Technische Universität München

Department of Mathematics

Course Project

Recherche rapide d'un triangle contenant un point dans un maillage

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Contents

Adj vis Map, Exp Gnu
O Prom, Adj vis List, find Sommets und Rec

1 Project description and aim

The aim of this project is to implement an algorithm which searches for a triangle in a convexe mesh covering a point (x, y). A linear complexity (in number of vertices of the mesh) is desired.

2 The template class T3 and the class Triangle

2.1 The template class T3

Objects of the class $T3$ represent elements of a three dimensional space in T. Hereby, the data type T is defined using a template. Within this project, either $T3 < int >$ for the definition of a triangle (see 2.2) or $T3 < double >$ for the definition of the coordinates of a vertex (see X) is used. The private members x, y and z of type T store the entries of the $T3$ vector. The class has a default constructor, a constructor by copy and a constructor creating a vector given the three vector entries. Moreover, the class has operators to access and modify the entries, add to elements of $T3$, multiply an element of $T3$ by a scalar of the type T and calculate the scalar product of two $T3$ vectors. The operator $<$ compares two elements of $T3$, $v_1 = \begin{pmatrix} x_1 & y_1 & z_1 \end{pmatrix}^T$ and $v_2 = \begin{pmatrix} x_2 & y_2 & z_2 \end{pmatrix}^T$, in the following way:

$$v_1 < v_2 \quad \Leftrightarrow \quad \begin{cases} x_1 < x_2 \text{ or} \\ x_1 = x_2 \text{ and } y_1 < y_2 \text{ or} \\ x_1 = x_2 \text{ and } y_1 = y_2 \text{ and } z_1 < z_2 \end{cases} .$$

This allows a lexicographic ordering of a list of $T3$ elements (see 3.2.2, $T3 < int >$). For three $T3$ vectors, which define the vertices of a triangle, the method *oriented_vol* computes the corresponding signed volume. Let a, b and c denote $T3$ vectors. The oriented volume A_{abc} is defined as

$$A_{abc} = \left(\overrightarrow{ab} \times \overrightarrow{bc}\right)^T \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (b_1 - a_1)(c_2 - b_2) - (b_2 - a_2)(c_1 - b_1)$$

The sign is positive for triangles oriented in trigonometric sense. This method is used in the method *promenade* 3.3 to determine a suitable 'walking' direction.

2.2 The class Triangle

The class Triangle inherits form the class T3. Its derived members x, y, z are specialized as integers representing the position of its defining vertices in the array of the points of the given mesh. This array is a member of the class mesh (SHOULD be changed from maillage to mesh...) in 3. In addition, it has three extra integer members *neighbor1, neighbor2, neighbor3* representing the position of the adjacent triangles in the array of triangles which is also a member of the class mesh. (MENTION THAT INDEXING STARTS AT 0?) An adjacent triangle $t^{'}$ is said to be *neighbor1* of a triangle t if x is the first vertex of t and x is not a vertex of $t^{'}$. The same holds for the indices 2, 3. Note that this relation is not symmetric, that is to say that $t^{'}$ is *neighbor1* does not imply that t is *neighbor1* of $t^{'}$. Note that this assignment facilitates the access to the following in the algorithm *promenade* (see 3.3).
At the creation of a new triangle the latter three members are initialized by -1 which means that a triangle has no neighbors when it is created. The neighbors are set via the functions *setAdjacencyViaMultimap* 3.2.1, respectively *setAdjacencyViaList* 3.2.2. After the execution of one of these two functions $neighbor1 = -1$ describes the case that there is no adjacent triangle on the opposite side of the first vertex. Since the neighbors are private members there are getter and setter in order to read and manipulate them.

