18.06 (Spring 14) Problem Set 5

This problem set is due Thursday, March 20, 2014 by 4pm in E17-131. The problems are out of the 4th edition of the textbook. This homework has 5 questions worth 50 points in total (plus 30 points for the questions not handed in with pset 4; 20 points for the MATLAB questions). Please WRITE NEATLY. You may discuss with others (and your TA), but you must turn in your own writing.

1. Least squares approximation: if $b = (5, 13, 17)$ at $t =$ (-1,1,2) find the best line $C+Dt$, and the vectors p and e. Check that $e^T p = 0$.

2. Least squares approximation: If b takes values 0, 1, 3, respectively 4 at the corners $(x, y) = \{(1, 0), (0, 1), (-1, 0), (0, -1)\}$ of a square, find the best least squares fit to b by a plane $C + Dx + Ey$.

3. Consider the reflection matrix $R = I - 2P$, where P is a projection.
   (a) Show that $R^2 = I$.
   (b) For vectors v in $C(P)$, what is Rv?
   (c) For vectors w orthogonal to $C(P)$, what is Rw?

4. If $q_1, q_2, \ldots, q_n$ are orthonormal vectors in $\mathbf{R}^n$, what combination of the q's produces a given vector v ? How do you know that the q's are a basis for $\mathbf{R}^n$?

5. (a) In class, you may have seen that in the projection matrix formula $P = A(A^T A)^{-1} A^T$, we always want A to have linearly independent columns. Why?
   (b) Given an $m \times n$ matrix A $(m > n)$, what is the projection matrix P_{null} onto the nullspace of A^T?

6. MATLAB problems: Please go to lms.mitx.mit.edu to finish this part.