MATH 1553
MIDTERM EXAMINATION 2

Name		**Section**	

1	2	3	4	5	6	Total

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 60 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Problem 1. [2 points each]

Suppose that A is an $n \times n$ matrix that is **not** invertible. Let $T(x) = Ax$ be the linear transformation associated to A. Which of the following can you conclude? Circle all that apply.

a) A has two identical columns

b) $\det(A) = 0$

c) A has a row of zeros

d) There are two different vectors u and v in $\mathbf{R}^n$ with $T(u) = T(v)$

One more true-false problem:

e) **T** **F** If A is a 2×5 matrix, then the solution set of $Ax = 0$ is a subspace of $\mathbf{R}^5$.

Problem 2.

Let

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 5 & 6 & 7 & 8 \\ 6 & 8 & 10 & 12 \\ -9 & -10 & -11 & -12 \end{pmatrix}.$$

a) [1 point] What three row operations are needed to transform A into B?

Operation 1:

Operation 2:

Operation 3:

b) [3 points] What are the elementary matrices for these three operations (in the same order)?

$E_1 =$ $E_2 =$ $E_3 =$

c) [3 points] Write an equation for B in terms of A and E_1, E_2, E_3.

$B =$

d) [3 points] Write an equation for A in terms of B and E_1, E_2, E_3.

$A =$

Problem 3.

Let

$$A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & -1 \end{pmatrix}.$$

Be sure to show your work in this problem.

a) [5 points] Find the inverse of A, or explain why A is not invertible.

b) [3 points] Find the determinant of A.

c) [2 points] Find the volume of the parallelepiped defined by the columns of A.

Problem 4.

Consider the following matrix and its reduced row echelon form:

$$A = \begin{pmatrix} 3 & 4 & 0 & 7 \\ 1 & -5 & 2 & -2 \\ -1 & 4 & 0 & 3 \\ 1 & -1 & 2 & 2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

a) [3 points] Find a basis for $\operatorname{Col} A$.

b) [3 points] Find a basis for $\operatorname{Nul} A$.

c) [2 points] What are $\operatorname{rank} A$ and $\dim \operatorname{Nul} A$?

d) [2 points] If B is any $m \times n$ matrix, then

$$\operatorname{rank} B + \dim \operatorname{Nul} B =$$

Problem 5.

Consider the vectors

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix} \quad v_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix} \quad v_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

and the subspace

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; w = 0 \right\}.$$

a) [4 points] Explain why $\mathcal{B}_1 = \{e_1, e_2, e_3\}$ is a basis for V.

b) [4 points] Explain why $\mathcal{B}_2 = \{v_1, v_2, v_3\}$ is a basis for V.

c) [2 points] Find $[v]_{\mathcal{B}_2}$ if $v = \begin{pmatrix} 4 \\ 4 \\ 4 \\ 0 \end{pmatrix}$.

Problem 6. [10 points]

Compute the determinant of the matrix

$$\begin{pmatrix} 1 & 3 & 2 & -4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{pmatrix}^2.$$

[Scratch work]