(1)

$$\begin{bmatrix} 3 & -1 & -3 \\ -3 & 3 & -1 \\ -4 & -3 & 3 \end{bmatrix}$$

Sol:

$$\begin{array}{cc} \begin{bmatrix} 3 & -1 & -3 \\ -3 & 3 & -1 \\ -4 & -3 & 3 \end{bmatrix} & \begin{array}{c} \text{Row min} \\ -3 \\ -3 \\ -4 \end{array} \\ \begin{array}{ccc} 3 & 3 & 3 \end{array} & \end{array}$$

Column max: 3 3 3

min max = min (Column max) = min(3 3 3) = 3

max min = max (Row min) = max(-3 -3 -4) = -3

$3 \neq -3$ maxmin $\neq$ minmax

$\underline{v} = 3$

$\bar{v} = -3$

$v \neq \bar{v}$

The game is not strictly determine

(2)

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix}$$

Sol:

$$\begin{array}{c} \\ \\ \\ \text{Column max} \end{array}\begin{array}{c}\begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix} \\ \begin{matrix} 1 & 5 & 1 \end{matrix}\end{array} \begin{array}{c} \text{Row min} \\ 1 \\ -4 \\ -1 \\ \\ \end{array}$$

$$\text{minmax} = \min(\text{Column max}) = \min(1\ 5\ 1) = 1$$

$$\text{maxmin} = \max(\text{Row min}) = \max(1\ -4\ -1) = 1$$

$$\text{minmax} = \text{maxmin}$$

$$1 = 1$$

$$\underline{v} = 1 \quad \bar{v} = 1$$

$$\underline{v} = \bar{v}$$

Saddle Point = 1

(3) Dominance Property

Player B

$$\begin{array}{c} \\ \text{Player A} \\ \\ \\ \text{Column max} \end{array}\begin{array}{c}\begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix} \\ \begin{matrix} 4 & 4 & 4 & 8 \end{matrix}\end{array} \begin{array}{c} \text{Row min} \\ 0 \\ 2 \\ 0 \\ 0 \\ \\ \end{array}$$

$$\text{max min} = \max(0\ 2\ 0\ 0) = 2$$

$$\text{min max} = \min(4\ 4\ 4\ 8) = 4$$

$$\text{maxmin} \neq \text{minmax}$$

It has no Saddle point

Hence it is a mixed Strategy

$$\begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 1 & 0 & 8 \end{bmatrix}$$

Row 1 $\leq$ Row 3

Delete Row 1

$$\begin{bmatrix} 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix}$$

$$R_2 \leq \frac{R_3 + R_4}{2} \Rightarrow$$

$$R_3 \leq \frac{R_3 + R_4}{2} \Rightarrow$$

$$R_4 \leq \frac{R_2 + R_3}{2} \Rightarrow$$

$$C_2 \geq \frac{C_3 + C_4}{2} \Rightarrow$$

$$\begin{array}{c c} & \begin{array}{cc} C_3 & C_4 \end{array} \\ \begin{array}{c} R_2 \\ R_3 \\ R_4 \end{array} & \begin{bmatrix} 2 & 4 \\ 4 & 0 \\ 0 & 8 \end{bmatrix} \end{array}$$

$$R_2 \leq \frac{R_3 + R_4}{2} \Rightarrow \text{Delete } R_2$$

Reduced matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$$

$a_{11} = 4$, $a_{12} = 0$, $a_{21} = 0$, $a_{22} = 8$

$$P_1 = \frac{8-0}{12-0} = \frac{8}{12} = \frac{2}{3} \qquad \boxed{P_1 = \frac{2}{3}}$$

$$P_2 = 1 - P_1 = 1 - \frac{2}{3} = \frac{1}{3}$$

$$q_1 = \frac{8-0}{12-0} = \frac{8}{12} = \frac{2}{3} \qquad \boxed{q_1 = \frac{2}{3}}$$

$$q_2 = 1 - q_1 = 1 - \frac{2}{3} = \frac{1}{3}$$

Strategy, $SA = [P_1 \ P_2] = \left[\frac{2}{3} \ \frac{1}{3}\right]$

Strategy, $SB = [q_1 \ q_2] = \left[\frac{2}{3} \ \frac{1}{3}\right]$

Value of the game, v

$$v = \frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{(4)(8) - 0}{12} = \frac{32}{12} = \frac{8}{3}$$

$$\boxed{v = \frac{8}{3}}$$

④ Dominance

$$\begin{array}{c c c} & \text{Player B} & \text{Row min} \\ \text{Player A} & \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 2 & 6 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 2 \end{matrix} \\ \text{Col max} & \begin{matrix} 6 & 7 & 7 \end{matrix} & \end{array}$$

maxmin = max (Row min) = max(1 2 2) = 2

minmax = min (Col max) = min (6 7 7) = 6

maxmin ≠ minmax

It has no saddle point

Hence it is mixed strategy

$$\begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 2 & 6 \end{bmatrix}$$

