Question 6

Part a:

```
x /. NSolve[x^8 - 4 x^3 == 3, x] // TableForm
```

```
-1.06168 - 0.849585 i
-1.06168 + 0.849585 i
-0.873104
0.304089 - 1.22508 i
0.304089 + 1.22508 i
0.500853 - 0.768325 i
0.500853 + 0.768325 i
1.38658
```

Part b:

```
Clear["Global`*"]
```

```
sqrt = Table[Sqrt[n], {n, 10^6, 1.5 × 10^6}];
```

```
Length[Select[sqrt, IntegerQ]]
```

```
225
```

Part c :

```
Integrate[(1 + x^n)^-m, {x, 0, Infinity}, Assumptions → {m > 0, n > 0, m n > 1}]
```

$$\frac{\text{Gamma}\left[m - \frac{1}{n}\right]\,\text{Gamma}\left[1 + \frac{1}{n}\right]}{\text{Gamma}[m]}$$

For the case of m=1:

```
Integrate[(1 + x^n)^-1, {x, 0, Infinity}, Assumptions → {n > 1}]
```

$$\frac{\pi\,\text{Csc}\left[\frac{\pi}{n}\right]}{n}$$

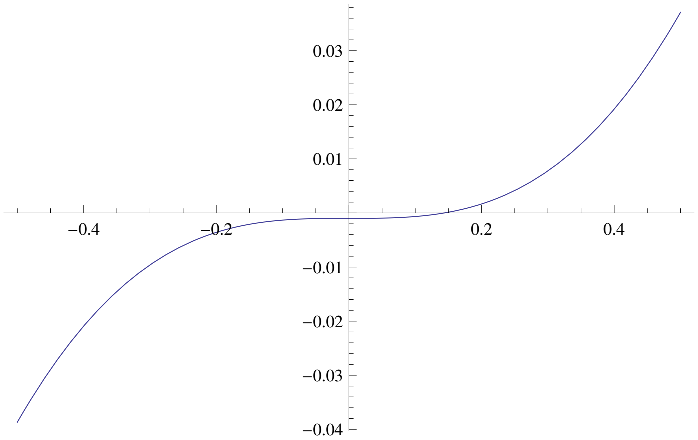
Question 7

```
h = 0.001; J = 1;
```

Part a:

```
T = 1;
```

```
Plot[m == Tanh[(J m + h) / T], {m, -.5, .5}]
```



The function was plotted in order to identify where to locate the root of the function.

```
sol = m /. FindRoot[{m == Tanh[(h + m) / T]}, {m, 1}][[1]]
```

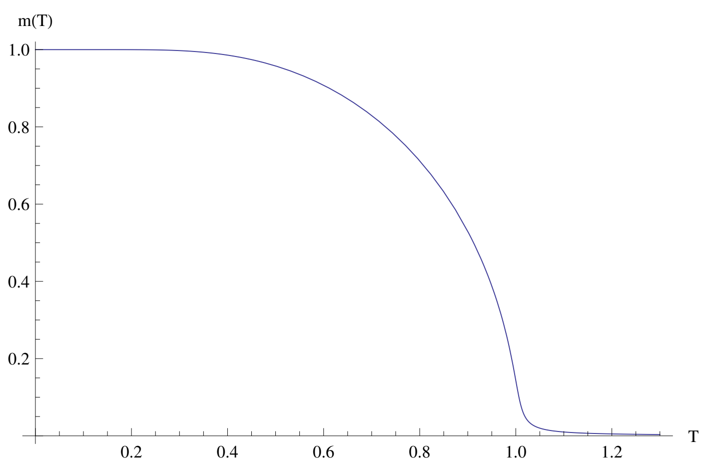
```
0.143626
```

Part b:

```
Clear["Global`*"];
```

```
sol := m /. FindRoot[{m == Tanh[(h + m) / T]}, {m, 1}][[1]]
```

```
Plot[{sol}, {T, 0, 1.3}, AxesLabel → {"T", "m(T)"}]
```



