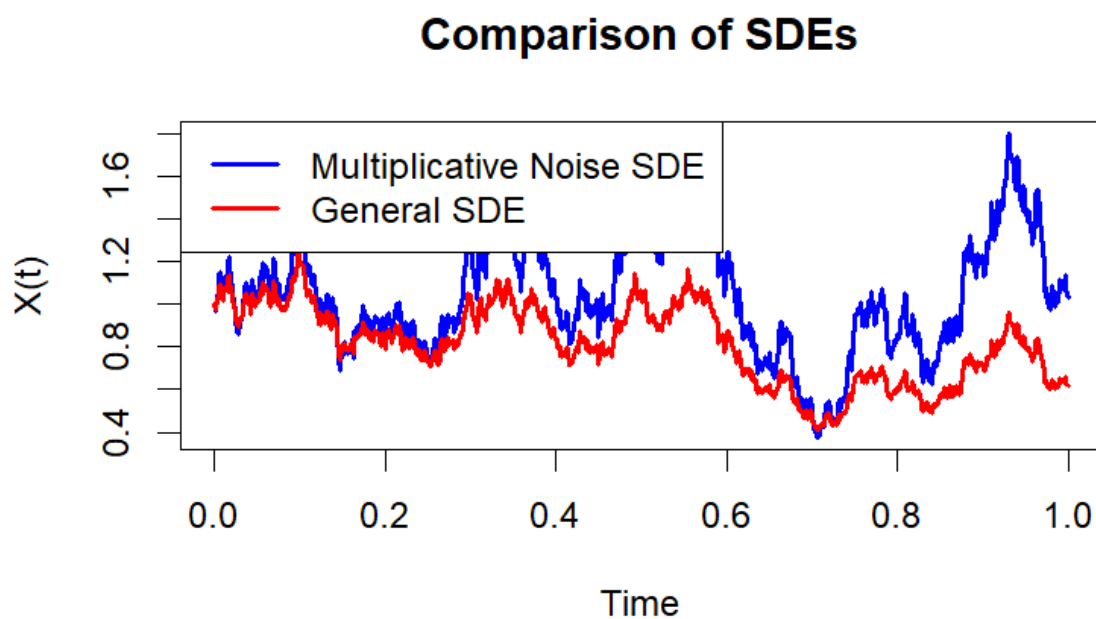# Comparison of SDEs

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Interpretation of Sample Paths

The plot shows sample paths of two stochastic differential equations (SDEs):
the general SDE and the SDE with multiplicative noise.

Comparison of SDEs

General SDE (Red Line)

$$dX_t = a_1(t)X_t\,dt + b_1(t)X_t\,dW_t$$

- **Behavior:** The general SDE exhibits relatively smoother behavior with less pronounced fluctuations compared to the SDE with multiplicative noise.

- **Influence:** The changes in X_t are solely influenced by the terms proportional to X_t, both in the drift $(a_1(t)X_t)$ and the diffusion $(b_1(t)X_t)$ components.

SDE with Multiplicative Noise (Blue Line)

$$dX_t = (a_1(t)X_t + a_2(t))\, dt + (b_1(t)X_t + b_2(t))\, dW_t$$

- **Behavior:** This SDE shows more volatility and higher amplitude fluctuations compared to the general SDE.

- **Influence:** The additional constant terms $a_2(t)$ and $b_2(t)$ add more variability to the path of X_t. The term $a_2(t)$ in the drift and $b_2(t)$ in the diffusion introduce extra variability and shifts, leading to more pronounced spikes and drops in the path.

Differences

- **Fluctuations:** The SDE with multiplicative noise (blue line) experiences larger and more frequent fluctuations due to the additional terms $a_2(t)$ and $b_2(t)$.

- **Range:** The path of the SDE with multiplicative noise spans a broader range of values compared to the general SDE, indicating higher variability.

- **Trend:** Both SDEs start from the same initial value but diverge over time. The path of the SDE with multiplicative noise responds more strongly to the random noise dW_t due to the additive diffusion term $b_2(t)$.