Lecture 11: Course Review

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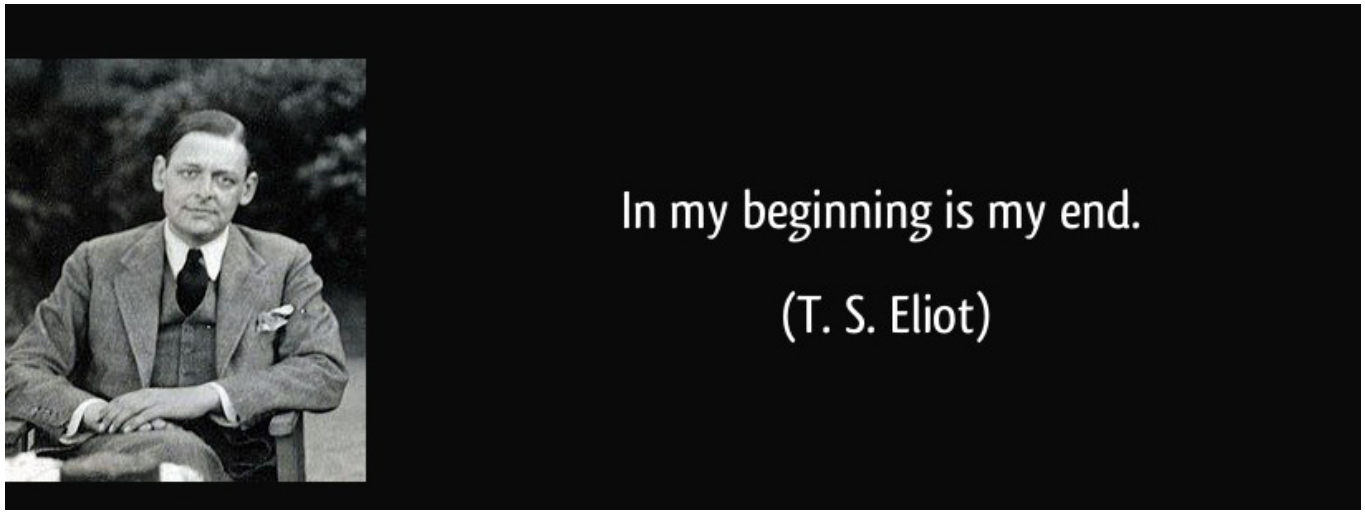
Final Exam

- Nine problems
- Closed book without cheating sheets
- 9:00-12:00 in the morning of June 25
- Teaching Center 201