Question 8

```
λ := 9 / 10
```

```
f[x_] := 4 λ x (1 - x);
```

Part a:

```
lyapunov[xinit_, n_, ninitialize_] :=
 (xlist = Drop[NestList[f, xinit, n], ninitialize + 1];
  Apply[Plus, Log[Abs[f'[xlist]]]] / Length[xlist])
```

```
lyapunov[0.5, 50 000, 20]
```

```
0.182963
```

Part b:

```
x0 = N[3 / 10, 8];
```

```
ans[iter_] := N[Nest[f, x0, iter], 8];
```

x1:

```
ans[1]
```

```
0.7560000
```

x2:

```
ans[2]
```

```
0.6640704
```

x5000:

```
N[Nest[f, .3, 5000]]
```

```
0.325115
```

```
Accuracy[0.325115]
```

```
16.4426
```

By using the accuracy function, I was able to find how many digits of precision the 5000th iteration yielded to the “true” answer.

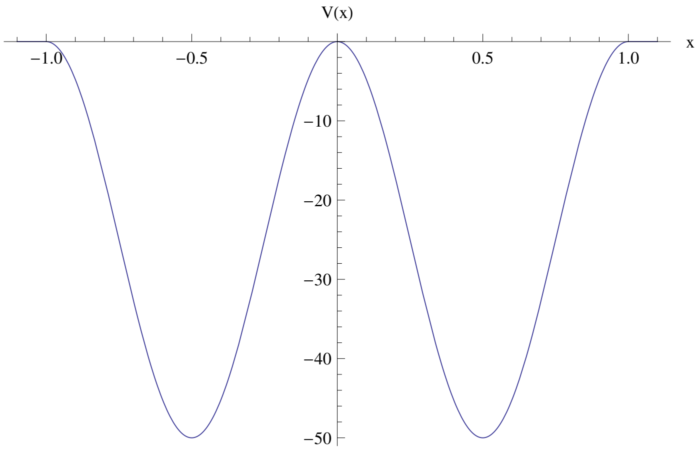
Question 6

```
L = 2; V0 = -50;
```

Part a:

```
v[x_] := V0 / 2 (1 - Cos[4 Pi x / L]) /; Abs[x] ≤ L / 2
v[x_] := 0 /; Abs[x] ≥ L / 2
```

```
Plot[v[x], {x, -.55 L, .55 L}, AxesLabel → {"x", "V(x)"}]
```



Part b:

```
B = 1;
```

```
ψl[x_] := A Exp[Abs[2 en]^(1/2) x]
ψr[x_] := B Exp[-Abs[2 en]^(1/2) x]
```

```
eqn[en_] := ψwell''[x] + 2 (en - v[x]) ψwell[x]
```

```
wavefuncwell[energy_] :=
  (en = energy; NDSolve[{eqn[energy] == 0, ψwell[-L/2] == ψl[-L/2],
     ψwell'[-L/2] == ψl'[-L/2]}, ψwell, {x, -L/2, 0}])
```

```
solwell[x_?NumericQ, en_?NumericQ] := ψwell[x] /. wavefuncwell[en][[1]]
```

```
solwellprime[x_?NumericQ, en_?NumericQ] := ψwell'[x] /. wavefuncwell[en][[1]]
```

The above code evaluates the Schrödinger equation with the given potential function. It also solves the boundary conditions necesarry to evaluate the eigenvalues for energy as well as the wave function.

Ground State-

Because this state has even parity:

```
A = B;
```

```
eval = energy /. FindRoot[solwellprime[0, energy], {energy, -50, -49}]
```

```
-35.7501
```

