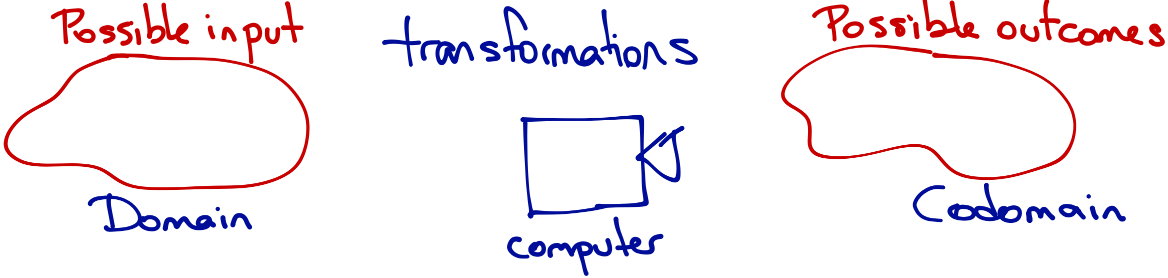
Review Session
Wednesday, Oct. 18

Poll is active:
You can participate now

Review:
- Matrix transformations
- Subspaces, basis, dimension
- Rank thm + Dimension thm



① Input : Student Name Output : Buzzard ID 9-digits

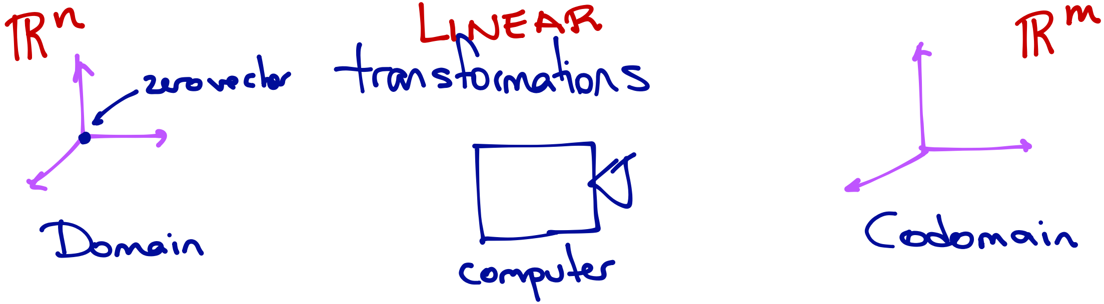
* It is not ambiguous : one-to-one
* One 9-digit code : 000-000-000 is not assigned to any student.

NOT onto

② Input : Student name Output : Subset of Courses taken in 2017

* Some student take all same classes NOT ONE-TO ONE

LINEAR transformations



How do we describe this transformations? $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_1 + x_3 \\ 2x_2 \end{pmatrix}$ linear.

Let suppose from now on all transformations are linear.

+ Through a basis ← most of the time use unit vectors

↳ suffice to describe all $x \in \mathbb{R}^n$

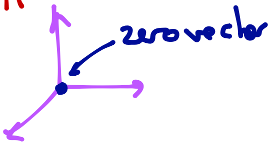
we need <u>all</u> $e_1, \dots, e_n$ to do that

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_1 e_1 + x_2 e_2 + x_3 e_3 \qquad \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = e_1 + e_2 \qquad \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = e_2 + e_3$$

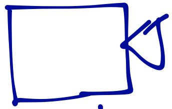
+ Through the standard matrix

$$A = \begin{pmatrix} | & | & & | \\ T(e_1) & T(e_2) & \cdots & T(e_n) \\ | & | & & | \end{pmatrix}$$

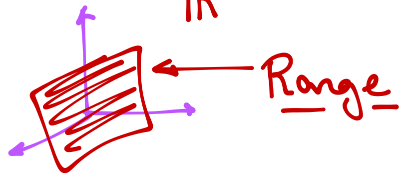
$\mathbb{R}^n$ LINEAR transformations $\mathbb{R}^m$



Domain



computer



Codomain

Subspaces: Why? To understand transformation at high-level

- We already know Range "is linear" = subspace * if it contains 0.

Our old example of subspace $\mathbb{R}^n$ with main features:

- dimension n
- basis: $e_1, e_2, \dots e_n$

Note: $\left\{\begin{pmatrix}0\\\vdots\\0\end{pmatrix}\right\}$ is a subspace

How to find that information about any other subspace?

- dimension n
- basis: $e_1, e_2, \dots e_n$

How to find that information about any other subspace?

Range of transformation

Col A ← look pivot columns { # pivot columns = dim; those columns in A = basis

Nul A ← look non pivot columns { # non-pivot col. = dim; basis = through param. vector solution set.

