Test 3, FORM A

1. Let $\vec{v}_1 = \begin{bmatrix} -2 \\ 2 \\ 1 \\ 4 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 4 \\ 1 \\ -2 \\ 2 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 1 \\ 4 \\ 2 \\ -2 \end{bmatrix}$. Note that $B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is an orthogonal set. Also, let W be the subspace spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

$$A = \begin{bmatrix} -2 & 4 & 1 \\ 2 & 1 & 4 \\ 1 & -2 & 2 \\ 4 & 2 & -2 \end{bmatrix}$$

   a. [15 points] Find the orthogonal projection of $\begin{bmatrix} -13 \\ 18 \\ 9 \\ 1 \end{bmatrix}$ into W, without inverting any matrices or solving any systems of linear equations.

   b. [10 points] Find an orthonormal basis for W.

2. Let W be the subspace spanned by $\left\{ \begin{bmatrix} 0 \\ -2 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \\ 7 \end{bmatrix} \right\}$. Note that this basis is **not** orthogonal.

a. [15 points] Find the vector in W closest to $\begin{bmatrix} -1 \\ 7 \\ -5 \\ -4 \end{bmatrix}$.

b. [15 points] Find an orthogonal basis for W.

3. Do the following, for the following set of data points: $(-4,-93)$, $(-1,-3)$, $(0,-1)$, $(4,27)$.

   a. [10 points] Find the parabola $y = ax^2 + bx + c$ which best fits these points.

$$A = \begin{bmatrix} 16 & -4 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \\ 16 & 4 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -93 \\ -3 \\ -1 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} \end{bmatrix} \qquad\qquad \begin{bmatrix}$$

   b. [10 points] Find the parabola $y = ax^2 + c$ with no linear term which best fits these points.

$$A = \begin{bmatrix} 16 & 1 \\ 1 & 1 \\ 0 & 1 \\ 16 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -93 \\ -3 \\ -1 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} \end{bmatrix} \qquad\qquad \begin{bmatrix} \end{bmatrix}$$

4. [10 points] Find the Least Squares Solution to the following system of linear equations:

$$\begin{aligned} -x_1 + 5x_2 - 3x_3 &= -2 \\ 2x_1 + 3x_2 - 4x_3 &= 0 \\ 3x_1 + x_2 - 3x_3 &= 2 \\ 3x_1 - 4x_2 + 5x_3 &= -4 \end{aligned}$$

5. [15 points] Find a basis for $W^{\perp}$, the orthogonal complement of W, if W is the subspace spanned by

$$\left\{ \begin{bmatrix} -4 \\ 2 \\ -4 \\ -4 \end{bmatrix} \right\}$$

Test 3, FORM B

1. Let $\vec{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \\ -2 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ -2 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$. Note that $B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is an orthogonal set. Also, let W be the subspace spanned by $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

$$A = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -2 & 0 \end{bmatrix}$$

a. [15 points] Find the vector in W closest to $\begin{bmatrix} -5 \\ 4 \\ -1 \\ 7 \end{bmatrix}$, without inverting any matrices or solving any systems of linear equations.

b. [10 points] Find an orthonormal basis for W.

2. Let W be the subspace spanned by $\left\{ \begin{bmatrix} -2 \\ 4 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ -15 \\ 0 \end{bmatrix} \right\}$. Note that this basis is **not** orthogonal.

a. [15 points] Find the vector in W closest to $\begin{bmatrix} 6 \\ -12 \\ -8 \\ 9 \end{bmatrix}$.

b. [15 points] Find an orthogonal basis for W.

3. Do the following, for the following set of data points: $(-5, -133)$, $(-4, -71)$, $(0, -3)$, $(3, 27)$.

   a. [10 points] Find the parabola $y = ax^2 + bx + c$ which best fits these points.

   b. [10 points] Find the parabola $y = ax^2 + c$ with no linear term which best fits these points.

4. [10 points] Find the Least Squares Solution to the following system of linear equations:

$$\begin{aligned} 4x_1 + 5x_2 &= -3 \\ x_1 + x_2 + x_3 &= 0 \\ -4x_3 &= 2 \\ -3x_1 + 3x_2 + 2x_3 &= -7 \end{aligned}$$

5. [15 points] Find a basis for $W^{\perp}$, the orthogonal complement of W, if W is the subspace spanned by

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ -4 \\ 4 \end{bmatrix} \right\}$$