18.06 (Spring 2014) Problem Set 2

These 8 problems are worth 80 points. MITx problems are worth 20 points. This problem set is due on Thursday, Feb 20, 2014 by 4pm in E17-131.

1. Imagine that the 2nd difference matrix S (with $1, -2, 1$ down three central diagonals) is INFINITE. Multiply S with these infinite vectors (infinite in both directions):

   (a) all-ones $(..., 1, 1, 1, 1, ...)$

   (b) linear $(..., -2, -1, 0, 1, 2, 3, ...)$

   (c) squares $(..., 4, 1, 0, 1, 4, 9, ...)$

   (d) cubes $(..., -8, -1, 0, 1, 8, 27, ...)$

   How do the answers match up with 2nd derivatives of $1, x, x^2, x^3$?

2. Find the inverse of the 4 by 4 backward difference matrix B (main diagonal of 1's and subdiagonal of -1's). Interpret as the fundamental theorem of calculus. The inverse of the derivative is _____.

3. If the permutation P has 1's on the antidiagonal (from the $(1, n)$ entry down to the $(n, 1)$ entry) is this an even or odd permutation (depending on n) ?

4. Find the pivots, multipliers, LU factors, and determinant of A:

$$A = \begin{pmatrix} 3 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}.$$

5. Now let

$$A = \begin{pmatrix} 4 & 10 & 0 \\ 8 & b & 4 \\ 4 & 0 & 1 \end{pmatrix}.$$

   What value of b interferes with normal elimination ? What should you do in this case? Which b makes the matrix singular ?

6. Problem 30 page 91 (Section 2.5)

7. Problem 40 page 92 (Section 2.5)

8.

   (a) Suppose every row of A adds up to zero. Why is A singular ?

   (b) Suppose every column of A adds to zero. Why is A singular ?

9. MATLAB problems (20 pts): please go to lms.mitx.mit.edu to complete the problems.