

Jin-Ao Olson Zhang
Z52ii4i4
Question i.

Suppose $\text{True}(i,j)$ which represents the number of ways to place parentheses between i and j that gives True and $\text{False}(i,j)$ represents the number of ways to place parentheses between i and j that gives False

$\text{True}(i,i) = 1$ if $i = T$

$\text{True}(i,i) = 0$ if $i = F$

$\text{False}(i,i) = 0$ if $i = T$

$\text{False}(i,i) = 1$ if $i = F$

So for every expression E with size e , it can be expressed as

$\text{Expression1}(i,k) \ \&/\ ||/\text{NAND}/\text{NOR} \ \text{expression2}(k+i,e) \ , \ i \leq k \leq e$

$\&$:

If $\text{expression1}(i,k) \ \& \ \text{expression2}(k+i,e) == \text{True}$

$\text{Expression1} == \text{expression2} = \text{True}$

Which is $\text{True}(i,k) \ \& \ \text{True}(k+i,e)$

So there are $\text{True}(i,k) * \text{True}(k+i,e)$ ways

And All - $\text{True}(i,k) * \text{True}(k+i,e)$ ways for $\text{expression1}(i,k) \ \& \ \text{expression2}(k+i,e) == \text{False}$

$||$:

If $\text{expression1}(i,k) \ \& \ \text{expression2}(k+i,e) == \text{True}$

It could be

$\text{True}(i,k) \ \& \ \text{True}(k+i,e)$

$\text{False}(i,k) \ \& \ \text{True}(k+i,e)$

$\text{True}(i,k) \ \& \ \text{False}(k+i,e)$

So there are $\text{True}(i,k) * \text{True}(k+i,e) + \text{False}(i,k) \ \& \ \text{True}(k+i,e) + \text{True}(i,k) \ \& \ \text{False}(k+i,e)$

And All-True ways for false

ways

NAND:

opposite to $\&$

ALL - $\text{True}(i,k) \ \& \ \text{True}(k+i,e) == \text{False}(i,k) * \text{False}(k+i,e) + \text{False}(i,k) \ \&$

$\text{True}(k+i,e) + \text{True}(i,k) \ \& \ \text{False}(k+i,e)$

And All-True ways for false

NOR:

Opposite to $||$

ALL - $\text{True}(i,k) * \text{True}(k+i,e) + \text{False}(i,k) \ \& \ \text{True}(k+i,e) + \text{True}(i,k) \ \& \ \text{False}(k+i,e)$

$= \text{False}(i,k) * \text{False}(k+i,e)$

And All-True ways for false

So recursively

$$T(i,j) = \sum_{k=i}^{j-1}$$

If operator at K = &

$$\text{True}(i,k) \ \& \ \text{True}(k+i,j)$$

If operator at K = ||

$$\text{True}(i,k)*\text{True}(k+i,j) + \text{False}(i,k) \ \& \ \text{True}(k+i,j) + \text{True}(i,k) \ \& \ \text{False}(k+i,j)$$

If operator at K = NAND

$$\text{False}(i,k)*\text{False}(k+i,j) + \text{False}(i,k) \ \& \ \text{True}(k+i,j) + \text{True}(i,k) \ \& \ \text{False}(k+i,j)$$

If operator at K = NOR

$$\text{False}(i,k)*\text{False}(k+i,j)$$

$$F(i,j) = \sum_{k=i}^{j-1}$$

If operator at K = &

$$\text{False}(i,k)*\text{False}(k+i,j) + \text{False}(i,k) \ \& \ \text{True}(k+i,j) + \text{True}(i,k) \ \& \ \text{False}(k+i,j)$$

If operator at K = ||

$$\text{False}(i,k)*\text{False}(k+i,j)$$

If operator at K = NAND

$$\text{True}(i,k) \ \& \ \text{True}(k+i,j)$$

If operator at K = NOR

$$\text{True}(i,k)*\text{True}(k+i,j) + \text{False}(i,k) \ \& \ \text{True}(k+i,j) + \text{True}(i,k) \ \& \ \text{False}(k+i,j)$$

And total way between i and j will be T(i,j)