3 The class Mesh

The class Mesh contains all necessary information of the mesh and the search of a vertex in (or outside) the mesh can be realized by its member functions. Its private members are pointers to the arrays of vertices and triangles of the mesh as integers who store the size of these two lists. In order to create an object of type Mesh the name of the desired .msh file has to be transmitted. The file is read by the functions *LoadVertices* and *LoadTriangles* 3.1 who then initialize the members *triangles, sommets, numbTriangles, numbSommets*. There are getter and setter for *numbTriangles* and *numbSommets* as well as getter for *triangles* and *sommets*. The functions *LoadTriangles* and *LoadVertices* are their setters.

l'intersection de 2 triangles distincts soit une arête commune, un sommet commun ou rien, we consider convexe meshes

3.1 LoadVertices and LoadTriangles

Both functions are called in the constructor of the class mesh setting the members *vertices* and *triangles*. They work basically the same way with the small difference that they create arrays of different data types and searching for different key words in the .msh file. At first the .msh file is opened according to its name. Then the functions search for the line indicating the number of vertices respectively the number of triangles. To make the code work the next line must include the according number which is then stored in *numbSommets*, respectively *numbTri*. The data is read as a string which is transformed to an integer by the command *astoi*. These numbers also define the size of the arrays of $typeT3 < double >$ and Triangle which are created by new. The following lines must include the coordinates of all vertices, respectively the positions of the triangles in the table of vertices. The lines are read as strings who are then Both functions finally return the arrays filled with the according vertices or triangles.

3.2 Find the adjacent triangle

3.2.1 setAdjacencyViaMultimap

The goal of this function is the initialization of the members *neighbor1, neighbor2, neighbor3* of the class Triangle. As a reminder they are defined by their position in the array of triangles. Their initialization is realized by the container *multimaps*. Each triangle $t = (v_{i_1}, v_{i_2}, v_{i_3})$ is represented by the sequence (i_1, i_2, i_3) where these indices indicate the position of the vertices in the array of vertices. For each t three pairs are added to the multimap, where the edges (v_{i_k}, v_{i_l}), $k, l \in \{1, 2, 3\},\ k \neq l$, stored by the data type $pair < int, int >$ (more precisely $pair < k, l >$), represent the keys, whereas the mapped value is an integer representing the position of the triangle in the array of triangles. Hence the initalization if the mulimap is realized in $3n_T$ loops, so in $O(n_T)$.
Essentially for the functioning of the method is the ordering of the of the indices i_1, i_2, i_3 in t. As in *setAdjacencyViaList* an edge $\{v_k, v_l\}$ has to be uniquely identified by the pair (k, l) which should not be mixed up with (l, k). This is achieved by demanding that for each triangle $t = (v_{i_1}, v_{i_2}, v_{i_3})$ the vertices are ordered, so $i_1 < i_2 < i_3$. Thus, an edge (v_{i_k}, v_{i_l}) can uniquely be identified by the pair (k, l), $k < l$.
We find the adjacent triangles by the following three steps:

1. We range over the multimap and in each step we save the current key (i_k, i_l), its position in the list (this is the current iterator) and the index i of the triangle $t = (v_{i_1}, v_{i_2}, v_{i_3})$. Then we move the iterator to the next element of the list and erase the pair $((i_k, i_l),\ i)$ of the multimap which is realized in constant time since the iterator belonging to $((i_k, i_l),\ i)$ was saved.

2. Now searching by the function $find$ for the stored key (i_k, i_l) allows us to determine if there is another index j associated to (i_k, i_l). If this is the case (the iterator returned by find does not point to the last element of the multimap) the triangles represented by i and j are adjacent. The function $find$ needs $O(\log(n_T))$ in order to return an iterator. Note that the pair $((i_k, i_l),\ j)$ is not erased. This would yield an error because the actual iterator would try to access that element.

3. The indices i and j are set as the neighbors of each other where j is set as neighbor $k \in \{1, 2, 3\}$ if the index k does not represent a vertex of the triangle represented by j. The same rule is applied to i as a neighbor of j. This assignment is realized in constant time.

The application of all three steps needs a time of $O(n_T)O(\log(n_T)) = O(n\log(n_T))$.