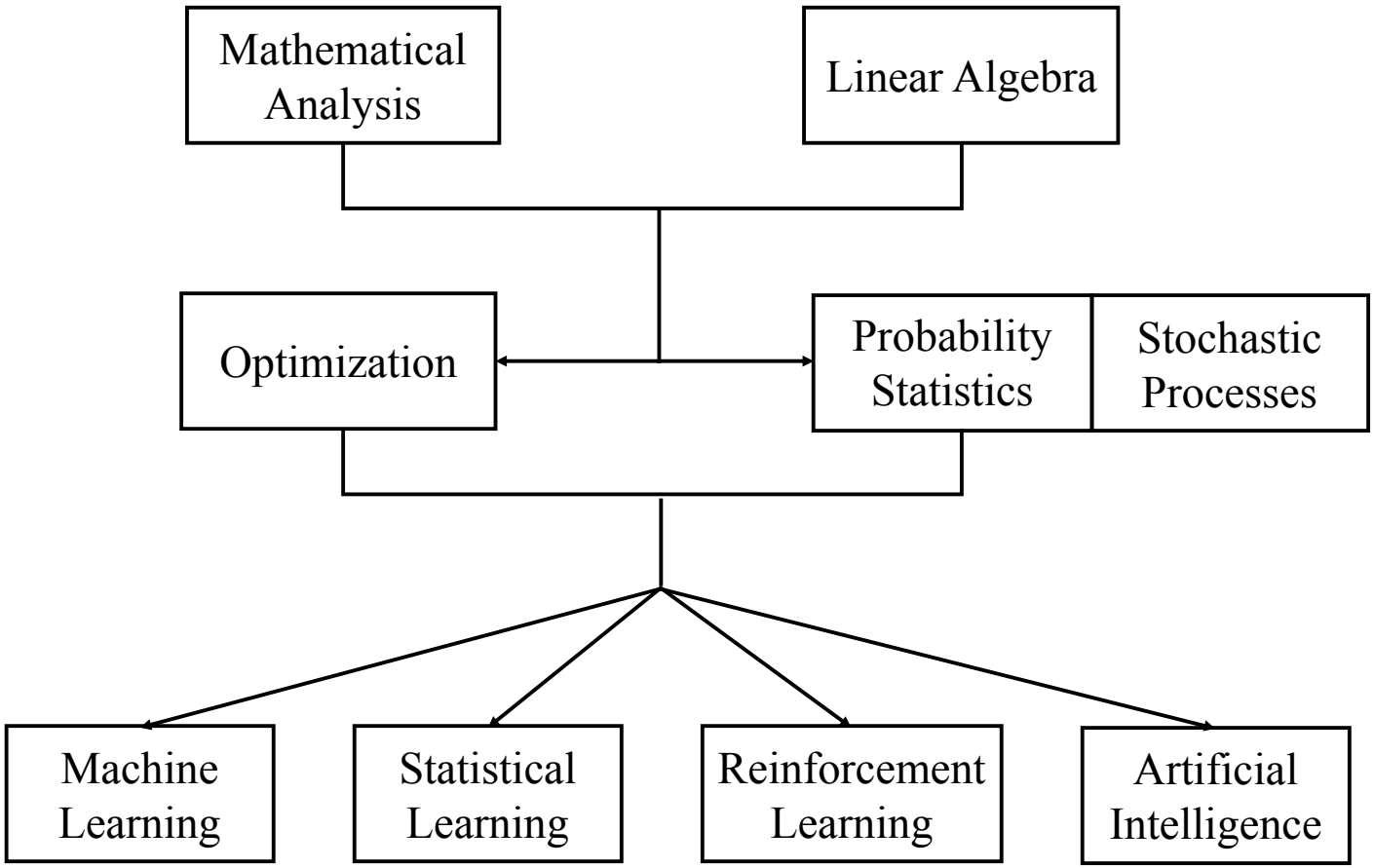
Course Reward

Final Exam

In My Beginning is My End



The Role of This Course



Teaching Philosophy of This Course

Probability
Math

