# Reminders

1. Hw 3.3, 3.4 due Friday 03/04 11:59 pm

2. Mid-Semester write up due 03/15 11:59 pm

3. Exam 02 on 03/29 ( Sections on Exam 2
   3.1, 3.2, 3.3, 3.4, 10.1, 10.2, 10.3, 10.5 )

4. Mid-Semester Survey due 03/04 (see D2L)

5. Studyguide for Exam 2 on Course page (See Notes on D2L)
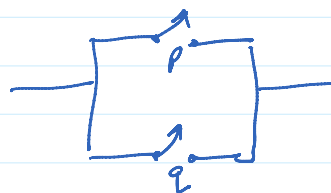
### Exercise from last time

recall

1. $P \rightarrow q \equiv \sim p \vee q$
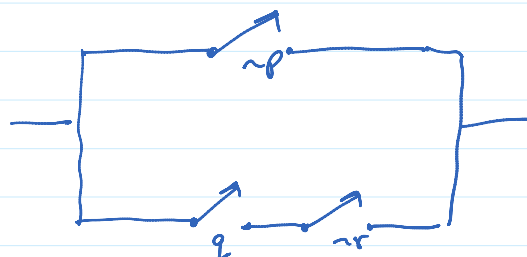
2. $p \wedge q$    Series Circuit



3. $P \vee q$    Parrallel circuit



Draw a Circuit diagram for

$$P \rightarrow (q \wedge \sim r) \equiv \sim p \vee (q \wedge \sim r)$$

More Exercise

Draw circuit diagrams for the following

1. $\sim q \rightarrow (\sim p \rightarrow q)$          Hint    $p \rightarrow q \equiv \sim p \vee q$
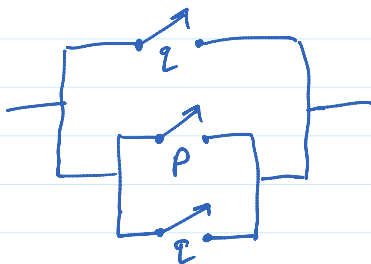
2. $[(p \vee q) \wedge r] \wedge \sim p$

Solution

① 1.  $\sim q \rightarrow (\sim p \rightarrow q)$          $\sim p \rightarrow q \equiv \sim(\sim p) \vee q$
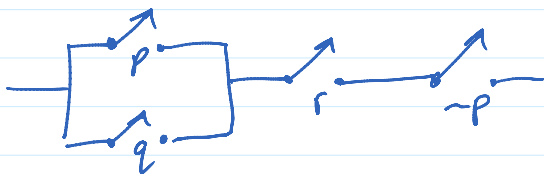
$\equiv \sim q \rightarrow (p \vee q)$                            $\equiv p \vee q$

$\equiv q \vee (p \vee q)$              $\sim q \rightarrow (p \vee q) \equiv \sim(\sim q) \vee (p \vee q)$

$\equiv q \vee (p \vee q)$



② $[(p \vee q) \wedge r] \wedge \sim p$
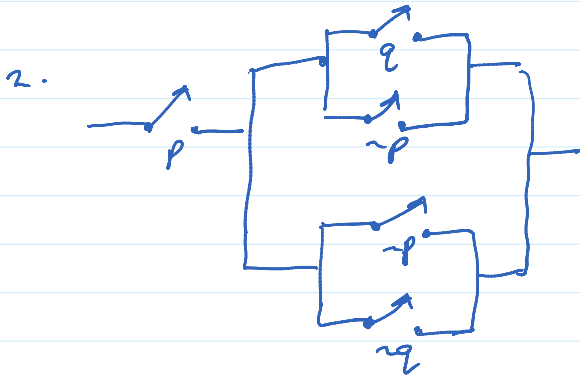


Write the statement from the circuit diagram

1.