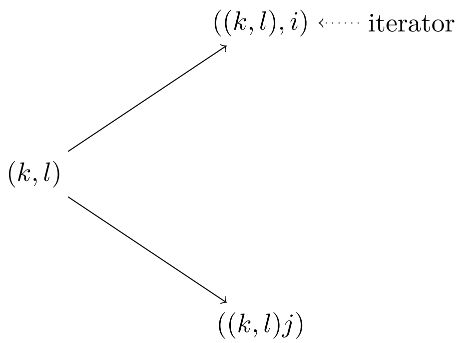


Figure 1: Initial situation for determing the neighbors

3.2.2 setAdjacencyViaList

As in setAdjacencyViaMultimap, this method sets the members, describing the neighbors, for each triangle in the considered mesh. In the same manner as before, the members are described using their position in the triangle array.
The idea is to store each triangle, in varying order, three times in a list. For the triangle t with the vertices (a_1, a_2, a_3), the triangles $(a_1, a_2, t), (a_1, a_3, t)$ and (a_2, a_3, t) are appended to the list. The creation of this list has complexity $O(3n_T)$.
After the creation, the list is sorted in lexicographic order. The complexity for sorting a list with n_T elements is $O(n_T log n_T)$.
Due to the lexicographical ordering, the list allows now to define adjacencies between triangles. Two consecutive triangles of the list are considered. The first two entries of the triangles are compared. If they are equal, the triangles share an edge and can be defined as adjacent. $(a, b, t_1), (a, b, t_2)$
complexity $O(3n_T)$
In summary, the method setAdjacencyViaList has complexity $O(n_T log n_T)$.
Laufzeitvergleich

3.3 L'algorithme promenade

Promenade is a method of the class Mesh which is given a triangle T and a point p of the type $T3 < double >$. As output a triangle in the mesh covering p is returned.
Denote the $T3 < double >$ vectors of the vertices of the triangle T by c_1, c_2 and c_3. Hereby, the vertices are numbered in trigonometric sense. Thus, the oriented volume of the triangle (c_1, c_2, c_3) is positive. Recall that *oriented_volume* is a method of the class $T3$ which takes two additional $T3$ vectors as input (see 2.1). (negierung in code) For each edge of the triangle T, a new triangle containing the edge and the point p is considered. We are interested in the oriented volumes of the triangles $(c_1, c_2, p), (c_2, c_3, p)$ and (c_3, c_1, p). If the oriented volume of one of those new triangles is positive, then p and the triangle T are contained in the half plane defined by the currently considered edge of T. Consequently, p os contained in T if the oriented volumes of the triangles $(c_1, c_2, p), (c_2, c_3, p)$ and (c_3, c_1, p) are all positive. In this case, the method *promenade* returns the input triangle T. On the other hand, if there is one triangle with negative oriented volume, T is not covering p. We choose a neighboring triangle N of T with negative oriented volume and call the method promenade recursively with starting triangle N.
the oriented volume of crossed edge is now positive, edge in other direction to consider neighboring triangle in trigonometric sense
head in direction of p, approach
pay attention for triangles on the border of the mesh. neighbor initialization with -1, just 2 neighboring triangles