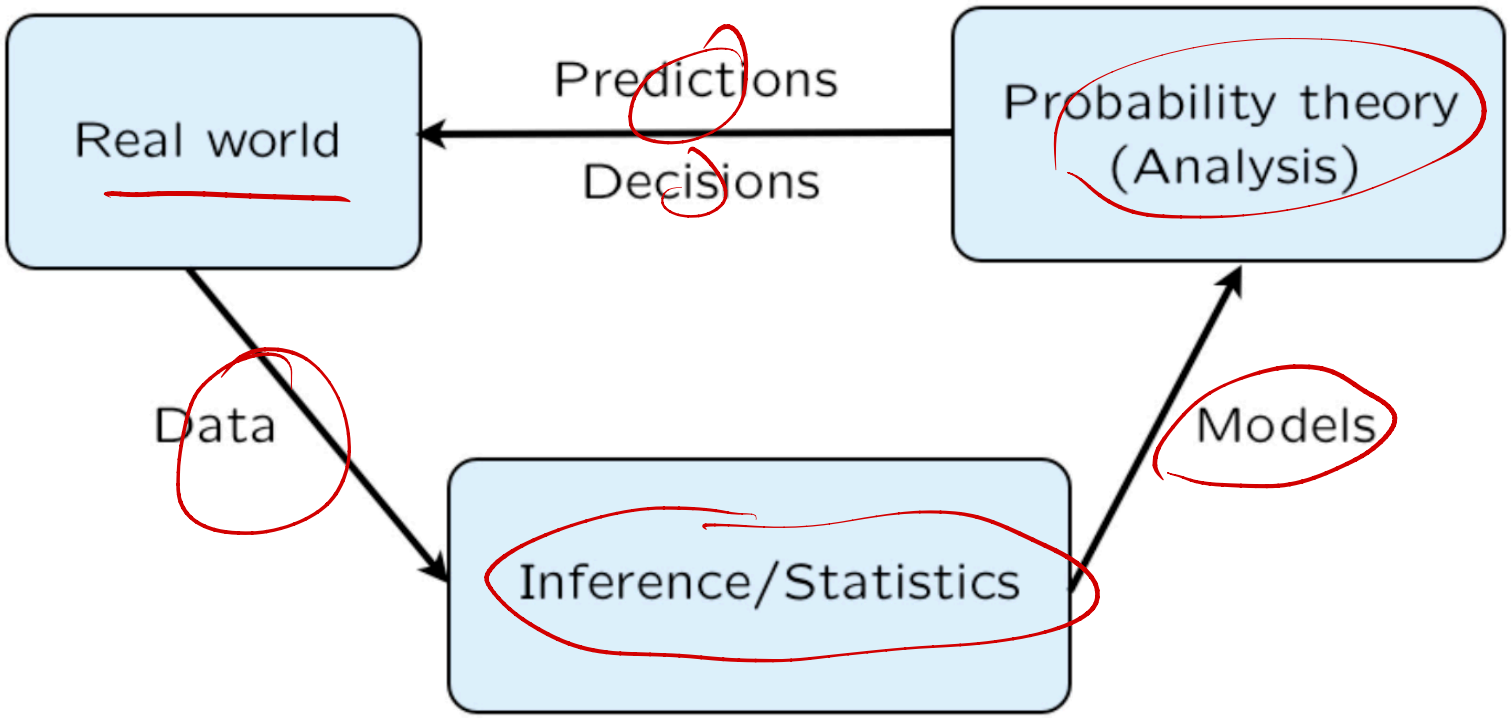
Statistics
Science

Monte Carlo
Computing

The Role of Probability & Statistics

A framework for analyzing phenomena with uncertain outcomes:

