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PHYS 115
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Assignment #6

Question 1

200!

```
788 657 867 364 790 503 552 363 213 932 185 062 295 135 977 687 173 263 294 742 533 244 359 449 \
  963 403 342 920 304 284 011 984 623 904 177 212 138 919 638 830 257 642 790 242 637 105 061 926 \
  624 952 829 931 113 462 857 270 763 317 237 396 988 943 922 445 621 451 664 240 254 033 291 864 \
  131 227 428 294 853 277 524 242 407 573 903 240 321 257 405 579 568 660 226 031 904 170 324 062 \
  351 700 858 796 178 922 222 789 623 703 897 374 720 000 000 000 000 000 000 000 000 000 000 000 \
  000 000 000 000 000
```

Question 2

N[50 000!]

$3.347320509597145 \times 10^{213\,236}$

Question 3

Input :

```
#include <stdio.h>
#include <math.h>

double f(double x)
{
    return log10(x);
}

main()
{
    int i, exponent, argument = 50000;
    double mantissa, logfactorial, logmantissa;

    //computes log base 10 of the factorial
    for(i = argument; i > 0; i--)
    {
        logfactorial += f(i);
    }


    printf("log(%d!) = %f  \n", argument, logfactorial);

    exponent = logfactorial;
    //computes the exponent of scientific notation
    logmantissa = logfactorial - exponent;
    //finds what's "left over" after subtracting exponent from log
    mantissa = pow(10, logmantissa);
    //raises  the "leftovers" by a power of 10 to find coefficient

    printf("%d! = %f + 10^%d\n", argument, mantissa, exponent);

return 0;
}
```

Output:

log(50000!) = 213236.524697
50000! = 3.347321 + 10^213236

Comment: The technique to solving this factorial in C is understanding the product property of logarithms, which allows the multiplied values in its argument to be seperated into the sum of their logarithmic values. After this sum is computed, the next step is to splt the value into a sum of an integer and a decimal. When these two additive values are raised by base ten, they then multiply. The integer will then be the power in which 10 is raised, while 10 raised to the decimal will take the position of the mantissa in floating point form.

Question 4

```
N[E^(Pi * √163), 29] // AccountingForm
```

262537412640768744.00000000000

```
N[E^(Pi * √163), 30] // AccountingForm
```

262537412640768743.999999999999

As shown above, the function rounds off to an integer until it’s output is evaluated to 30 decimal places.

Question 5

```
TableForm[x /. NSolve[x^7 + x^5 + 2 x^3 + x^2 + 1 == 0]]
```

-0.812432
-0.640787 - 1.07931 i
-0.640787 + 1.07931 i
0.254825 - 0.700968 i
0.254825 + 0.700968 i
0.792178 - 0.881387 i
0.792178 + 0.881387 i

Question 6

```
NIntegrate[HermiteH[4, x]^2 Exp[-x^2], {x, -Infinity, Infinity}]
```

