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Question2:

Suppose we have M roads from warehouses to shops, let's sort all the roads by their time cost d_i
Complexity $O(M \log(M))$

First create a bipartite graph, let n warehouse and n shops be the vertices (all warehouses in the same side and shops in the other side)

For every road i from warehouse j to shop k , link an edge from warehouse j to shop k , edge cost is d_i .

Create a super source linked to all the warehouses, only one truck located at one warehouse, $c_i = 1$.

Create a super sink linked to all the shops, $c_i = 1$

Use binary search to find the first d_i

First $d_i = d(M/2)$:

1. If there exists a road j has $d_j < d_i$, $c_j = 1$ else $c_j = 0$
2. Use max-flow algorithm to find the max-flow, if $\text{max-flow} == n$, which means time d_i can send all the trucks to all the shops, but that time may not be the shortest cost. so if equals to n , keep binary search the smaller half, else keep binary search the larger half
3. Use binary search to get the next d_i and repeated step 1,2 until find the smallest time (if only three elements a, b, c in the binary search and $\text{max-flow } b == n$, if $\text{max-flow } a \neq n$, b is the smallest)
4. That time is the shortest time until all shops are supplied

Complexity max-flow algorithm * binary search = $O(n^3) * O(\log(M)) = O(n^3 \log(M))$

Total time complexity $O(M \log(M)) + O(n^3 \log(M))$