Test 1, FORM A

1. [15 points] Use Cramer's Rule to solve the following system of linear equations for x.

$$\begin{aligned} -3x - 2y &= -1 \\ 2x - 2y &= 3 \end{aligned}$$

2. [15 points] How many solutions does each of the following systems of linear equations have? Circle the entries which led you to your conclusion.

(a) $$\left[\begin{array}{rrrrr|r} 1 & \mathbf{-1} & 0 & \mathbf{-1} & 0 & -4 \\ 0 & \mathbf{0} & 1 & \mathbf{-3} & 0 & -3 \\ 0 & \mathbf{0} & 0 & \mathbf{0} & 1 & 2 \\ 0 & \mathbf{0} & 0 & \mathbf{0} & 0 & 0 \end{array}\right]$$

(b) $$\left[\begin{array}{rrrrr|r} 1 & -2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{-1} \end{array}\right]$$

(c) $$\left[\begin{array}{rrr|r} \mathbf{1} & 0 & 0 & 2 \\ 0 & \mathbf{1} & 0 & 3 \\ 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

3. [20 points] Find the inverse of the matrix $\begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$ using Gauss-Jordan Elimination. (Other methods may result in the loss of points.)

4. [15 points] Solve the system of linear equations

$$\begin{array}{rcrcrcrcl} 60x_1 & - & 30x_2 & - & 11x_3 & + & 3x_4 & = & 0 \\ 198x_1 & - & 104x_2 & - & 39x_3 & + & 12x_4 & = & 2 \\ -52x_1 & + & 27x_2 & + & 10x_3 & - & 3x_4 & = & 4 \\ 15x_1 & - & 8x_2 & - & 3x_3 & + & x_4 & = & 2 \end{array}$$

using the fact that $\begin{bmatrix} 60 & -30 & -11 & 3 \\ 198 & -104 & -39 & 12 \\ -52 & 27 & 10 & -3 \\ 15 & -8 & -3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 3 & 1 & 9 & 6 \\ -2 & -3 & -12 & 6 \\ 3 & -1 & 6 & 22 \end{bmatrix}$. (Other methods may result in the loss of points.)

5. [15 points] Parameterize the solutions to the system of linear equations whose matrix, in reduced row echelon form, is below. Assume that the original variables were x_1, x_2, x_3, x_4, and x_5.

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 0 & -2 \\ 0 & 1 & 3 & -3 & 1 & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Test 1, FORM B

1. [15 points] Use Cramer's Rule to solve the following system of linear equations for y.

$$\begin{aligned} 2x + 2y &= 2 \\ 3x - 3y &= 0 \end{aligned}$$

2. [15 points] How many solutions does each of the following systems of linear equations have? Circle the entries which led you to your conclusion.

(a) $\left[\begin{array}{ccc|c} \mathbf{1} & 0 & 0 & -3 \\ 0 & \mathbf{1} & 0 & -1 \\ 0 & 0 & \mathbf{1} & -2 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & \mathbf{1} & 0 & 2 \\ 0 & \mathbf{0} & 1 & -3 \\ 0 & \mathbf{0} & 0 & 0 \\ 0 & \mathbf{0} & 0 & 0 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 0 & -2 & -3 \\ 0 & 1 & -2 & 2 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{-2} \\ 0 & 0 & 0 & 0 \end{array}\right]$

3. [20 points] Find the inverse of the matrix $\begin{bmatrix} 3 & 3 & -1 \\ 1 & -2 & 1 \\ 0 & -2 & 1 \end{bmatrix}$ using Gauss-Jordan Elimination.

4. [15 points] Solve the system of linear equations

$$\begin{array}{rcrcrcrcr} -8x_1 & + & 6x_2 & - & 7x_3 & - & 5x_4 & = & 2 \\ -4x_1 & + & 2x_2 & + & 3x_3 & & & = & 0 \\ 3x_1 & - & 2x_2 & & & + & x_4 & = & 1 \\ 5x_1 & - & 3x_2 & - & x_3 & + & x_4 & = & 3 \end{array}$$

using the fact that $\begin{bmatrix} -8 & 6 & -7 & -5 \\ -4 & 2 & 3 & 0 \\ 3 & -2 & 0 & 1 \\ 5 & -3 & -1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & -3 & -3 & -2 \\ -2 & -7 & -3 & -7 \\ 0 & 1 & -2 & 2 \\ -1 & -5 & 4 & -8 \end{bmatrix}$. (Other methods may result in the loss of points.)

