CSE 211: Discrete Mathematics **(Due: 17/11/20)**

Homework #1

Instructor: Dr. Zafeirakis Zafeirakopoulos
Assistant: Gizem Süngü

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1: Conditional Statements (5+5+5=15 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows tonight, then I will stay at home.
(Solution)

Converse: If I stay at home ,then it snows tonight

Contrapositive: If I don't stay at home,then it won't snow tonight

Inverse: If it doesn't snow tonight ,then I won't stay at home.

(b) I go to the beach whenever it is a sunny summer day.
(Solution)

Converse: It is a sunny summer day whenever I go to the beach.

Contrapositive: It isn't a sunny summer day whenever I don't go to the beach.

Inverse: I dont' go to the beach whenever it isn't a suny summer day.

(c) If I stay up late, then I sleep until noon.
(Solution)

Converse: If I sleep until noon,then I stay up late.

Contrapositive: If I don't sleep until moon,then I don't stay up late.

Inverse: If I don't stay up late,then I don't sleep until noon.

Problem 2: Truth Tables For Logic Operators (5+5+5=15 points)

Construct a truth table for each of the following compound propositions.
(a) $(p \oplus \neg q)$
(Solution)

p	q	¬ q	p ⊕ ¬ q
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

(b) $(p \iff q) \oplus (\neg p \iff \neg r)$
(Solution)

p	q	¬ q	p ⟺ q	r	¬ r	¬ p ⟺ ¬ r	p ⟺ q ⊕ ¬ p ⟺ ¬ r
T	T	F	T	F	T	F	T
T	F	F	F	F	T	F	F
F	T	T	F	F	T	T	T
F	F	T	T	F	T	T	F
T	T	F	T	T	F	T	F
T	F	F	F	T	F	T	T
F	T	T	F	T	F	F	F
F	F	T	T	T	F	F	T

(c) $(p \oplus q) \Rightarrow (p \oplus \neg q)$
(Solution)

p	q	¬ q	p ⊕ q	p ⊕ ¬ q	p ⊕ q ⇒ p ⊕ ¬ q
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Problem 3: Predicates and Quantifiers (21 points)

There are three predicate logic statements which represent English sentences as follows.

- P(x): "x can speak English."
- Q(x): "x knows Python."
- H(x): "x is happy."

Express each of the following sentences in terms of P(x), Q(x), H(x), quantifiers, and logical connectives or vice versa. The domain for quantifiers consists of all students at the university.

(a) There is a student at the university who can speak English and who knows Python.
(Solution)
$\exists x(P(x) \wedge Q(x))$
(b) There is a student at the university who can speak English but who doesn't know Python.
(Solution)
$\exists x(P(x) \wedge \neg Q(x))$
(c) Every student at the university either can speak English or knows Python.
(Solution)
$\forall x(P(x) \vee Q(x))$
(d) No student at the university can speak English or knows Python.
(Solution)
$\forall x(\neg P(x) \vee \neg Q(x))$
(e) If there is a student at the university who can speak English and know Python, then she/he is happy.
(Solution)
$\exists x((P(x) \wedge Q(x)) \Rightarrow H(x))$
(f) At least two students are happy.
(Solution)
$\exists x H(x)$
(g)$\neg\forall x(Q(x) \wedge P(x))$
(Solution)
No student at the university can speak English and know python.

Problem 4: Mathematical Induction (21 points)

Prove that $3 + 3\ .\ 5 + 3\ .\ 5^2 + . . . + 3\ .\ 5^n = \frac{3(5^{n+1}-1)}{4}$ whenever n is a nonnegative integer.
(Solution)

Problem 4:

$$3+3.5+3.5^2+...+3.5^n = \frac{3(5^{n+1}-1)}{4}$$

basis step: Apply $n=1$ on the equation

$$3+3.5 = \frac{3.(5^2-1)}{4}$$

$$2=2$$

We prove that the equation is true for $n=1$

inductive step: Apply $n=k+1$ on the equation

$$\underbrace{3+3.5+3.5^2+....+3.5^k}_{\frac{3.(5^{k+1}-1)}{4}} + 3.5^{k+1} = \frac{3(5^{k+2}-1)}{4}$$

$$\frac{3.(5^{k+2}-1)}{4} - \frac{3.(5^{k+1}-1)}{4} = 3.5^{k+1}$$

$$\frac{\cancel{3}.5^{k+1}.\cancel{(5-1)}}{\cancel{4}} = \cancel{3}.5^{k+1}$$

$$5^{k+1} = 5^{k+1} \checkmark$$

Problem 5: Mathematical Induction (20 points)

Prove that n^2 - 1 is divisible by 8 whenever n is an odd positive integer.
(Solution)

Problem 5 :

a be an odd integer.

$a = 2b+1 \Rightarrow a^2 = (2b+1)^2 \Rightarrow a^2 = 4b^2+4b+1$

$\Rightarrow a^2 = 4(b^2+b)+1 \ \dots (1)$

$a^2 - 1 = 4(b^2+b)+1-1 = 4b(b+1) \ \dots (3)$

Now, we discriminate the following two cases:

1) if b is even:

m is positive integer

$\boxed{b = 2m}$

$a^2 - 1 = 4b(b+1) = 4.(2m)(2m+1) = \boxed{8m(2m+1)}$ ✓ divisible by 8.

2) if b is odd:

r is positive integer

$\boxed{b+1 = 2r}$

$a^2 - 1 = 4b(b+1) = 4b(2r) = \boxed{8qr}$ ✓ divisible by 8.

Problem 6: Sets (8 points)

Which of the following sets are equal? Show your work step by step.

(a) {t : t is a root of x^2 – 6x + 8 = 0}

(b) {y : y is a real number in the closed interval [2, 3]}

(c) {4, 2, 5, 4}

(d) {4, 5, 7, 2} - {5, 7}

(e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

(Solution)

(a) root1= 2 , root2= 4
t is a root of this equation.
so t={2,4}

(b) y is **real** number in the closed interval [2,3]
y has a infinite value.

(c) {4,2,5,4}

(d) {4,5,7,2} - {5,7} = {4,2}

(e) number of sides of rectangle = 4
number of digits in any integer between 11 and 99 = 2
q = {4,2}

so that a,d and e are equal sets.

Problem Bonus: Logic in Algorithms (20 points)

Let p and q be the statements as follows.

- **p:** It is sunny.
- **q:** The flowers are blooming.

circuit.png

Figure 1: Combinational Circuit

In Figure 1, the two statements are used as input. The circuit has 3 gates as AND, OR and NOT operators. It has also a 2x1 multiplexer[1] which provides to select one of the two options.
(a) Write the sentence that "result" output has.
(Solution)

(b) Convert Figure 1 to an algorithm which you can write in any programming language that you prefer (including pseudocode).

[1] https://www.geeksforgeeks.org/multiplexers-in-digital-logic/

(Solution)

(a) It is sunny or the flowers aren't blooming.

(b)

```c
void logic(int multiplexer){

    char p[]="It is sunny.";
    char q[]="The flowers are blooming.";
    char p_not[]="It is not sunny.";
    char q_not[]="The flowers are not blooming.";

    if(multiplexer==1){
        printf("%s or %s",p,q_not); // p or q_not
    }
    else if(multiplexer==0){
        printf("%s and %s",p,q);    // p and q
    }
}
```