

1.No, they are different.

Let  $\forall x P(x) \wedge Q(x)$  be True then  $\forall x P(x)$  is True and  $Q(x)$  is True

from  $\forall x P(x)$  is True we got  $P(x)$  is True

then  $\exists x(P(x) \wedge Q(x))$  Is True

but  $\forall x P(x) \wedge Q(x)$  is different to  $\exists x(P(x) \wedge Q(x))$

2.  $\exists y \forall x P(x, y)$  imply  $\forall x \exists y P(x, y)$ ,

We suppose  $\exists y \forall x P(x, y)$  is true, then for any constant  $c$

$\exists y P(c, y)$  is true so  $\forall x \exists y P(x, y)$  is true

3.wp's of the assignment statement is

$(x > y) \rightarrow (\text{temp} := x; x := y; y := \text{temp};) = \{x < y\} \wedge \neg (x > y) \rightarrow \{\text{nothing}\} = \{x < y\}$

For  $(x > y) \rightarrow (\text{temp} := x; x := y; y := \text{temp};) = \{x < y\}$ ,  $x$  in post condition will becomes to  $y$  and  $y$  will becomes to  $x$  so  $((x > y) \rightarrow (y < x))$  and  $\neg (x > y) \rightarrow \{\text{nothing}\} = \{x < y\}$  no change which is  $\neg (x > y) \rightarrow (x < y)$ .

$((x > y) \rightarrow (y > x))$  also can write to  $((x > y) \rightarrow (x > y))$  which is True because  $A \rightarrow A$  will always be True.

$\neg (x > y) \rightarrow (x < y)$  can be write to  $(x > y) \vee (x < y)$ .

$Wp = \neg (x > y) \rightarrow (x < y) \wedge ((x > y) \rightarrow (y > x))$

$= (x > y) \vee (x < y) \vee \text{True}$

$= (x > y) \vee (x < y)$

4.(a)  $\exists x(P(x) \rightarrow Q(x))$

$\equiv \exists x(\neg P(x) \vee Q(x))$  ----- Implication law

$\equiv \exists x \neg P(x) \vee \exists x Q(x)$  ----- associative

$\equiv \neg \exists x P(x) \vee \exists x Q(x)$  ----- Commutativity?

$\equiv \forall x P(x) \rightarrow \exists x Q(x)$  ----- De Morgan's laws and Implication law

(b) p: people L(p): people have license D(p): people drive car

we want to prove

$(\forall p (D(p) \rightarrow L(p)) \wedge (\exists p (\neg L(p))) \rightarrow \exists p (\neg D(p)))$  is correct

From LHS

Let  $(\forall p (D(p) \rightarrow L(p)) \wedge (\exists p (\neg L(p)))$  be true

$\equiv \neg(\forall p (D(p) \wedge \neg L(p)) \wedge (\exists p (\neg L(p)))$

-----Implication law and associative

$\equiv (\neg \forall p D(p) \vee \forall p L(p)) \wedge (\exists p \neg L(p))$  ----- Associativity

$\equiv (\exists p \neg D(p) \wedge \exists p \neg L(p)) \vee (\forall p L(p) \wedge (\exists p \neg L(p)))$

-----De Morgan's laws and Distributive rules

$\equiv (\exists p \neg D(p) \wedge \exists p \neg L(p)) \vee \text{False}$  -----negation law

$\equiv \exists p \neg D(p) \wedge \exists p \neg L(p)$  -----Identity

So  $(\exists p \neg D(p))$  is true

so  $(\forall p (D(p) \rightarrow L(p)) \wedge (\exists p (\neg L(p))) \rightarrow \exists p (\neg D(p)))$  is correct

(c) s: statements F(s): Statements is false K(s): speaker knows it is false

Lies =  $K(s) \wedge F(s)$

So the description can be write as

$\exists s (\neg K(s) \wedge F(s)) \rightarrow \exists s (F(s) \wedge \neg (K(s) \wedge F(s)))$ , to prove it's correct

Let  $\exists s (\neg K(s) \wedge F(s))$  be true

RHS can be simplify,

$\exists s (F(s) \wedge \neg (K(s) \wedge F(s)))$

$\equiv \exists s (F(s) \wedge (\neg K(s) \vee \neg F(s)))$  ----- De Morgan's laws

$\equiv \exists s ((F(s) \wedge \neg K(s)) \vee (F(s) \wedge \neg F(s)))$  -----Distributive rules

$\equiv \exists s ((F(s) \wedge \neg K(s)) \vee \text{False})$  -----Negation

$\equiv \exists s (F(s) \wedge \neg K(s))$  -----Identity

which is same to LHS

So let LHS be True,  $\exists s (\neg K(s) \wedge F(s)) \rightarrow \exists s (F(s) \wedge \neg (K(s) \wedge F(s)))$  is correct

5.

(a). Returns the location of the last 0 in the list

If there is no 0 in the array, return -1

(b). Input must not smaller than 1,

return value must in domain(-1,n-1)

all the index values after the index of return value in array should not equals to 0

the index value of return value in array should equals to 0.

(c)

We want to prove

$$r \in [-1, n-1] \wedge (\forall i \in [r+1, n-1] A[i] \neq 0) \wedge \neg (r \neq -1 \ \&\& \ A[r] \neq 0)$$

$\equiv$

$$r \in [-1, n-1] \wedge (\forall i \in [r+1, n-1] A[i] \neq 0) \wedge (r \in [0, n-1] \rightarrow A[r] = 0)$$

so we have to prove  $\neg (r \neq -1 \ \&\& \ A[r] \neq 0) \equiv (r \in [0, n-1] \rightarrow A[r] = 0)$

$$\neg (r \neq -1 \ \&\& \ A[r] \neq 0)$$

$$\equiv r == -1 \ || \ A[r] == 0$$

$$\equiv r \neq -1 \rightarrow A[r] == 0$$

And if  $r \in [-1, n-1]$  is True then

$$\neg (r \neq -1 \ \&\& \ A[r] \neq 0) \equiv (r \in [0, n-1] \rightarrow A[r] = 0)$$

And whatever we suppose

$$\equiv r \in [0, n-1] \rightarrow A[r] == 0 \text{ LHS or RHS is True, } r \in [-1, n-1] \text{ will also be True}$$

So

$$r \in [-1, n-1] \wedge (\forall i \in [r+1, n-1] A[i] \neq 0) \wedge \neg (r \neq -1 \ \&\& \ A[r] \neq 0)$$

$\equiv$

$$r \in [-1, n-1] \wedge (\forall i \in [r+1, n-1] A[i] \neq 0) \wedge (r \in [0, n-1] \rightarrow A[r] = 0)$$