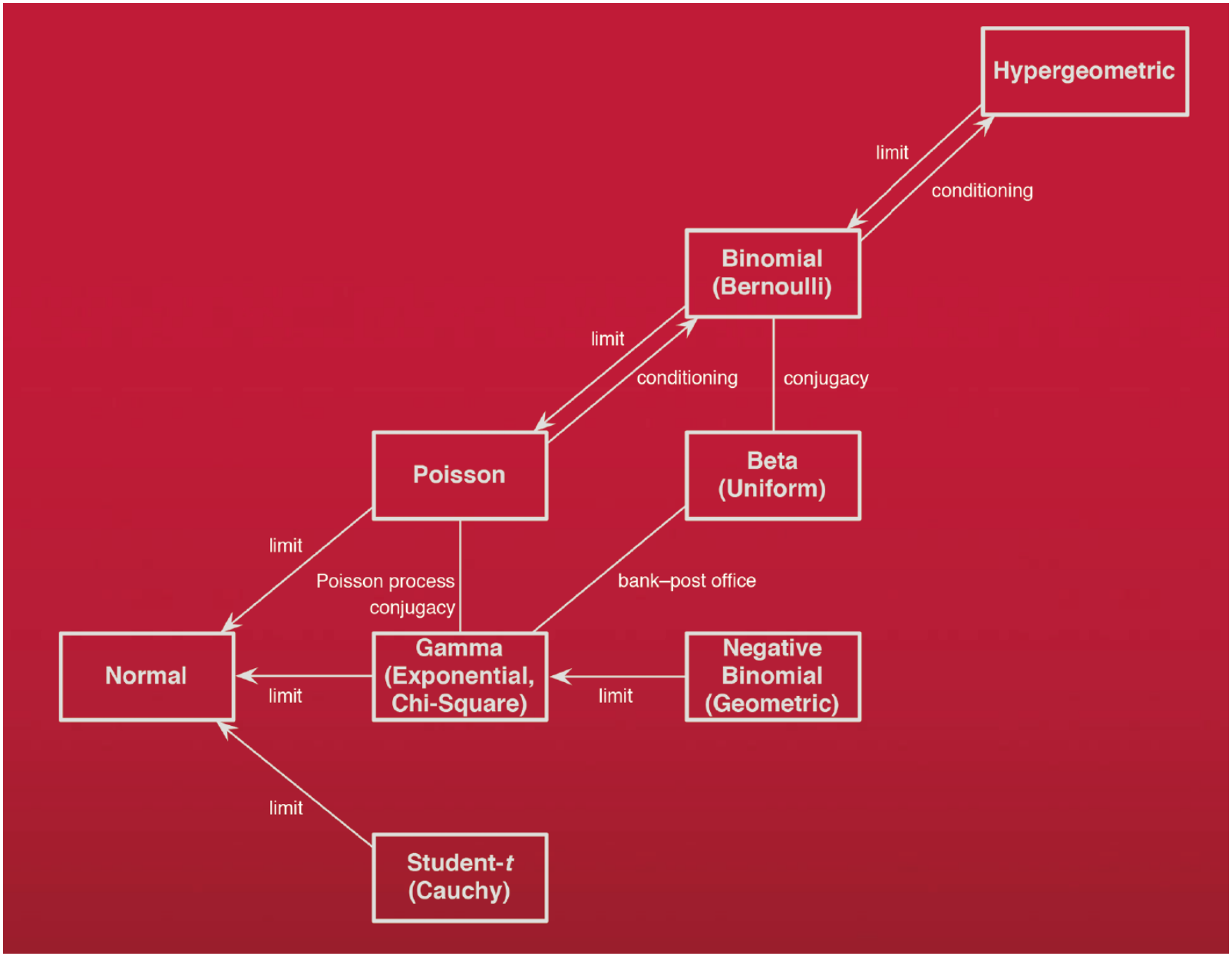
- Rules for consistent reasoning
- Used for predictions and decisions



Typical Distributions

Name	Param.	PMF or PDF	Mean	Variance
Bernoulli	p	$P(X=1)=p, P(X=0)=q$	p	pq
Binomial	n,p	$\binom{n}{k}p^k q^{n-k}$, for $k \in \{0,1,\dots,n\}$	np	npq
FS	p	pq^{k-1}, for $k \in \{1,2,\dots\}$	$1/p$	q/p^2
Geom	p	pq^k, for $k \in \{0,1,2,\dots\}$	q/p	q/p^2
NBinom	r,p	$\binom{r+n-1}{r-1}p^r q^n, n \in \{0,1,2,\dots\}$	rq/p	rq/p^2
HGeom	w,b,n	$\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$, for $k \in \{0,1,\dots,n\}$	$\mu = \frac{nw}{w+b}$	$(\frac{w+b-n}{w+b-1})n\frac{\mu}{n}(1-\frac{\mu}{n})$
Poisson	λ	$\frac{e^{-\lambda}\lambda^k}{k!}$, for $k \in \{0,1,2,\dots\}$	λ	λ
Uniform	$a<b$	$\frac{1}{b-a}$, for $x \in (a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal	μ,σ^2	$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}$	μ	σ^2
Log-Normal	μ,σ^2	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-(\log x-\mu)^2/(2\sigma^2)}, x>0$	$\theta = e^{\mu+\sigma^2/2}$	$\theta^2(e^{\sigma^2}-1)$
Expo	λ	$\lambda e^{-\lambda x}$, for $x>0$	$1/\lambda$	$1/\lambda^2$
Gamma	a,λ	$\Gamma(a)^{-1}(\lambda x)^a e^{-\lambda x}x^{-1}$, for $x>0$	a/λ	a/λ^2
Beta	a,b	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$, for $0<x<1$	$\mu = \frac{a}{a+b}$	$\frac{\mu(1-\mu)}{a+b+1}$
Chi-Square	n	$\frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2}$, for $x>0$	n	$2n$
Student-t	n	$\frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)}(1+x^2/n)^{-(n+1)/2}$	0 if $n>1$	$\frac{n}{n-2}$ if $n>2$