Row 3 is dominated by Row 2

delete row 3 ($R_3 \le R_2$)

$$\begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \end{bmatrix}$$

Col 3 is dominated by Col 1

delete Col 3 ($C_3 \ge C_1$)

$$\begin{bmatrix} 1 & 7 \\ 6 & 2 \end{bmatrix} \quad \begin{matrix} a_{11} = 1 & a_{21} = 6 \\ a_{12} = 7 & a_{22} = 2 \end{matrix}$$

$$P_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{2-6}{3-13} = \frac{2}{5}$$

$$P_2 = 1 - P_1 = 1 - \frac{2}{5} = \frac{3}{5}$$

$$q_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{2-7}{-10} = \frac{-4}{-10} = \frac{2}{5}$$

$$q_2 = 1 - q_1 = 1 - \frac{2}{5} = \frac{3}{5}.$$

Strategy, $S_A = [P_1 \quad P_2] = \left[\frac{2}{5} \quad \frac{3}{5}\right]$

Strategy, $S_B = [q_1 \quad q_2] = \left[\frac{2}{5} \quad \frac{3}{5}\right]$

Value of the game

$$\vartheta = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$$

$$= \frac{2-42}{-10} = \frac{\cancel{-40}^{\,4}}{\cancel{-10}} = 4$$

$$\boxed{\vartheta = 4}$$

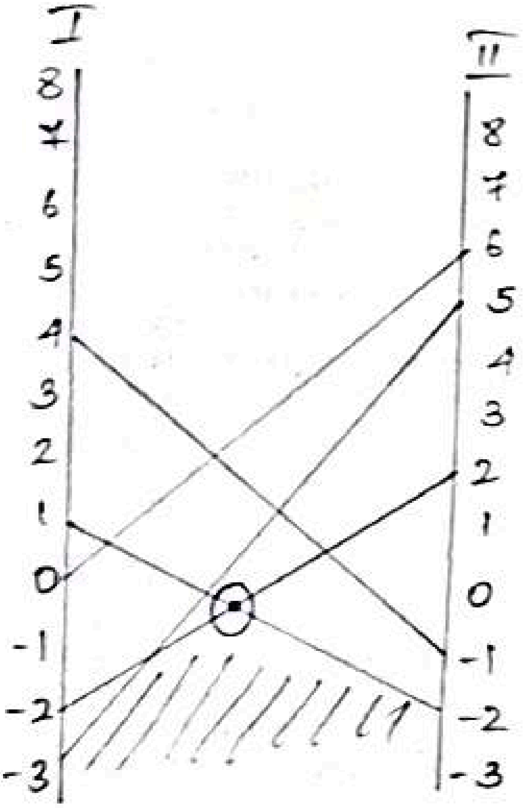
5. Graphically 2×5

Player B

Player A $\begin{bmatrix} 2 & -1 & 5 & -2 & 6 \\ -2 & 4 & -3 & 1 & 0 \end{bmatrix}$

Axis I $\quad -2 \quad 4 \quad -3 \quad 1 \quad 0$

Axis II $\quad 2 \quad -1 \quad 5 \quad -2 \quad 6$



Player B

Player A $\begin{bmatrix} 2 & -2 \\ -2 & 1 \end{bmatrix}$

$a_{11} = 2$, $a_{12} = -2$, $a_{21} = -2$, $a_{22} = 1$

$$P_1 = \frac{1-(-2)}{3-(-4)} = \frac{1+2}{3+4} = \frac{3}{8}$$

$$P_2 = 1 - P_1 = \frac{5}{8}$$

$$q_1 = \frac{1+2}{8} = \frac{3}{8} \qquad \text{Strategy } SA = [P_1\ P_2] = \left[\frac{3}{8}\ \frac{5}{8}\right]$$

$$q_2 = 1 - q_1 = \frac{5}{8} \qquad \text{Strategy } SA = [q_1\ q_2] = \left[\frac{3}{8}\ \frac{5}{8}\right]$$

