

Reminders

1. HW 2.3, 2.4 due on 02/11, 11:59 pm
2. Exam #1 on 02/15
(see study guide)

De Morgan's Law (Continuation of 2.3)

$$1. A' \cup B' = (A \cap B)'$$

$$2. A' \cap B' = (A \cup B)'$$

We will use Venn diagrams to verify De Morgan's law

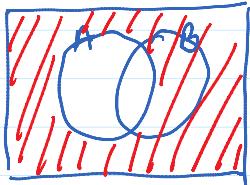
Prove that

$$A' \cup B' = (A \cap B)'$$

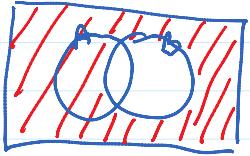
using Venn diagrams

PF

L.H.S



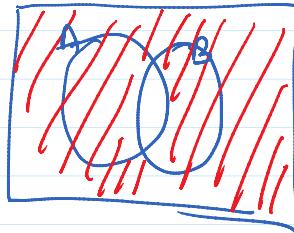
A' is shaded
 $A' = \{x | x \in U \text{ and } x \notin A\}$



B' is shaded
 $B' = \{x | x \in U \text{ and } x \notin B\}$

$\cap \dots \cap$

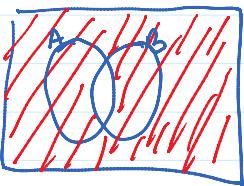
R.H.S



$$(A \cap B)' = \{x | x \in U \text{ and } x \notin (A \cap B)\}$$

$$(A \cap B)'$$

.....



$$A' \cup B'$$

Take union of
A' and B' above
 $A' \cup B' = \{x | x \notin A \text{ or } x \notin B\}$

