Test 2, FORM A

1. [30 points] For the matrix A below, find a basis for the null space of A, a basis for the row space of A, a basis for the column space of A, the rank of A, and the nullity of A. The reduced row echelon form of A is the matrix R given below.

$$A = \begin{bmatrix} 5 & 15 & 5 & 0 & 4 \\ 4 & 12 & 4 & 5 & -3 \\ -2 & -6 & -2 & 0 & -2 \\ -2 & -6 & -2 & 1 & -5 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Let B be the (ordered) basis $\left(\begin{bmatrix}-1\\-1\\-3\end{bmatrix}, \begin{bmatrix}1\\2\\5\end{bmatrix}, \begin{bmatrix}-2\\-3\\-7\end{bmatrix}\right)$ and C the basis $\left(\begin{bmatrix}-1\\-3\\-2\end{bmatrix}, \begin{bmatrix}-3\\-8\\-3\end{bmatrix}, \begin{bmatrix}1\\3\\3\end{bmatrix}\right)$.

a. [10 points] Find the coordinates of $\begin{bmatrix}-2\\0\\0\end{bmatrix}$ with respect to the basis B.

b. [10 points] If the coordinates of $\vec{u}$ with respect to B are $\begin{bmatrix}1\\2\\3\end{bmatrix}$, what are the coordinates of $\vec{u}$ with respect to C?

3. [15 points] Let $\vec{v}_1 = \begin{bmatrix} 5 \\ 0 \\ 1 \\ 2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 3 \\ 1 \\ -3 \\ 5 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 24 \\ 3 \\ -6 \\ 21 \end{bmatrix}$, and $\vec{v}_4 = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 2 \end{bmatrix}$. Is the vector $\begin{bmatrix} 0 \\ -2 \\ 2 \\ -5 \end{bmatrix}$ in the span of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$? Justify your answer.

4. [15 points] Find a basis for the subspace spanned by $\left\{ \begin{bmatrix} 1 \\ 3 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -4 \\ 3 \end{bmatrix}, \begin{bmatrix} -18 \\ 20 \\ -4 \\ 16 \end{bmatrix}, \begin{bmatrix} 15 \\ -11 \\ -8 \\ -7 \end{bmatrix} \right\}$, and the dimension of that subspace.

5. The eigenvalues of the matrix $A = \begin{bmatrix} 1 & -2 & -4 \\ 8 & 11 & 16 \\ -2 & -2 & -1 \end{bmatrix}$ are 3 (with multiplicity 2) and 5 (with multiplicity 1). (You do not need to find these.) Do the following for the matrix A:

a. [10 points] Find a basis for the eigenspace of **each** eigenvalue.

Solution: The eigenspace of an eigenvalue λ is the null space of $A - \lambda I$. So, if $\lambda = 3$,

b. [10 points] Is the matrix A diagonalizable? If so, find matrices D and P such that $A = PDP^{-1}$ and D is a diagonal matrix. If A is not diagonalizable explain carefully why it is not.

Test 2, FORM B

1. [30 points] For the matrix A below, find a basis for the null space of A, a basis for the row space of A, a basis for the column space of A, the rank of A, and the nullity of A. The reduced row echelon form of A is the matrix R given below.

$$A = \begin{bmatrix} 0 & 3 & -3 & 9 & 3 & -6 \\ 2 & 3 & 2 & 4 & -27 & 4 \\ 5 & 5 & 5 & 10 & -60 & 10 \\ 4 & 0 & 3 & 11 & -32 & 6 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & 0 & 0 & 5 & -5 & 0 \\ 0 & 1 & 0 & 0 & -3 & 0 \\ 0 & 0 & 1 & -3 & -4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Let B be the (ordered) basis $\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -2 \end{bmatrix} \right)$ and C the basis $\left(\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} \right)$.

a. [10 points] Find the coordinates of $\begin{bmatrix} 14 \\ -50 \\ 16 \end{bmatrix}$ with respect to the basis C.

b. [10 points] If the coordinates of $\vec{u}$ with respect to C are $\begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$, what are the coordinates of $\vec{u}$ with respect to B?

3. [15 points] Let $\vec{v}_1 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ -3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 5 \\ 4 \\ -3 \\ 0 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} -3 \\ 4 \\ -7 \\ 12 \end{bmatrix}$, and $\vec{v}_4 = \begin{bmatrix} 5 \\ 4 \\ 2 \\ -1 \end{bmatrix}$. Is the vector $\begin{bmatrix} 3 \\ 8 \\ 4 \\ -20 \end{bmatrix}$ in the span of $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$? Justify your answer.

4. [15 points] Find a basis for the subspace spanned by $\left\{ \begin{bmatrix} 1 \\ -1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ 6 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ -3 \\ 0 \end{bmatrix} \right\}$, and the dimension of that subspace.

5. The eigenvalues of the matrix $A = \begin{bmatrix} 3 & 30 & 60 \\ 0 & -15 & -36 \\ 0 & 6 & 15 \end{bmatrix}$ are 3 (with multiplicity 2) and -3 (with multiplicity 1). (You do not need to find these.) Do the following for the matrix A:

a. [10 points] Find a basis for the eigenspace of **each** eigenvalue.

b. [10 points] Is the matrix A diagonalizable? If so, find matrices D and P such that $A = PDP^{-1}$ and D is a diagonal matrix. If A is not diagonalizable explain carefully why it is not.