

Name:

Recitation Section: J

Math 1553 J1-J3 Quiz : Sections 3.1,3.2

Solutions

The quiz has a total of 10 points and you have 10 minutes. Read carefully and clearly show your work.

1. [2 points each]

There is exactly one error in each of the following determinant computations.

Circle the error and **write down a correction** (do not carry on with the rest of the computations):

E.g. By the cofactor expansion along the first row:

$$\det \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & -1 & 1 \end{pmatrix} = 1 \cdot \underbrace{\det \begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}}_{\text{should be } \det \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}} = 2(-1) - 3 \cdot 1 = -5$$

(1) Using the 2×2 formula:

$$\det \begin{pmatrix} 2 & -3 \\ -1 & -4 \end{pmatrix} = 2 \cdot (-4) - (-3)(-1) = -8 \underbrace{\boxed{+3}}_{\text{should be } -3} = -5$$

(2) Using cofactor expansion along first row:

$$\begin{aligned} \det \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & -1 \\ -1 & 0 & 2 \end{pmatrix} &= 1 \cdot \det \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix} \underbrace{\boxed{-1}}_{\text{should be } -(-1)} \cdot \det \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} + 0 \cdot \det \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix} \\ &= [3 \cdot 2 - (-1 \cdot 0)] - [2 \cdot 2 - (-1)(-1)] + 0 = 6 - [4 - 1] = 3 \end{aligned}$$

Turn the page!

(3) Using cofactor expansion along last column:

$$\det \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & -1 \\ -1 & 0 & 2 \end{pmatrix} = 0 \cdot \det \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix} + 1 \cdot \det \begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} + 2 \cdot \underbrace{\det \begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix}}_{\text{should be } \det \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}}$$

$$= 0 + [1 \cdot 0 - (-1)(-1)] + 2[1 \cdot 0 - (-1)(-1)] = -1 - 2 = -3$$

(4) Using the properties of determinant:

$$\det \begin{pmatrix} 1 & 0 & 7 \\ -1 & 3 & -7 \\ 0 & -3 & 4 \end{pmatrix} = \det \begin{pmatrix} 1 & 0 & 7 \\ 0 & 3 & 0 \\ 0 & -3 & 4 \end{pmatrix} = \det \begin{pmatrix} 1 & 0 & 7 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \underbrace{1 + 3 + 4}_{\text{should be } 1 \cdot 3 \cdot 4} = 8$$

(5) Using the properties of determinant:

$$\det \begin{pmatrix} 3 & 6 \\ 3 & 0 \end{pmatrix} = \det \left(3 \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} \right) = \underbrace{\boxed{3}}_{\text{should be } 9} \det \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix} = 3(1 \cdot 0 - 2 \cdot 1) = 3(-2) = -6$$