"Observations that are always true" = Theorem

<u>Rank thm</u> $\dim \operatorname{Col} A + \dim \operatorname{Nul} A = n$

T F $\dim \operatorname{Col} A + \dim \operatorname{Nul} A = \underline{m}$

↳ # columns

Basis thm: If already know $\dim V$: check only either condition for basis

Some exercises:

$V = \text{Span}\left\{ \underset{w_1}{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}}, \underset{w_2}{\begin{pmatrix}1\\1\\0\\0\end{pmatrix}}, \underset{w_3}{\begin{pmatrix}1\\0\\1\\0\end{pmatrix}} \right\} \subseteq \mathbb{R}^4$ find another basis $\{v_1, v_2, v_3\}$ where v_1, v_2, v_3 are not multiples of w_1, w_2, w_3

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}1\\1\\0\\0\end{pmatrix} \in V$$

then basis $\left\{ \begin{pmatrix}2\\1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\-1\\0\\0\end{pmatrix}, \begin{pmatrix}2\\0\\1\\0\end{pmatrix} \right\}$

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} - \begin{pmatrix}1\\1\\0\\0\end{pmatrix} \in V$$

$$-\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}1\\1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \in V$$ do not use in basis any more

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}1\\0\\1\\0\end{pmatrix} = \begin{pmatrix}2\\0\\1\\0\end{pmatrix} \in V$$

Find a basis $\{v_1, v_2\}$ of $\text{Span}\left\{\begin{pmatrix}3\\1\\2\\5\end{pmatrix}, \begin{pmatrix}-1\\2\\0\\-2\end{pmatrix}\right\}$ so that $v_1 = \begin{pmatrix}*\\1\\*\\0\end{pmatrix}$ and $v_2 \begin{pmatrix}*\\0\\*\\1\end{pmatrix}$

Hint for Webwork difficult problem

If $v_1 \in \text{Span}\left\{\begin{pmatrix}3\\1\\2\\5\end{pmatrix}, \begin{pmatrix}-1\\2\\0\\-2\end{pmatrix}\right\}$ then find a and b such that $s\begin{pmatrix}3\\1\\2\\5\end{pmatrix} + t\begin{pmatrix}-1\\2\\0\\-2\end{pmatrix} = \begin{pmatrix}a\\1\\b\\0\end{pmatrix}$ is consistent

If $v_1 \in \text{Span}\left\{\begin{pmatrix}3\\1\\2\\5\end{pmatrix}, \begin{pmatrix}-1\\2\\0\\-2\end{pmatrix}\right\}$ then find c and d such that $s\begin{pmatrix}3\\1\\2\\5\end{pmatrix} + t\begin{pmatrix}-1\\2\\0\\-2\end{pmatrix} = \begin{pmatrix}c\\0\\d\\1\end{pmatrix}$ is consistent

Conclusion:

The matrix $A = \begin{pmatrix}1 & -a & 0 & -c\\ 0 & -b & 1 & -d\\ 0 & 0 & 0 & 0\end{pmatrix}$ has solution set for $Ax=0$: $x_2 \cdot v_1 + x_4 \cdot v_2$

$$\left(\begin{array}{cc|c}3 & -1 & a\\ 1 & 2 & 1\\ 2 & 0 & b\\ 5 & -2 & 0\end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c}1 & 2 & 1\\ 3 & -1 & a\\ 2 & 0 & b\\ 5 & -2 & 0\end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c}1 & 2 & 1\\ 0 & -7 & a-3\\ 0 & -4 & b-2\\ 0 & -12 & -5\end{array}\right) \xrightarrow{R_4 \leftrightarrow R_2} \left(\begin{array}{cc|c}1 & 2 & 1\\ 0 & -12 & -5\\ 0 & -4 & b-2\\ 0 & -7 & a-3\end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c}1 & 2 & 1\\ 0 & 12 & -5\\ 0 & 12 & -3b+6\\ 0 & 12 & -\frac{12}{7}(a-3)\end{array}\right)$$

For this system to be consistent: $-3b+6=-5$, $-\frac{12}{7}(a-3)=-5$ $\Rightarrow$ $b = 11/3$, $a = \frac{7\cdot 5}{12} + 3$

$$\left(\begin{array}{cc|c}3 & -1 & c\\ 1 & 2 & 0\\ 2 & 0 & d\\ 5 & -2 & 1\end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c}1 & 2 & 0\\ 3 & -1 & c\\ 2 & 0 & d\\ 5 & -2 & 1\end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c}1 & 2 & 0\\ 0 & -7 & c\\ 0 & -4 & d\\ 0 & -12 & 1\end{array}\right) \rightsquigarrow \left(\begin{array}{cc|c}1 & 2 & 0\\ 0 & -12 & 1\\ 0 & -12 & 3d\\ 0 & -12 & \frac{12}{7}c\end{array}\right)$$

Consistent if $3d = 1$, $12c/7 = 1$ $\Rightarrow$ $d = 1/3$, $c = 7/12$