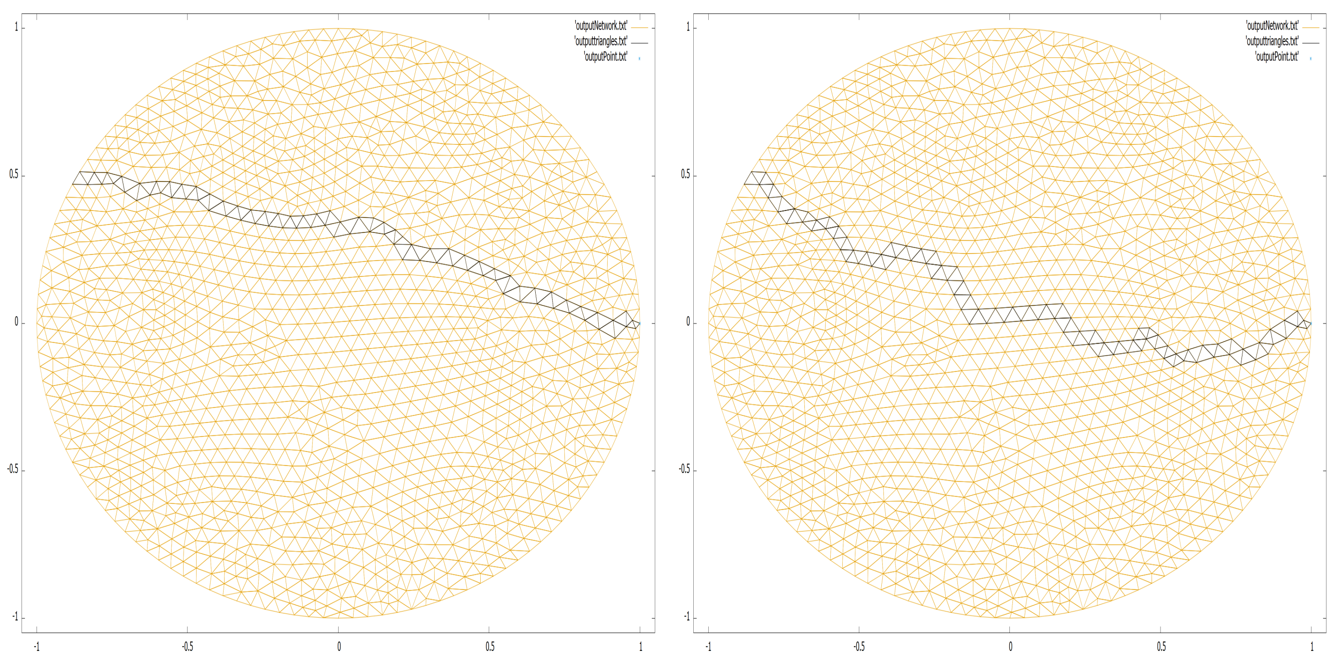
Optionally, the method promenade takes also a vector with entries of the class Triangle, called path, as input variable. The current triangle is pushed back on the vector and, due to the recursion, the vector stores the consecutively passed triangle when the algorithm terminates. The path is used to visualize the results of the method promenade in **??**.
terminates because?

3.3.1 Random neighbor selection

random_neg
random number generation

3.3.2 Selection of neighbor with minimal oriented volume

min_neg
same path length for multiply execution with same starting triangle and point p in most cases
only case in which neighbor selection is not uniquely determined: p on half plane



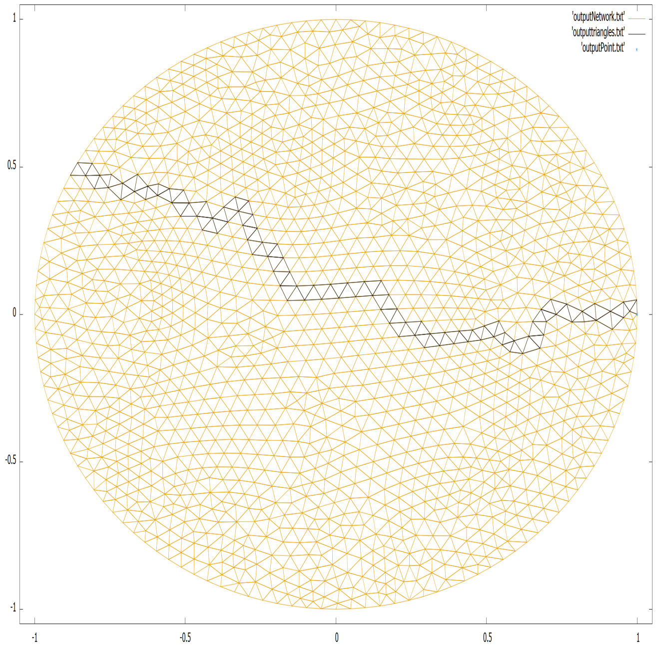
As shown in figure 3.3.2 a random choice of the following triangle can be better. If the algorithm starts with the red triangle and is searching for the green point it has to decide if it takes the neighbor lying relatively higher in the plane or the one lying relatively lower. Deciding this by the smaller oriented volume the lower one will be chosen.

