1) a) $2^n + n^3 \leq c_1 \cdot 4^n$ c_1 and n_0 positive constant

$n \geq n_0$

$\rightarrow$ Statement is true.

b) A function $f(n) \in \Omega(g(n))$ if there exist positive constants c and n_0 s.t. $c \cdot g(n) \leq f(n)$ whenever $n \geq n_0$

$\sqrt{10n^2 + 7n + 3} \geq c \cdot n$

$\rightarrow$ in order for that to be true, for any c, we have to be able to find an n_0 that makes $f(n) \geq c \cdot g(n)$ asymptotically true. For example $c = 5$ we will be able to find an n_0 (Try $n_0 = 1000$) conjecture appears to be correct

this is true.

c) $n^2+n < c.n^2$ c and n_0 Positive constant

$n \geq n_0$

For example $c = \frac{1}{100} \Rightarrow n^2+n < \frac{n^2}{100}$

Only negtive values will work.

Therfore $n^2+n \notin o(n^2)$

n^2 is not a loose upper-bound on n^2+n.

d) $c_1 . \log_2 n^2 \leq 3\log_2^2 n \leq c_2 . \log_2 n^2$ $\exists c_1, c_2$ s.t.

for $n \geq n_0$ $g(n) = \log_2 n^2$

$c_1 \leq \frac{3}{2}\log_2 n \leq c_2$

$3\log_2^2 n \notin \Theta(\log_2 n^2)$ This is False

e) $(n^3+1)^6 \leq c.n^3$

$g(n) = n^3$ if $c=1$, then no value of n satisfies the equality.

Therfore,

$(n^3+1)^6 \notin O(n^3)$ This is false.

2)

a) $2n\log(n+2)^2 + (n+2)^2\log\left(\frac{n}{2}\right) \Rightarrow n^2.\log(n) > 4n\log n$

$g(n) = n^2.\log n$

$c_1.n^2.\log n \leq 2n\log(n+2)^2 + (n+2)^2\log\left(\frac{n}{2}\right) \leq c_2.n^2.\log n$

b) $0{,}001n^4 + 3n^3 + 1$

$g(n) = n^4$

$c_1.n^4 \leq 0{,}001n^4 + 3n^3 + 1 \leq c_2.n^4$

3)

a) $\lim_{n\to\infty} \frac{n^{1,5}}{n^{\log n}} \Rightarrow \lim_{n\to\infty} n^{1,5-\log n} = \infty^{-\infty} = 0$

so that $n^{\log n} > n^{1,5}$

$\lim_{n\to\infty} \frac{n^{1,5}}{\log n} = \frac{\infty}{\infty}$ L'hospital $\lim_{n\to\infty} \frac{(n^{1,5})'}{(\log n)'}$

$\frac{1,5\sqrt{n}}{\frac{1}{n}.\ln 10} = \frac{1,5.n.\sqrt{n}}{\ln 10} = \infty$ so that $n^{1,5} > \log n$

$n^{\log n} > n^{1,5} > \log n$

b) $n! \approx \sqrt{2\pi n}\left[\frac{n}{e}\right]^n$

$\lim_{n\to\infty} \frac{\sqrt{2\pi n}\left[\frac{n}{e}\right]^n}{2^n} \Rightarrow \lim_{n\to\infty} \sqrt{2\pi n}.\left[\frac{n}{2e}\right]^n = \infty$

$n! \in w(2^n)$

$\lim_{n\to\infty} \frac{(2^n)'}{(n^2)'} \Rightarrow \lim_{n\to\infty} \frac{2^n.\ln 2}{2n} \Rightarrow \frac{2^n(\ln 2)^2}{2} = \infty$

$n! > 2^n > n^2$

c) $\lim_{n\to\infty} \frac{n\log n}{\sqrt{n}} \Rightarrow \lim_{n\to\infty} n^{\frac{1}{2}}.\log n = \infty$

$n\log n > \sqrt{n}$

d) $\lim\limits_{n\to\infty} \frac{n.2^n}{3^n} \Rightarrow 0$ $\quad 3^n > n.2^n$

e) $\lim\limits_{n\to\infty} \frac{(\sqrt{n+10})'}{(n^3)'} \Rightarrow \lim\limits_{n\to\infty} \frac{1}{2\sqrt{n+10}\,.\,3n^2} = 0$

$n^3 > \sqrt{n+10}$

4)

a) basic operation of the algorithm if(B[i,j] != B[j,i])

b) $$\sum_{i=0}^{n-2}\sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2}((n-1)-(i+1)+1) = \sum_{i=0}^{n-2}(n-1)-i = \frac{(n-1).n}{2}$$

$$= \frac{n.(n-1)}{2} = \frac{n^2-n}{2}$$

c) $F(n) = \frac{n^2-n}{2} \Rightarrow \frac{n^2-n}{2} \in O(n^2)$

5)

a) Basic operation of the algorithm $C[i,j] = 0.0$ and $C[i,j] = C[i,j] + A[i,k] \cdot B[k,j]$.

b) $$\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-1} 1 = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1} n = \sum_{i=0}^{n-1} n^2 = n^3 \Rightarrow F(n) = n^3$$

c) $F(n) = n^3 \Rightarrow n^3 \in \Theta(n^3)$

6) algorithm(A[0,1,... n-1], int dest)

```
for i=0 to n-1 do
    for j=i+1 to n-1 do
        if(A[i]*A[j] == dest)
            Print(A[i], A[j])
```

$$\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} 1 = \sum_{i=0}^{n-1} n = n.n = n^2$$

$$T(n) = n^2 \Rightarrow n^2 \in \Theta(n^2)$$