Linear Algebra

Chapter 6 Eigenvalues and Eigenvectors

Q1. **(Eigenvalues and eigenvectors)** Determine whether the vectors $\mathbf{v}_1 = (1, 2),\ \mathbf{v}_2 = (3, 4),\ \mathbf{v}_3 = (5, 6)$ are eigenvectors of the given matrix $\mathbf{A}$. If so, what are the eigenvalues?

$$\mathbf{A} = \begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix}.$$

Q2. **(Diagonalization)** Verify the vectors $\mathbf{v}_1 = (1, -1, 1),\ \mathbf{v}_2 = (1, 1, 0),\ \mathbf{v}_3 = (1, 0, -1)$ are eigenvectors of the given matrix $\mathbf{A}$. Then find a diagonalization of the matrix.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}.$$

Q3. **(Diagonalizability)** Show that if $\mathbf{A}$ is diagonalizable, then $\mathbf{A}^2$ is also diagonalizable.

Q4. **(Eigenvalues and eigenvectors)**

(a) Find an example of 3×3 matrices $\mathbf{A}$ and $\mathbf{B}$, such that $\mathbf{A}$ and $\mathbf{B}$ have the same eigenvectors but distinct eigenvalues.

(b) Find an example of 3×3 matrices $\mathbf{A}$ and $\mathbf{B}$, such that $\mathbf{A}$ and $\mathbf{B}$ have the same eigenvalues but distinct eigenvectors.

Q5. **(Diagonalizable matrix)** Consider

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 \\ 4 & 4 & 4 & 4 & 0 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix}, \qquad \mathbf{P} = \begin{bmatrix} 1 & 0.1 & 0.01 & 0.001 & 0.0001 \\ 0 & 1 & 10 & 100 & 1000 \\ 0 & 0 & e & e^2 & e^3 \\ 0 & 0 & 0 & \pi & \sqrt{\pi} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Is $\mathbf{A}$ diagonalizable? Find the eigenvalues of $\mathbf{P}^{-1}\mathbf{A}^{2006}\mathbf{P}$.

Q6. **(Diagonalizable matrix)** Find eigenvalues and eigenvectors of the matrix $\mathbf{A} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$. Write down the diagonalization of $\mathbf{A}$ if it is diagonalizable.

Q7. **(Diagonalizable matrix)** Find eigenvalues and eigenvectors of the matrix $\mathbf{A} = \begin{bmatrix} 2 & 2 & -2 \\ -5 & 1 & 2 \\ -2 & 4 & -1 \end{bmatrix}$. Write down the diagonalization of $\mathbf{A}$ if it is diagonalizable.

Q8. **(Non-diagonalizable matrix)** Find eigenvalues and eigenvectors of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. Write down the diagonalization of $\mathbf{A}$ if it is diagonalizable.