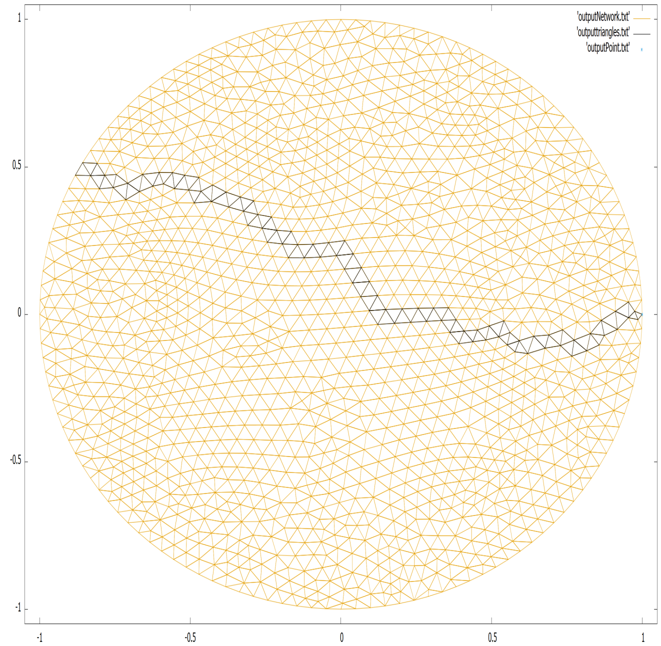
3.4 The visualization via gnuplot

The visualization is done by the function *exportGnuplot* being able to visualize two different things:

1. The result of the algorithm promenade, that is a sequence of adjacent triangles from a starting triangle to a triangle covering the searched point. In this case the input variable $triangles_path$ contains the sequence of triangles and the input variable *points* contains just one point, hence the input *intnumbpoints* is 1.

2. Given a mesh and the vertices of another mesh it can visualize the set of the covering triangles (a subset of triangles of the first mesh) of the points of the second mesh. In this case the input variable *triangles_path* contains the covering triangles and the *points* (IN VERTICES UMBENENNEN?) are the points of the second mesh.





The functions writes four text files, one for all triangles of the mesh, one for the triangles in $triangles\_path$, one for the points in *points* and one for the commands which shall be executed by gnuplot. This is all realized by a simple $ofstream$ variable, allowing to create and manipulate a text file.
The script for gnuplot contains a line which keeps the plot open until some key is hit in the terminal. The actual execution in the terminal is achieved by the command *system* which executes its input string in the terminal.

4 5 find covering Triangles

5 Testumgebung - main

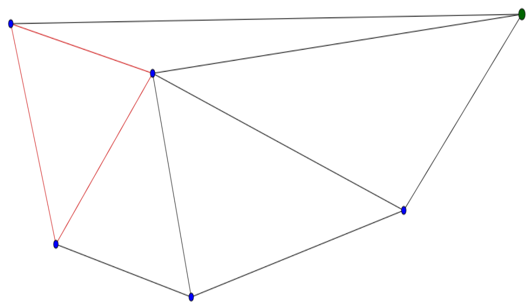


Figure 2: An example of a mesh where a random choice of the consecutive triangle is better.

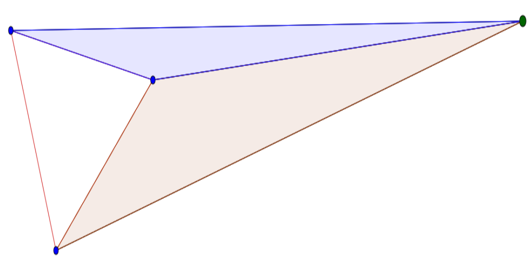


Figure 3: If the choice of the next triangle is not random it is determined according to the size of the blue and red surfaces.