6 Inner product space
Selected Exercise

List of Exercise

Exercise 6.1.5. In C^2, show that $\langle x,\ y\rangle = xAy^*$ is an inner product, where

$$A = \begin{pmatrix} 1 & i \\ -i & 2 \end{pmatrix}.$$

Compute $\langle x,\ y\rangle$ for $x = (1-i,\ 2+3i)$ and $y = (2+i,\ 3-2i)$.

Exercise 6.1.9. Let β be a basis for a finite-dimensional inner product space.

(a) Prove that if $\langle x,\ z\rangle$ for all $z \in \beta$, then $x = \mathit{0}$.

(b) Prove that if $\langle x,\ z\rangle = \langle y,\ z\rangle$ for all $z \in \beta$, then $x = y$.

Exercise 6.2.2. In each part, apply the Gram-Schmidt process to the given subset S of the inner product space V to obtain an orthogonal basis for $\text{span}(S)$. Then normalize the vectors in this basis to obtain an orthonormal basis β for $\text{span}(S)$, and compute the Fourier coefficients of the given vector relative to β. Finally, use **Theorem 6.5**. to verify your result.

(a) $\mathsf{V} = \mathsf{R}^3$, $S = \{(1, 0, 1), (0, 1, 1), (1, 3, 3)\}$ and $x = (1, 1, 2)$

(f) $\mathsf{V} = \mathsf{R}^4$, $S = \{(1, -2, -1, 3), (3, 6, 3, -1), (1, 4, 2, 8)\}$, and $x = (-1, 2, 1, 1)$.

Exercise 6.2.3. In R^2, let

$$\beta = \left\{ \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right), \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right) \right\}$$

Find the Fourier coefficients of $(3, 4)$ relative to β.

Exercise 6.3.2. For each of the following inner product spaces V (over F) and linear transformations $g : \mathsf{V} \longrightarrow F$, find a vector y such that $\mathsf{g}(x) = \langle x, y \rangle$ for all $x \in \mathsf{V}$.

(a) $\mathsf{V} = \mathsf{R}^3$, $\mathsf{g}(a_1, a_2, a_3) = a_1 - 2a_2 + 4a_3$

Exercise 6.3.3. For each of the following inner product spaces V and linear operatos T on V, evaluate T^* at the given vector in V.

(a) $\mathsf{V} = \mathsf{R}^2$, $\mathsf{T}(a, b) = (2a + b, a - 3b)$, $x = (3, 5)$.