680.622

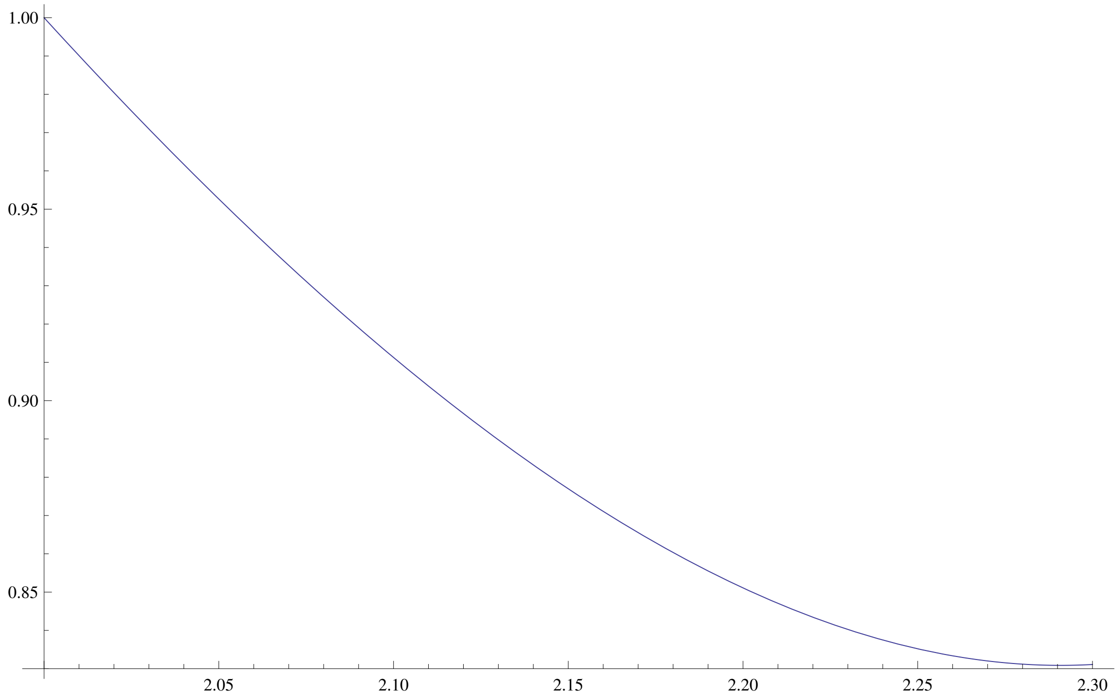
Question 7

```
ans[x_] =
  y[x] /.
   NDSolve[
     {y''[x] == 2 x + y[x] + 3 y'[x], y[2] == 1, y'[2] == -1}, y, {x, 2.0, 2.3}][[1]];
```

```
ans[2.2]
```

0.851094

```
Show[Plot[ans[x], {x, 2.0, 2.3}], ImageSize → Large]
```



Question 8

```
TableForm[{{x, y} /. Solve[{4 x + 5 y == 5, 6 x + 7 y == 7}, {x, y}][[1]]},
 TableHeadings → {None, {"x", "y"}}]
```

x	y
0	1

Question 9

```
x /. Solve[{Sqrt[x + 2] + 4 == x}, {x}][[1]]
```

7

Question 10

```
Limit[1 / (E^x - E^(x - x^(-2))), x → Infinity]
```

0

Question 11

```
y[x] /. DSolve[y''[x] - 6 y'[x] + 13 y[x] == (E^x) Cos[x], y, x][[1]] // Simplify
```

$$\frac{1}{65} e^{x} \left(65\, e^{2x}\, C[2]\, \mathrm{Cos}[2x] - 4\, \mathrm{Sin}[x] + \mathrm{Cos}[x] \left(7 + 130\, e^{2x}\, C[1]\, \mathrm{Sin}[x]\right)\right)$$

