Convergence of Monte Carlo Methods

Okemwa.Sharon

July 2024

The plot reveals the convergence of various Monte Carlo methods for option pricing as the number of simulations increases.

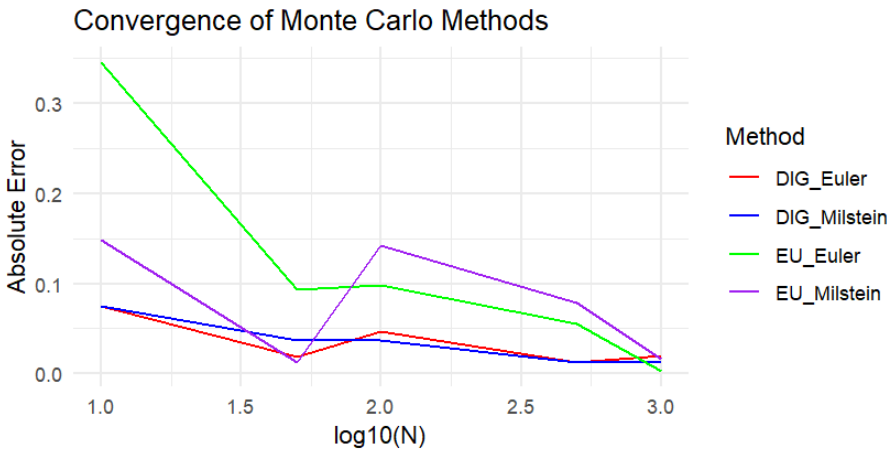

Figure 1: Convergence of different Monte Carlo methods for pricing options

Axes and Labels

- **X-axis (log10(N))**: These represent base 10 logarithms of numbers of simulations.log10(N) = 1 is N = 10 simulations; log10(N) = 2 is N = 100 simulations; and log10(N) = 3 is N = 1000 simulations.
- **Y-axis (Absolute Error)**: It represents the absolute error between the theoretically Black-Scholes-priced option and the Monte Carlo estimated price. This error shows how close the Monte Carlo estimate is to an actual value.

Legend

- **DIG_Euler** (Red): Digital option price by Euler Scheme.
- **DIG_Milstein** (Blue): Digital option priced with the Milstein scheme.
- **EU_Euler** (Green): European option priced using the Euler method.
- **EU_Milstein** (Purple): European option priced using the Milstein scheme.

Interpretation

- **High Errors for Initials**: For the low values of log10(N), essentially the set with fewer simulations (N 10), the absolute errors across all methods are comparably high. This is because the reduced number of simulations will give a weak point estimate for the option prices.
- **Convergence**: As the number of simulations grows, the absolute errors decrease. This implies that higher numbers of simulations allow for better estimations.
  - **DIG_Euler** (Red line): The error starts pretty low and falls as the number of simulations increases. It is linearly improved.
  - **DIG_Milstein** (Blue line): Similar to the DIG_Euler, error is high initially, but decreases subsequently. The rate of convergence is almost similar in both methods for the digital option.
  - **EU_Euler** (Green line): Error starts above the digital options and is more spread but generally decreases with a growing number of simulations. The method fluctuates for a lower number of simulations.
  - **EU_Milstein** (Purple line): Error is very high to start with and oscillates, but there is a clear trend to go downwards by increasing the number of simulations. This shows the most significant drop in error.

Observations

1. **Digital Options**: both Euler and Milstein types (DIG_Euler and DIG_Milstein) start with much more minor initial errors and converge much faster. It means that, as far as the number of simulations is concerned, digital options can be estimated with less computational effort.
2. **European Options**: In both methods adopted for the European options—EU_Euler and EU_Milstein—the initial errors are higher and much more variable, yet reveal convergence as the number of simulations increases.

3. **Scheme Comparison**: The Milstein scheme performs with more minor errors than the Euler scheme for both types of options. his is because the Milstein scheme has one more term to offset the diffusion process.

- An increase in the number of simulations leads to better precision in pricing options.
- The Milstein scheme is somewhat more accurate than the Euler one.
- Digital options have a more accurate estimated price relative to European options, for an equal amount of simulations.