Relationship Among Distributions



Part I: Univariate Distribution

- Probability and Counting: Definition of Probability, Counting, Bose-Einstein Model, Birthday Match Problem, Hash Table.
- Conditional Probability: Bayes'rule, LOTP, Conditioning, Gambler's ruin, Simpson's Paradox, Monty Hall.
- Random Variables and Distributions: Bernoulli, Binomial, Story for distributions, entropy.
- Expectation: Indicator r.v., Geometric, Coupon Collector, Poisson, Probability Generating Function.
- Continuous Random Variables: Universality of the Uniform, Normal, Exponential, Memoryless, Moment Generating Function. Central Limit Theorem.

Part II: Multivariate Distribution

- Joint Distributions: Joint/Marginal distribution, chicken-egg model, Poisson, meaning of conditioning on zero-probability event, Four Forms of Bayes' Rule, Four Forms of LOTP, Covariance, Correlation, Multivariate Normal Distribution.
- Transformations: Change of Variables, Jacobian Matrix, Convolution, ~~Order Statistics~~

Part III: Monte Carlo Statistical Methods

- Random Variable Generation: Inverse Transform Method, Acceptance-Rejection Method
- Monte Carlo Integration: Sample Average, Importance Sampling
- Asymptotic Analysis: Law of Large Numbers, night model of Smartphone
- Inequality:Cauchy-Schwarz Inequality, Jensen's Inequality, Kullback-Leibler Divergence (Entropy), Markov's Inequality, Chebyshev's Inequality, Chernoff's Inequality (related to MGF), Chernoff's Technique, Hoeffding Bound, Parameter Estimation (confidence interval), Monte Carlo Method for Estimation π.

Part IV: Statistical Inference

- Frequency Perspective: Maximum Likelihood Estimation
- Conditional Expectation: Conditional Expectation Given An Event , Conditional Expectation Given An R.V., LOTE, Adam's Law, Eve's law, Projection Interpretation, Minimum Mean Square Error Estimator (MMSE), Prediction Perspective (MMSE).
- Bayesian Perspective: Bayesian Inference and the Posterior Distribution, The Maximum A Posteriori Probability (MAP) Rule, Conjugate-Prior, Beta-Binomial Conjugacy, Dirichlet-Multinomial Conjugacy, Bayesian Ranking.

Part V: Markov Chain

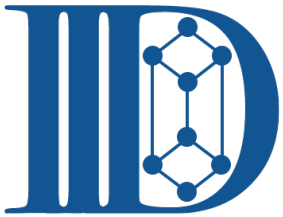
- Basic Conception: Stochastic Processes, Markov Model, Markov Chain (Discrete-Time & Discrete State Space)
- Markov Chain: Markov property, Time-Homogeneous Markov Chains, Graphical Representation of Markov Chain (State-Transition Diagram), Matrix Representation of Markov Chain (Transition Matrix), n-step Transition Probability, Chapman-Kolmogorov Relationship
- Classification: Recurrent and Transient States, Irreducible and Reducible Chain, Period, Periodic & Aperiodic Markov Chain
- Long-Term Property: Stationary Distribution, Reversibility, Detailed Balance Equation, Random Walk on Undirected Graph, Google PageRank

In My End is My Beginning

- Now this is not the end. It is not even the beginning of the end.
- But it is, perhaps, the end of the beginning.

The End of Adventure in Probability & Statistics

A New Adventure: Welcome to Join IID Lab!



Laboratory for Intelligence Information and Decision

智 能 信 息 与 决 策 实 验 室

In My End is My Beginning