Question 12

```
xlist = Table[n, {n, 0, 10}]
```

```
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

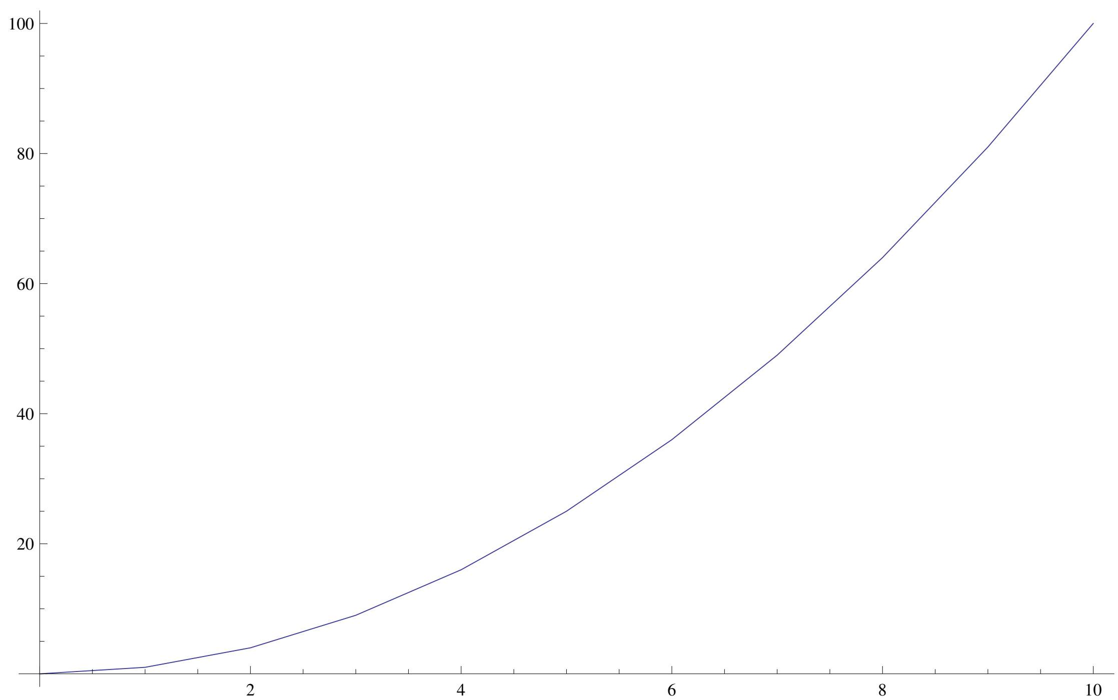
```
ylist = Table[n^2, {n, 0, 10}]
```

```
{0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

```
Transpose[{xlist, ylist}]
```

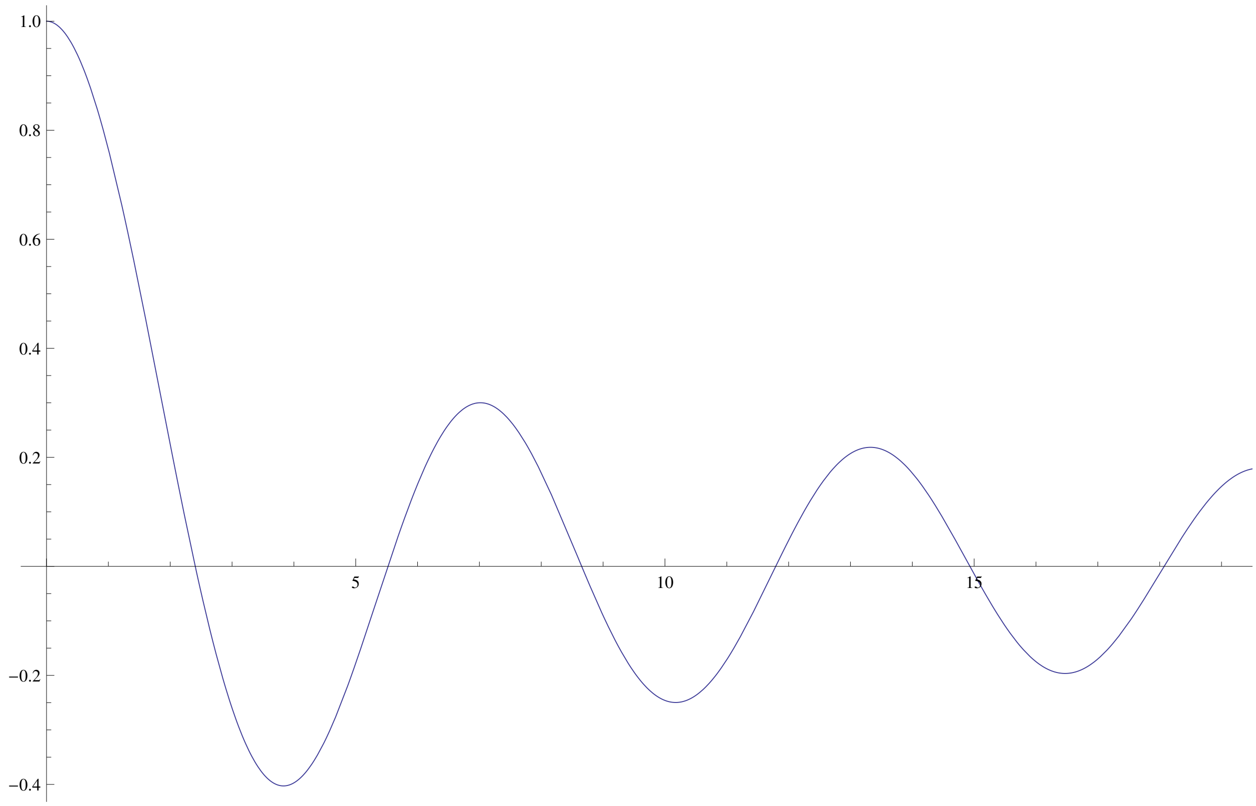
```
{{0, 0}, {1, 1}, {2, 4}, {3, 9}, {4, 16},
 {5, 25}, {6, 36}, {7, 49}, {8, 64}, {9, 81}, {10, 100}}
```

```
ListPlot[{{0, 0}, {1, 1}, {2, 4}, {3, 9}, {4, 16}, {5, 25},
  {6, 36}, {7, 49}, {8, 64}, {9, 81}, {10, 100}}, Joined → True]
```