          $p \wedge (r \vee q)$

1.



$$p \wedge (r \vee q)$$

2.



$$p \wedge \left[ (q \vee \sim p) \vee (\sim p \vee \sim q) \right]$$

## 3.4

Conditional, Converse, Inverse, Contrapositive, bi-conditional

| | | |
|---|---|---|
| Conditional | $p \rightarrow q$ | (if $p$, then $q$) |
| Converse | $q \rightarrow p$ | (if $q$, then $p$) |
| Inverse | $\sim p \rightarrow \sim q$ | (if not $p$, then not $q$) |
| Contrapositive | $\sim q \rightarrow \sim p$ | (if not $q$, then not $p$) |

$$q \rightarrow p \quad \equiv \quad \sim p \rightarrow \sim q$$

Converse          Inverse

$$p \rightarrow q \quad \equiv \quad \sim q \rightarrow \sim p$$

Conditional        Contrapositive

Prove that $\sqrt{2}$ is irrational

## pf

Suppose $\sqrt{2}$ is rational

If $\sqrt{2}$ is rational, then $\sqrt{2} = \frac{p}{q}$ $\left(\begin{array}{l} p, q \text{ are integers} \\ \text{and } \frac{p}{q} \text{ is in} \\ \text{its simplest form} \end{array}\right)$

by Contraposition, I can re-write the above statement as follows

If $\sqrt{2} \neq \frac{p}{q}$ $\left(\begin{array}{l} p, q \text{ are integers} \\ \text{and } \frac{p}{q} \text{ is in} \\ \text{its simplest form} \end{array}\right)$ then $\sqrt{2}$ is irrational

Suppose $\boxed{\sqrt{2} = \frac{p}{q}}$

$$2 = \frac{p^2}{q^2}$$

$$2q^2 = p^2 \quad ——— ①$$

$p^2$ must be even $\quad$ ($p$ must also be even)

If $p$ is even, & I can write $p = 2 \cdot k$

then ① becomes $\quad 2q^2 = (2k)^2$

$$2q^2 = 4k^2$$

$$q^2 = 2k^2$$

$q^2$ must also be even $\quad$ ($q$ must be even)

## More on Conditional Statements

$$P \rightarrow q \quad \left\{ \quad \text{i. if } P, \text{ then } q \right.$$

$P \to q$

1. If $P$, then $q$
2. If $P$, $q$
3. $P$ implies $q$
4. $P$ only if $q$
5. $P$ is sufficient for $q$
6. $q$ is necessary for $P$
7. All $P$ are $q$
8. $q$ if $P$

## Exercise

Bi - Conditional

$(P$ if and only if $q)$

Draw a truth table for $\qquad (q \to P) \wedge (P \to q) \equiv P \leftrightarrow q$

| P | q | $q \to P$ | $P \to q$ | $(q \to P) \wedge (P \to q)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | T | F | F |
| F | T | F | T | F |
| F | F | T | T | T |

| P | q | $P \leftrightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

## Exercise

write in words

1. If you lead, then I will follow          (conditional)

2. If I will follow, then you will lead          (converse)

3. If you don't lead, then I will not follow  (Inverse)

4. If I will not follow, then you will not lead  (contrapositive)

## Some equivalent Statements

verify using truth table

| P | T | P∨T |
|---|---|-----|
| T | T | T |
| F | T | T |

1.   $P \vee T \equiv T$

2.   $P \wedge F \equiv F$

3.   $P \vee \sim P \equiv T$

4.   $P \wedge \sim P \equiv F$

5.   $P \vee P \equiv P$

6.   $P \wedge P \equiv P$

7.   $P \vee (q \wedge r) \equiv (P \vee q) \wedge (P \vee r)$

8.   $P \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge r)$

9.   $P \rightarrow q \equiv \sim q \rightarrow \sim P$

10.  $P \rightarrow q \equiv \sim P \vee q$

11.  $\sim (P \wedge q) \equiv \sim P \vee \sim q$

12.  $\sim (P \vee q) \equiv \sim P \wedge \sim q$

In       (        )

10. ( Counting )