Value of a game

$$V = \frac{a_{22}\,a_{11} - a_{12}\,a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$$

$$= \frac{2-4}{8} = \frac{-1}{4} \qquad \boxed{V = \frac{-1}{4}}$$

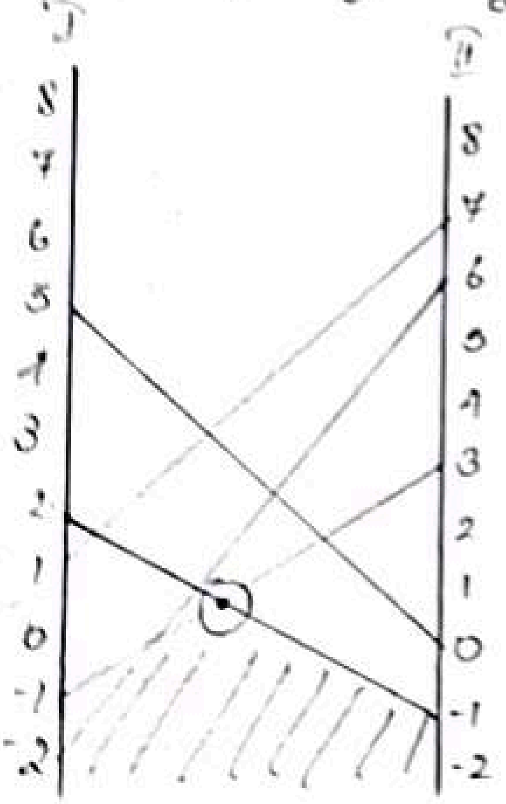
6. Graphically 2×5

Player B

$$\text{Player A} \begin{bmatrix} 3 & 0 & 6 & -1 & 7 \\ -1 & 6 & -2 & 2 & 1 \end{bmatrix}$$

Axis I : -1 5 -2 2 1

Axis II : 3 0 6 -1 7



Reduced matrix

$$\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

$a_{11} = 3$, $a_{12} = -1$, $a_{21} = -1$

$a_{22} = 2$

$P_1 = \frac{3}{5+2} = \frac{3}{5}$ $P_2 = 1 - P_1 = \frac{4}{7}$

$q_1 = \frac{2+1}{7} = \frac{3}{7}$ $q_2 = 1 - q_1 = \frac{4}{7}$

Strategy, $SA = [P_1 \; P_2] = \left[\frac{3}{7} \; \frac{4}{7}\right]$

Strategy, $SB = [q_1 \; q_2] = \left[\frac{3}{7} \; \frac{4}{7}\right]$

Value of a game

$$V = \frac{a_{22}\, a_{11} - a_{12}\, a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$