Question 13

```
Plot[BesselJ[0, x], {x, 0, 20}]
```




```
TableForm[{x /. FindRoot[BesselJ[0, x], {x, {2, 5, 9, 12, 15}}]},
 TableHeadings → {{Roots}, {x1, x2, x3, x4, x5}}]
```

	x1	x2	x3	x4	x5
Roots	2.40483	5.52008	8.65373	11.7915	14.9309

Question 14

```
u0 = 2^(-0/2) (Pi 0!^2)^(-1/4) HermiteH[0, x] Exp[-(x^2)/2];

u2 = 2^(-2/2) (Pi 2!^2)^(-1/4) HermiteH[2, x] Exp[-(x^2)/2];

u3 = 2^(-3/2) (Pi 3!^2)^(-1/4) HermiteH[3, x] Exp[-(x^2)/2];

u4 = 2^(-4/2) (Pi 4!^2)^(-1/4) HermiteH[4, x] Exp[-(x^2)/2];
```

Part a

```
Integrate[u3^2, {x, -Infinity, Infinity}]
```

1

Part b

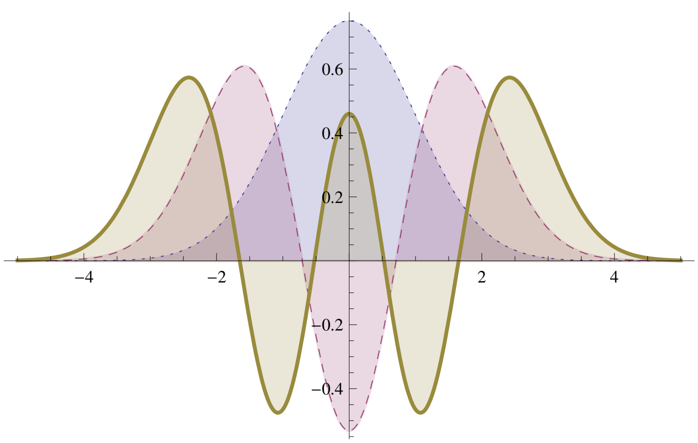
```
Integrate[u3 u2, {x, -Infinity, Infinity}]
```

0

Part c

```
Plot[{u0, u2, u4}, {x, -5, 5},
 PlotStyle → {Dotted, Dashed, Thick}, Filling → Axis]
```

