(p)} = n^2 + 8n + 20}$$

Step 5 :

$$f(n) = c_1 \cdot (2)^n + c_2 (2)^n \cdot n + n^2 + 8n + 20$$

$$f(0) = c_1 + 20 = 2$$

$$\boxed{c_1 = -18}$$

$$f(1) = 2c_1 + 2c_2 + 29 = 5$$

$$2c_2 = 12$$

$$\boxed{c_2 = 6}$$

Step 6 :

$$\boxed{f(n) = -18(2)^n + 6 \cdot n \cdot (2)^n + n^2 + 8n + 20}$$

3)

a) **step 1:** $a_n = a_n^{(h)}$ homo

**step 2:** $a_{n-2} = 1$

$a_{n-1} = r$

$a_n = r^2$

**step 3:** $r^2 = 2r - 2$

$r^2 - 2r + 2 = 0$ characteristic equation

$\Delta = b^2 - 4ac$

$\Delta = -4$

$r_{1,2} = \dfrac{-b \pm \sqrt{\Delta}}{2a} \longrightarrow \dfrac{2 \pm \sqrt{-4}}{2} \longrightarrow \boxed{\begin{matrix} r_1 = 1+i \\ r_2 = 1-i \end{matrix}}$ characteristic roots

$\sqrt{-4} \to 2i$

b) $a_n^{(h)} = c_1(1+i)^n + c_2(1-i)^n$ homogeneous solution

**Step 2:** $a_0 = 1 \Rightarrow \boxed{c_1 + c_2 = 1}$

$a_1 = 2 \Rightarrow \boxed{(1+i)c_1 + (1-i)c_2 = 2}$

$c_1 = \dfrac{1-i}{2}$

$c_2 = \dfrac{1+i}{2}$

$$\Rightarrow \boxed{a_n = \left(\frac{1-i}{2}\right)(1+i)^n + \left(\frac{1+i}{2}\right)(1-i)^n}$$