18.06 (Spring 14) Problem Set 4

This problem set is due Thursday, Mar. 13, 2014 by 4pm in E17-131. The problems are out of the 4th edition of the textbook. This homework has 10 questions worth 100 points in total. Please WRITE NEATLY. You may discuss with others (and your TA), but you must turn in your own writing.

1. How far is the point $b = (1, 2, 2, 4)$ from the line through the origin in the direction of the vector $a = (1, 7, 7, 1)$?

2. How far is that point b from the line that goes through $c = (4, 1, 2, 2)$ in the direction of the same vector a ?

3. How far is the point $b = (1, 2, 2)$ from the plane $3x + 2y + 6z = 0$?

4. How far is $b = (1, 2, 2)$ from the plane $Ax + By + Cz = D$?

5. Suppose that $A'Ax = 0$. Show that $Ax = 0$.
   Hint: One approach is in Problem 4.1.9: We are told that Ax is in the nullspace of A'. Which space does Ax also lie in?

6. Find the 3 by 3 matrix P that projects every vector b in $\mathbb{R}^3$ onto the plane $x+2y+2z = 0$. Pb is the closest point in the plane to b.

7. Problem 13 of Section 4.2

8. Problem 31 of Section 4.2

9. Construct matrices with the following properties. Write None if no such matrix can be constructed and explain why. (Explanation should be in the form of a matrix type equation.)

   (a) Column space contains $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$, nulspace contains $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

   (b) row space contains $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$, nulspace contains $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

   (c) $A\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ has solution and $A^T \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

   (d) Every row is orthogonal to every column. (A is not the zero matrix.)

   (e) Columns add up to a column of zeros. Rows add to a row of 1s.

10. (a) Suppose you are given nonzero column vectors $\mathbf{r}, \mathbf{n}, \mathbf{c}, \mathbf{l}$ in $\mathbb{R}^2$. Explain how to determine if these vectors can form bases for the 4 fundamental subspaces, row space, nullspace, column space and left nullspace respectively, and outputs a matrix with those 4 fundamental subspaces if possible. Hint: Think about orthogonality of the four subspaces. Express matrix in terms of the vectors above.

    (b) Suppose that I give you 4 matrices whose columns are all vectors in $\mathbb{R}^{10}$. $R = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \cdots & \mathbf{r}_i \end{bmatrix}$, $N = \begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \cdots & \mathbf{n}_j \end{bmatrix}$, $C = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_i \end{bmatrix}$, $L = \begin{bmatrix} \mathbf{l}_1 & \mathbf{l}_2 & \cdots & \mathbf{l}_n \end{bmatrix}$. The columns of R form a basis for the row space, the columns of N form a basis for the nullspace, the columns of C form a basis for the column space, and the columns of L form a basis for the left nullspace. Explain how to determine if these vectors form the basis for the four fundamental subspaces, ad output a matrix that has those four subspaces.

    (c) Suppose that I give you 4 matrices: $R = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \cdots & \mathbf{r}_i \end{bmatrix}$, $N = \begin{bmatrix} \mathbf{n}_1 & \mathbf{n}_2 & \cdots & \mathbf{n}_j \end{bmatrix}$, $C = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_m \end{bmatrix}$, $L = \begin{bmatrix} \mathbf{l}_1 & \mathbf{l}_2 & \cdots & \mathbf{l}_n \end{bmatrix}$. The dimensions are not specified. Do the same things as above. Note the dimension of the output matrix.

11. MATLAB problems: Please go to lms.mitx.mit.edu to finish this part.