0



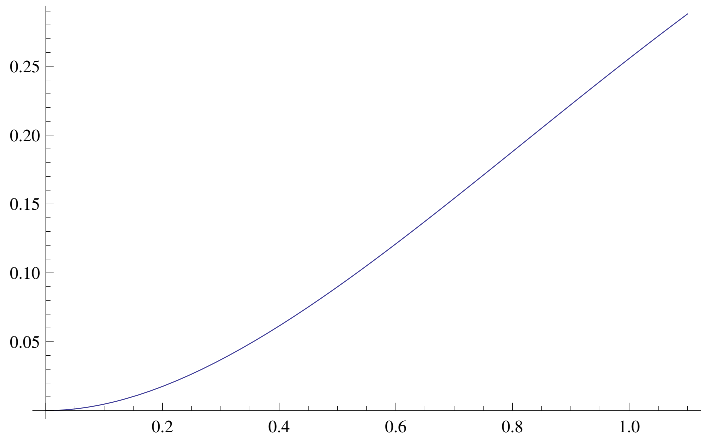
u0: Blue,   u2: Magenta, u4: Brown

Question 15

```
nans[t_] = i[t] /.
  NDSolve[{i'[t] + 2 i[t] == Sin[t], i[0] == 0}, i[t], {t, 0, 1.1}][[1]]
```

InterpolatingFunction[{{0., 1.1}}, <>][t]

```
Plot[nans[t], {t, 0, 1.1}]
```




```
aans[t_] = i[t] /.
   DSolve[{i'[t] + 2 i[t] == Sin[t], i[0] == 0}, i, t][[1]] // Simplify
```

$$\frac{1}{5}\left(e^{-2\,t} - \mathrm{Cos}[t] + 2\,\mathrm{Sin}[t]\right)$$

```
numeric = Table[nans[t],  {t, 0, 1, .1}];
```

```
analytic = Table[aans[t], {i, 0, 9}, {t, i / 10, i / 10 + .1}];
```

```
TableForm[Table[{j, numeric[[j]], analytic[[j]]}, {j, 1, 10}],
 TableHeadings → {None, {"t", "Numeric", "Analytic"}}]
```

t	Numeric	Analytic
1	0.	0
2	0.0046787	$\frac{1}{5}\left(\frac{1}{e^{1/5}} - \mathrm{Cos}\left[\frac{1}{10}\right] + 2\,\mathrm{Sin}\left[\frac{1}{10}\right]\right)$
3	0.0175184	$\frac{1}{5}\left(\frac{1}{e^{2/5}} - \mathrm{Cos}\left[\frac{1}{5}\right] + 2\,\mathrm{Sin}\left[\frac{1}{5}\right]\right)$
4	0.0369031	$\frac{1}{5}\left(\frac{1}{e^{3/5}} - \mathrm{Cos}\left[\frac{3}{10}\right] + 2\,\mathrm{Sin}\left[\frac{3}{10}\right]\right)$
5	0.0614209	$\frac{1}{5}\left(\frac{1}{e^{4/5}} - \mathrm{Cos}\left[\frac{2}{5}\right] + 2\,\mathrm{Sin}\left[\frac{2}{5}\right]\right)$
6	0.0898296	$\frac{1}{5}\left(\frac{1}{e} - \mathrm{Cos}\left[\frac{1}{2}\right] + 2\,\mathrm{Sin}\left[\frac{1}{2}\right]\right)$
7	0.121029	$\frac{1}{5}\left(\frac{1}{e^{6/5}} - \mathrm{Cos}\left[\frac{3}{5}\right] + 2\,\mathrm{Sin}\left[\frac{3}{5}\right]\right)$
8	0.154038	$\frac{1}{5}\left(\frac{1}{e^{7/5}} - \mathrm{Cos}\left[\frac{7}{10}\right] + 2\,\mathrm{Sin}\left[\frac{7}{10}\right]\right)$
9	0.18798	$\frac{1}{5}\left(\frac{1}{e^{8/5}} - \mathrm{Cos}\left[\frac{4}{5}\right] + 2\,\mathrm{Sin}\left[\frac{4}{5}\right]\right)$
10	0.222069	$\frac{1}{5}\left(\frac{1}{e^{9/5}} - \mathrm{Cos}\left[\frac{9}{10}\right] + 2\,\mathrm{Sin}\left[\frac{9}{10}\right]\right)$