Energy = -35.7501

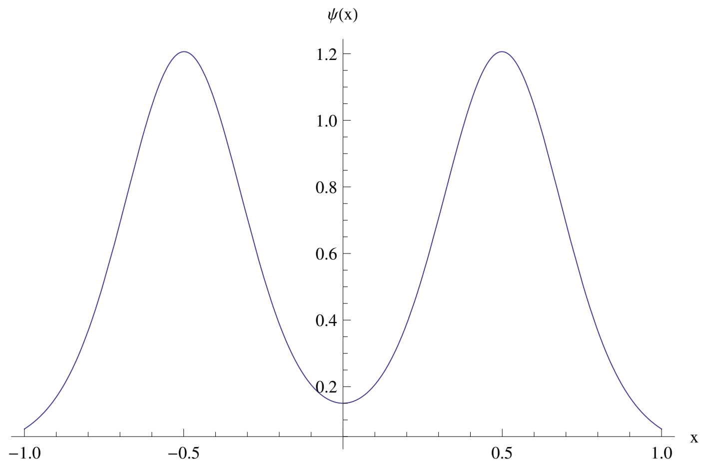
```
efuncwell[x_] = ψwell[x] /. wavefuncwell[eval][[1]];
```

```
ψ[x_] := efuncwell[x] /; -L/2 ≤ x ≤ 0
ψ[x_] := efuncwell[-x] /; 0 ≤ x ≤ L/2
ψ[x_] := ψl[x] /; x < -L/2
ψ[x_] := ψr[x] /; x > L/2
```

```
norm = Sqrt[NIntegrate[ψ[x]^2, {x, -Infinity, Infinity}]];

ψnorm[x_] := ψ[x] / norm

Plot[ψnorm[x], {x, -.5 L, .5 L}, PlotRange → All, AxesLabel → {"x", "ψ(x)"}]
```




```
Clear["Global`*"]
```

1st Excited State-

Because this state has odd parity:

```
A = -B;

eval = energy /. FindRoot[solwell[0, energy], {energy, -50, -49}]
-35.5663
```

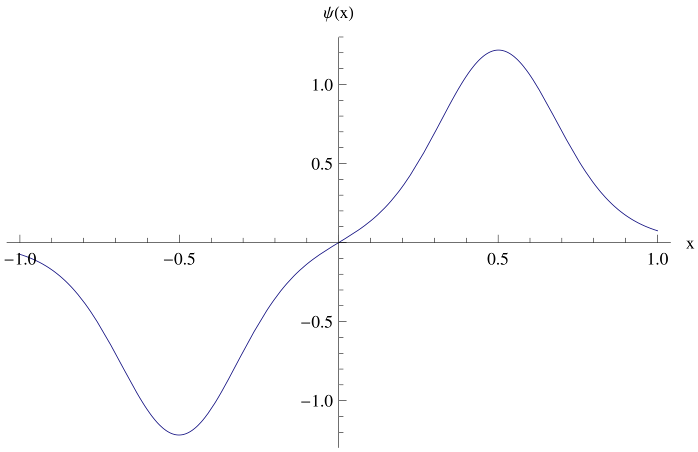
Energy = -35.5663

```
ψ[x_] := efuncwell[x] /; -L / 2 ≤ x ≤ 0
ψ[x_] := -efuncwell[-x] /; 0 ≤ x ≤ L / 2
ψ[x_] := ψl[x] /; x < -L / 2
ψ[x_] := ψr[x] /; x > L / 2

norm = Sqrt[NIntegrate[ψ[x]^2, {x, -Infinity, Infinity}]];

ψnorm[x_] := ψ[x] / norm
```

```
Plot[ψnorm[x], {x, -.5 L, .5 L}, PlotRange → All, AxesLabel → {"x", "ψ(x)"}]
```



2nd Excited State-

Because this state has even parity:

```
A = B;
```

```
eval = energy /. FindRoot[solwellprime[0, energy], {energy, -10, -0}]
```

```
-11.7515
```

Energy = -11.7515

```
ψ[x_] := efuncwell[x] /; -L / 2 ≤ x ≤ 0
ψ[x_] := efuncwell[-x] /; 0 ≤ x ≤ L / 2
ψ[x_] := ψl[x] /; x < -L / 2
ψ[x_] := ψr[x] /; x > L / 2
```

```
norm = Sqrt[NIntegrate[ψ[x]^2, {x, -Infinity, Infinity}]];
```

```
ψnorm[x_] := ψ[x] / norm
```

```
Plot[ψnorm[x], {x, -.5 L, .5 L}, PlotRange → All, AxesLabel → {"x", "ψ(x)"}]
```