$$\text{So L.H.S} = \text{R.H.S}$$

Let us prove the second half of De Morgan's law

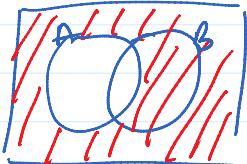
prove that

$$A' \cap B' = (A \cup B)'$$

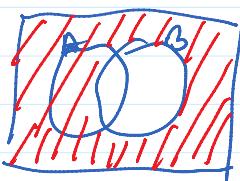
using Venn diagrams

pf

L.H.S



A' is shaded

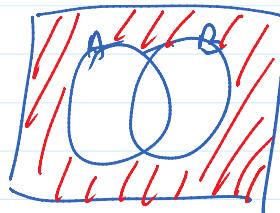


B' is shaded



$A' \cap B'$ is shaded

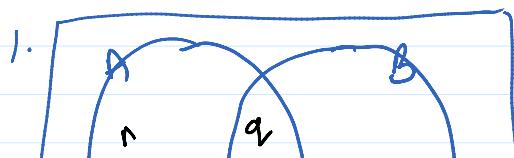
R.H.S



$(A \cup B)'$ is shaded

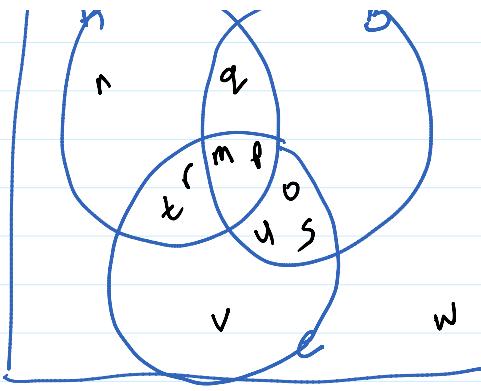
$$\text{L.H.S} = \text{R.H.S}$$

Exercise on Venn diagram



$$U = \{m, n, o, p, q, r, s, t, u, v, w\}$$

$$A = \{m, n, p, q, r, s\}$$



$$A = \{m, r, p, x, t, s\}$$

$$B = \{n, o, s, q, r, x\}$$

$$C = \{d, v, t, f, s, x, u\}$$

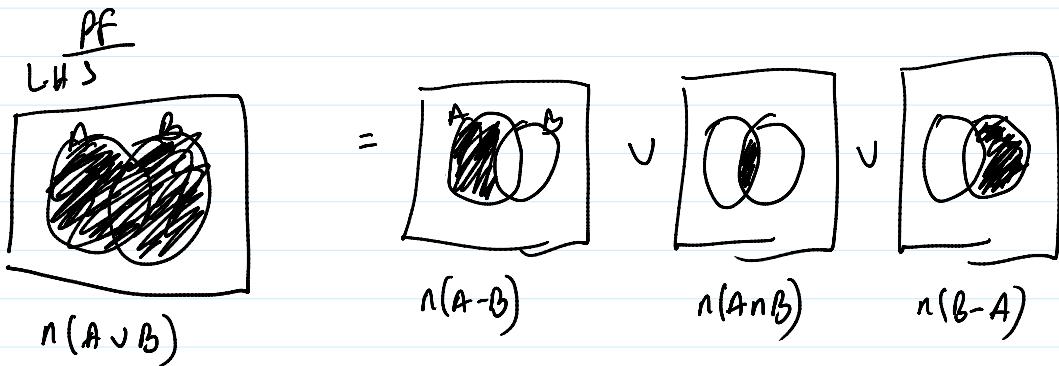
$$D = \{w, u, v, x, p, q\}$$

2.4 Surveys and Cardinal Numbers

Remark 1

Let A, B be sets

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



So

$$n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A) - (1)$$

Now,

$$n(A) = n(A - B) + n(A \cap B)$$

$$n(A) - n(A \cap B) = n(A - B) \quad \text{--- } (2)$$

Similarly for

$$n(B) = n(B - A) + n(A \cap B)$$

$$n(B) - n(A \cap B) = n(B - A) \quad \text{--- } (3)$$

Let us substitute ② and ③ into ①

$$\begin{aligned} n(A \cup B) &= n(A) - n(\cancel{A \cap B}) + \cancel{n(A \cap B)} + n(B) - n(\cancel{A \cap B}) \\ &= n(A) + n(B) - n(A \cap B) \end{aligned}$$

R.H.S



Exercise

- Find the value of $n(A \cup B)$ if

$$n(A) = 12, \quad n(B) = 14, \quad n(A \cap B) = 5$$

Hint

$$\begin{aligned} \text{use } n(A \cup B) &= n(A) + n(B) - n(A \cap B) \\ &= 12 + 14 - 5 \\ &= 26 - 5 \\ &= 21 \end{aligned}$$

- Draw a Venn diagram and use the information to fill in the number of elements in each region

④ $n(A) = 19 \quad n(B) = 13 \quad n(A \cup B) = 25$

$$n(A') = 11$$

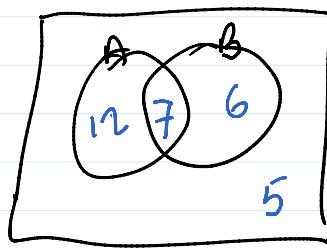
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$25 = 19 + 13 - n(A \cap B)$$

$$25 = 32 - n(A \cap B)$$



$$n(A \cap B) = 32 - 25$$



$$= 7$$

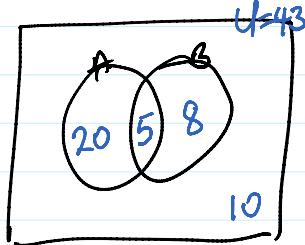
(b)

$$n(U) = 43$$

$$n(A) = 25$$

$$n(A \cap B) = 5$$

$$n(B') = 30$$



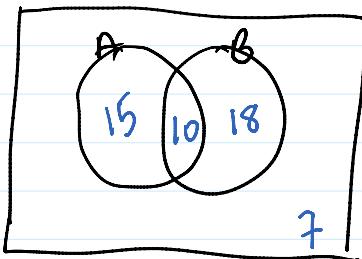
(c)

$$n(A') = 25$$

$$n(B) = 28$$

$$n(A' \cup B') = 40$$

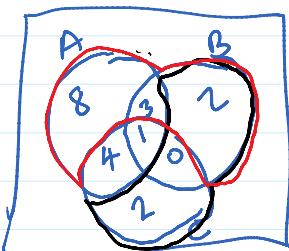
$$n(A \cap B) = 10$$



Hint
use De Morgan's law
 $n(A' \cup B') = n(A \cap B)'$

Exercise

The cardinality of each region is given in the Venn diagram ~~to~~ below



(a) $n(A \cap B \cap C) = 1$

(b) $n(A \cap B' \cap C) = 4$

A', C

(c) $n(A' \cap B' \cap C) = 2$

(d) $n(A' \cap B \cap C') = 2$

Exercise

		Facilities			
		B	R	O	Totals
Performed	C	12	29	54	95
	W	4	5	6	15
	E	374	71	285	730
Totals		390	105	345	840

$$n(C) = 95$$

$$n(W) = 15$$

$$n(E) = 730$$

$$n(B) = 390$$

$$n(R) = 105$$

$$n(O) = 345$$

$$n(C \cap B) = 12$$

$$n(C \cup B) = n(C) + n(B) - n(C \cap B)$$

$$= 95 + 390 - 12$$

$$= 473$$

Find the following

$$1. n(W \cap O)$$

$$2. n(R' \cup W')$$

$$3. n((C \cap B) \cup (E \cap O))$$

$$4. n((C \cup W) \cap (B \cup R))$$

$$5. n(B \cap (W \cup R)')$$