A Linearly implicit predictor-corrector scheme for pricing European options

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1 Problem Description

We considered the the Black-Scholes equation and boundary conditions for a European call with value $C(S,t)$

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0 \tag{1}$$

With

$$C(0,t) = 0, \qquad C(S,t) \sim S \qquad as \qquad S \to \infty \qquad C(S,T) = max(S-E,0)$$

letting

$$S = Eexp(x), \qquad t = T - \frac{\tau}{\frac{1}{2}\sigma^2}, \qquad C = Eexp(x,\tau)$$

transforms (1) into a forward in time equation in dimensionless form (Wilmott er al., 1995, Section 5.4) which, using parameter values from Neftci (2000), becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + (c-1)\frac{\partial u}{\partial x} - cu \qquad t_0 \le t \le t_f, \qquad a \le x \le b,$$

where

$$c = \frac{r}{\frac{1}{2}\sigma^2}, \qquad r = 0.065, \qquad \sigma = 0.8, \qquad a = ln(\frac{2}{5}) \qquad b = ln(\frac{7}{5}), \qquad t_0 = 0, \qquad t_f = 5$$

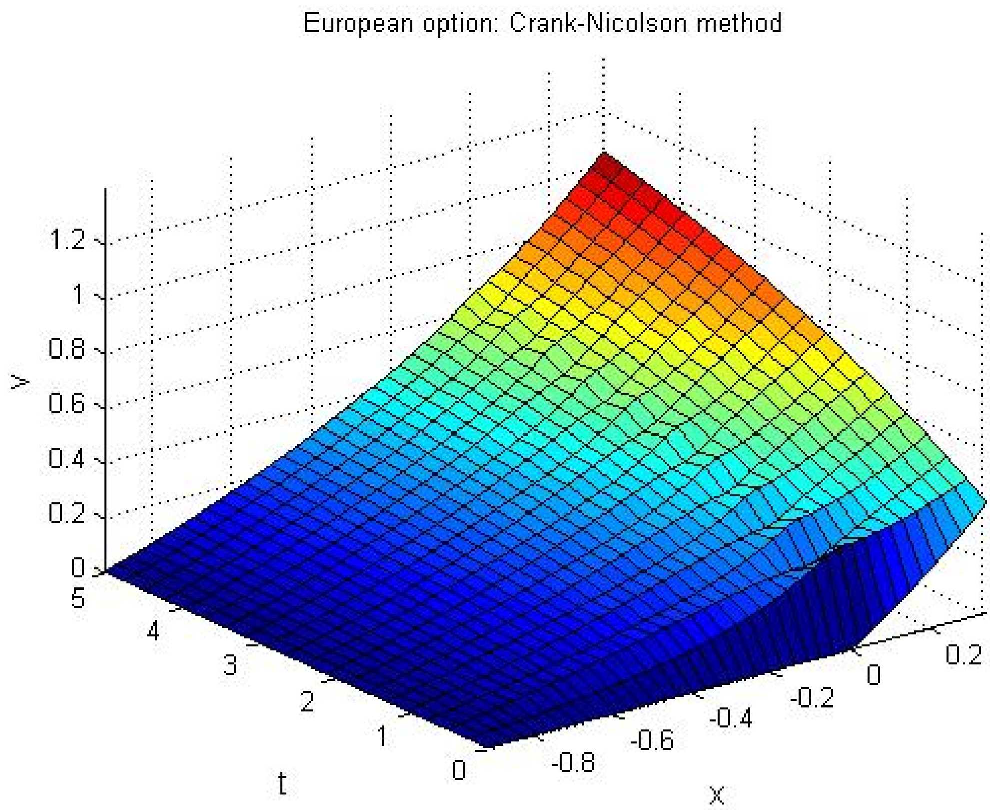
with initial condition

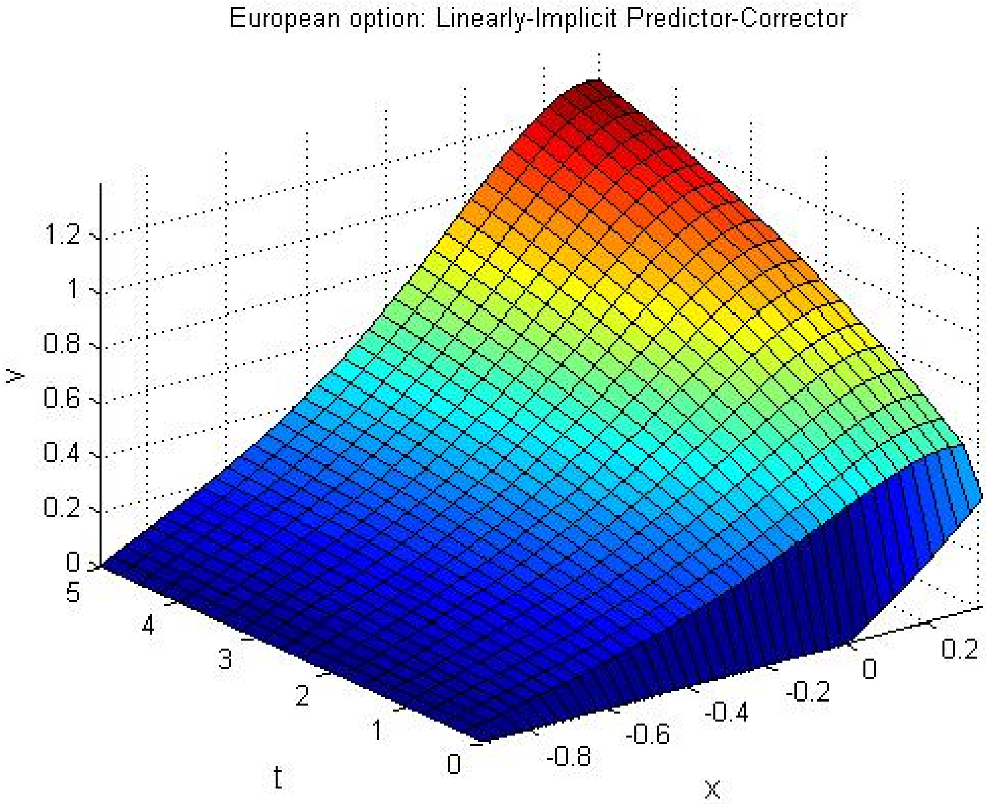
$$u(x,0) = max(exp(x) - 1, 0)$$

and boundary conditions

$$u(a,t) = 0, \qquad u(b,t) = \frac{7 - 5exp(-ct)}{5}$$

We solved numerically with the ETD-Crank-Nicolson method and the linearly implicit method using a space mesh consisting of 40 points and 20 equally spaced time levels. We observe that the Crank-Nicolson method gives oscillatory behavior at $x = 0$. The linearly implicit method on the otherhand is oscillation-free everywhere.


European option: Crank-Nicolson method
v
1.2
1
0.8
0.6
0.4
0.2
0
5
4
3
2
1
0
t
-0.8
-0.6
-0.4
-0.2
0
0.2
x


European option: Linearly-Implicit Predictor-Corrector
v
1.2
1
0.8
0.6
0.4
0.2
0
5
4
3
2
1
0
t
-0.8
-0.6
-0.4
-0.2
0
0.2
x