5. [15 points] Parameterize the solutions to the system of linear equations whose matrix, in reduced row echelon form, is below. Assume that the original variables were x_1, x_2, x_3, and x_4.

$$\left[\begin{array}{rrrr|r} 1 & -1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Test 1, FORM C

1. [15 points] Use Cramer's Rule to solve the following system of linear equations for x.

$$\begin{aligned} -2x - 3y &= 1 \\ -2x + y &= 3 \end{aligned}$$

2. [15 points] How many solutions does each of the following systems of linear equations have? Circle the entries which led you to your conclusion.

(a) $\left[\begin{array}{ccc|c} \mathbf{1} & 0 & 0 & 2 \\ 0 & \mathbf{1} & 0 & 1 \\ 0 & 0 & \mathbf{1} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & \mathbf{3} & 0 & 0 \\ 0 & \mathbf{0} & 1 & -1 \\ 0 & \mathbf{0} & 0 & 0 \\ 0 & \mathbf{0} & 0 & 0 \end{array}\right]$

(c) $\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & -3 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{3} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$

3. [20 points] Find the inverse of the matrix $\begin{bmatrix} 0 & -3 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ using Gauss-Jordan Elimination.

4. [15 points] Solve the system of linear equations

$$\begin{aligned}
-9x_1 - 2x_2 - 4x_3 \phantom{{}-x_4} &= 2\\
-10x_1 - 3x_2 - 2x_3 - x_4 &= 0\\
4x_1 + x_2 + x_3 \phantom{{}-x_4} &= 1\\
-x_1 - x_2 + x_3 - x_4 &= 3
\end{aligned}$$

using the fact that $\begin{bmatrix} -9 & -2 & -4 & 0 \\ -10 & -3 & -2 & -1 \\ 4 & 1 & 1 & 0 \\ -1 & -1 & 1 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 & -2 & 2 \\ -3 & 7 & 9 & -7 \\ -1 & 1 & 0 & -1 \\ 1 & -4 & -7 & 3 \end{bmatrix}$. (Other methods may result in the loss of points.)

5. [15 points] Parameterize the solutions to the system of linear equations whose matrix, in reduced row echelon form, is below. Assume that the original variables were x_1, x_2, x_3, and x_4.

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 2 & 4 \\ 0 & 1 & -2 & 1 & -8 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Test 1, FORM D

1. [15 points] Use Cramer's Rule to solve the following system of linear equations for y.

$$\begin{aligned} 3x \; - \; y &= 0 \\ -2x \; - \; 2y &= 3 \end{aligned}$$

2. [15 points] How many solutions does each of the following systems of linear equations have? Circle the entries which led you to your conclusion.

(a) $\left[\begin{array}{ccc|c} \mathbf{1} & 0 & 0 & -2 \\ 0 & \mathbf{1} & 0 & 3 \\ 0 & 0 & \mathbf{1} & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{3} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 0 & \mathbf{1} & 1 \\ 0 & 1 & \mathbf{2} & 0 \\ 0 & 0 & \mathbf{0} & 0 \\ 0 & 0 & \mathbf{0} & 0 \\ 0 & 0 & \mathbf{0} & 0 \end{array}\right]$

3. [20 points] Find the inverse of the matrix $\begin{bmatrix} 3 & 1 & -1 \\ 4 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix}$ using Gauss-Jordan Elimination.

4. [15 points] Solve the system of linear equations

$$\begin{array}{rcrcrcrcl} 53x_1 & + & 10x_2 & - & 27x_3 & + & 11x_4 & = & 0 \\ -20x_1 & - & 3x_2 & + & 10x_3 & - & 4x_4 & = & 2 \\ 10x_1 & + & 2x_2 & - & 5x_3 & + & 2x_4 & = & 4 \\ -4x_1 & - & x_2 & + & 2x_3 & - & x_4 & = & 2 \end{array}$$

using the fact that $\begin{bmatrix} 53 & 10 & -27 & 11 \\ -20 & -3 & 10 & -4 \\ 10 & 2 & -5 & 2 \\ -4 & -1 & 2 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & -1 & 3 & -1 \\ 0 & 1 & 2 & 0 \\ -2 & -2 & 5 & -4 \\ 0 & -1 & -4 & -5 \end{bmatrix}$. (Other methods may result in the loss of points.)

5. [15 points] Parameterize the solutions to the system of linear equations whose matrix, in reduced row echelon form, is below. Assume that the original variables were x_1, x_2, x_3, and x_4.

$$\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 8 \\ 0 & 1 & 2 & 0 & 5 \\ 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$