$$= \frac{(2)(3) - 1}{7} = \frac{6-1}{7} = \frac{5}{7}$$

$$\boxed{V = \frac{5}{7}}$$

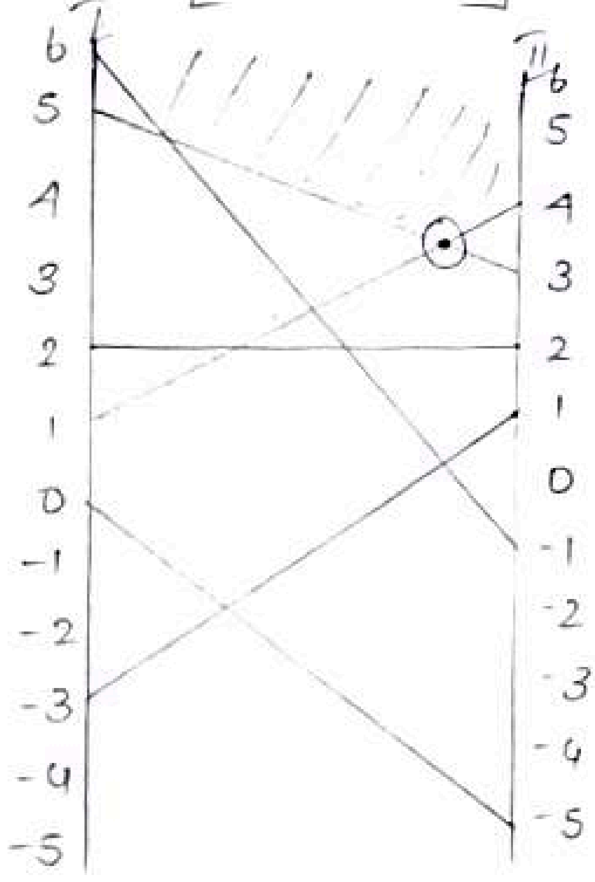
7 Graphically 6×2

Player B

$$\text{Player A} \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix}$$

Axis I = -3 5 6 1 2 0

Axis II = 1 3 -1 4 2 -5



Reduced matrix

$$\begin{bmatrix} 3 & 5 \\ 4 & 1 \end{bmatrix}$$

$a_{11} = 3$, $a_{12} = 5$,

$a_{21} = 4$, $a_{22} = 1$

$$P_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{12})}$$

$$= \frac{1-4}{4-9} = \frac{3}{5}$$

$$P_2 = 1 - P_1 = \frac{3}{5}$$

$$q_1 = \frac{1-5}{-5} = \frac{4}{5}$$

$$q_2 = 1 - q_1 = \frac{4}{5}$$

Strategy, $SA = [P_1 \; P_2]$

$= \left[\frac{3}{5} \; \frac{3}{5}\right]$

Strategy, $SA = [q_1 \; q_2]$

$= \left[\frac{4}{5} \; \frac{4}{5}\right]$

Value of the game

$$V = \frac{a_{22}a_{11} - a_{12}a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}$$

$$= \frac{3-20}{-5} = 7/5$$

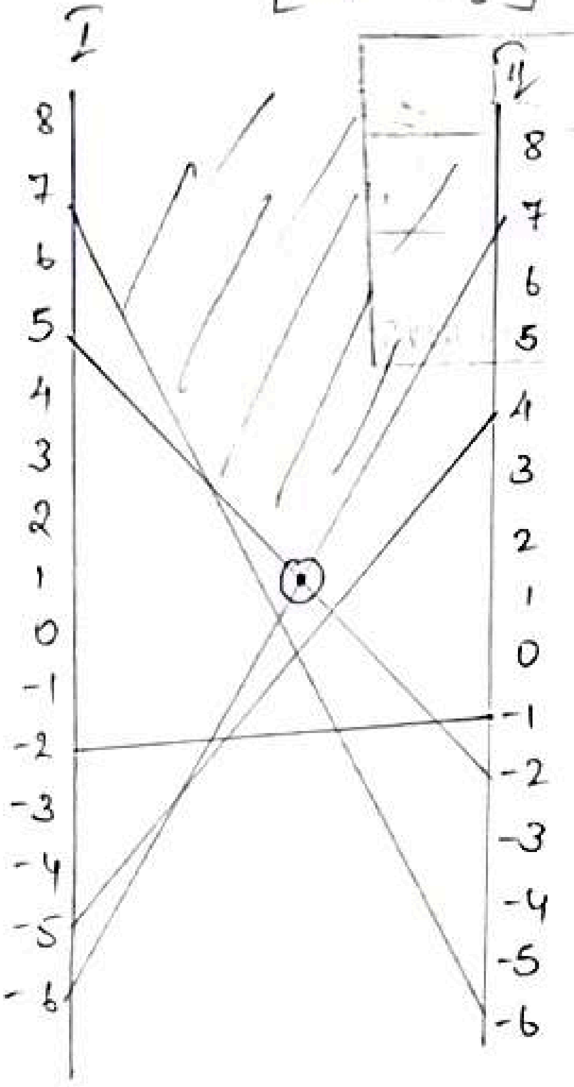
$\boxed{V = 7/5}$

8. Graphically 5×2

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} -6 & 7 \\ 4 & -5 \\ -1 & -2 \\ -2 & 5 \\ 7 & -6 \end{bmatrix} \end{array}$$

Axis I = 7 −5 −2 5 −6

Axis II = −6 4 −1 −2 7



Reduced matrix

$$\begin{bmatrix} -2 & 5 \\ 7 & -6 \end{bmatrix}$$

$a_{11} = -2 \qquad a_{12} = 5$

$a_{21} = 7 \qquad a_{22} = -6$

$$P_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} = \frac{-6-7}{(-8)-12}$$

$$= \frac{+13}{+20} = \frac{13}{20}$$

$$P_2 = 1 - P_1 = \frac{7}{20}$$

$$q_1 = \frac{-2-5}{-20} = \frac{+7}{+20} = \frac{7}{20}$$

$$q_2 = 1 - q_1 = \frac{13}{20}$$

Strategy, $SA = [P_1 \;\; P_2] = [13/20 \;\; 7/20]$

Strategy, $SB = [q_1 \;\; q_2] = [7/20 \;\; 13/20]$

Value of the game

$$v = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})} = \frac{-2(-6) - 7(5)}{-20} = \frac{+23}{+20}$$

$$\boxed{v = 23/20}$$