797N - Macro 3

Fall 2015

Problem set 9

1. (Final 2013, question 2) Kaldor (1940) assumed that both investment and saving depend on output Y and the capital stock K; $I = I(Y, K)$ and $S = S(Y, K)$. Non-linearities in the investment and / or saving functions could, he argued, generate persistent fluctuations in output.

   (a) Briefly describe the intuition behind the cycles and discuss the plausibility of the key assumptions.

   (b) Now consider the following model of investment and saving:

   $$\begin{aligned}\dot{Y} &= \lambda[I(Y,\pi) - S(Y,\pi)] \\ \dot{\pi} &= f(Y)\end{aligned}$$

   where π is the profit share. Assume that $I_Y > S_Y, I_\pi < S_\pi, f\,' > 0$.

   i. Show that the model has a unique stationary point and that the stationary point is locally asymptotically unstable.

   ii. Draw a phase diagram and briefly discuss how non-linearities in the investment function may convert the local instability into persistent fluctuations. [Hint: Consider modifications such that

   $$\begin{aligned}I_Y &> S_Y \text{ for } Y_0 < Y < Y_1 \\ I_Y &< S_Y \text{ for } Y < Y_0 \text{ or } Y > Y_1\end{aligned}$$

   where $Y_0 < f^{-1}(0) < Y_1$.]

   (c) Briefly (i) relate your analysis in b) to Kaldor's original model and (ii) discuss whether the modified model is subject to the same criticisms as Kaldor's original model. Does the modified model have other important weaknesses?

2. Let the consumption function be

   $$C = aY + bA$$

   where A is wealth. Disregarding capital gains, the change in wealth is

   $$\dot{A} = Y - C$$

   Analyze the evolution of the wealth income ratio A/Y, assuming that income grows at a constant rate, $\hat{Y} = g$.

3. Let the consumption function be

$$C = a(1 - s_f\pi)Y + bA$$

where A is wealth; s_f and π are the retention rate and the profit share, respectively, and both are assumed constant. There is no inflation and the price level is normalized at one. Wealth takes the form of equity holdings or money (bank deposits) and, for simplicity, assume that the interest rate on deposits is zero and that households want to hold a constant fraction of their total wealth in money:

$$\begin{aligned} A &= M + vN \\ \frac{M}{A} &= \lambda \end{aligned}$$

Assume that net issues of new equity are zero ($\hat{N} = 0$).

(a) Show that the budget constraint can be written

$$C + \dot{M} = (1 - s_f\pi)Y$$

(b) Now consider a steady growth path with $\hat{Y} = \hat{C} = g$. Find an expression for the steady-growth ratio of wealth to income (A/Y).

(c) Do households have capital gains? If so, find an expression for these capital gains and explain why the capital gains did not seem to enter the analysis under a) and b).