姓名：

学院：

学号：

上海科技大学

2024-2025 学年第 1 学期本科生期末考试卷

开课单位：信息学院

授课教师：邵子瑜、文鼎柱

考试科目：《面向信息科学的概率论与数理统计》

课程代码：SI 140A

考试时间：2025 年 1 月 10 日 14 点 00 分 –17 点 00 分。

考试成绩录入表：

题目	1	2	3	4	5	6	7	8	9	总分
计分										
复核										

评卷人签名：　　　　复核人签名：

日期：　　　　日期：

编写说明：

1. 要求评卷人和复核人不能是同一人。
2. 试卷内页和答题纸编排格式由各学院和出题教师根据实际需要自定，每页须按顺序标注页码（除封面外），要求排版清晰、美观，便于在页面左侧装订。为方便印刷归档，建议使用 A4 双面印刷（学校有印刷一体机提供）。
3. 主考教师编写试卷时尽可能保证试题科学、准确、合理，如考试过程中发现试题有误，主考教师需负责现场解释，此类情况学校将作为教学评估记录的一部分。

Probability & Statistics for EECS
Fall 2024
Final
2025/01/10
Time Limit: 180 Minutes

Name (Print): ____________________

Advisor Name ____________________

This exam contains 11 pages (including this cover page) and 9 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

Try to answer as many problems as you can. The following rules apply:

- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	10	
4	15	
5	5	
6	5	
7	5	
8	10	
9	10	
Total:	100	

1. (20 points) Given two i.i.d. random variables X and Y satisfying distribution Expo(1).
   (a) (10 points) Find $P(\max(X, Y) \geq 1)$.
   (b) (10 points) Find $P(\min(X, Y) \leq 1)$.

2. (20 points) Let X and Y be two continuous random variables with joint PDF

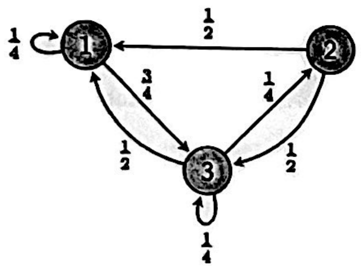
$$f_{X,Y}(x,y) = \begin{cases} c & \text{if } 0 < x < 2,\ 0 < y < 2x, \\ 0 & \text{otherwise.} \end{cases}$$

(a) (10 points) Find the value of constant c.

(b) (5 points) Find the marginal distribution of X.

(c) (5 points) Find the marginal distribution of Y.

3. (10 points) Given a time-homogeneous Markov chain with state-transition diagram shown as follows:



(a) (2 points) Find $P(X_5 = 3|X_4 = 2)$ and $P(X_4 = 1|X_3 = 2)$.

(b) (2 points) Find the stationary distribution of the chain.

(c) (2 points) Find the period of state 2 and the period of the chain.

(d) (2 points) If $P(X_0 = 3) = \frac{1}{5}$, find $P(X_0 = 3, X_1 = 2, X_2 = 1)$.

(e) (2 points) Find $E(X_5|X_3 = 1)$ and $\mathrm{Var}(X_5|X_3 = 1)$

4. (15 points) Given a coin with the probability p of landing heads. p is unknown and we need to estimate its value through data. In our data collection model, we have n independent tosses, result of each toss is either Head or Tail. Let X denote the number of heads in the total n tosses. Now we conduct experiments to collect data and find $X = k$. Then we need to find $\hat{p}$, the estimation of p.

   (a) (5 points) Assume p is an unknown constant. Find $\hat{p}$ through the MLE (Maximum Likelihood Estimation) rule.

   (b) (5 points) Assume p is a random variable with a prior distribution $p \sim \text{Beta}(1,1)$. Find $\hat{p}$ through the MAP (Maximum a Posterior Probability) rule.

   (c) (5 points) Assume p is a random variable with a prior distribution $p \sim \text{Unif}(0,1)$. Find $\hat{p}$ through the MMSE (Minimal Mean Squared Error) rule.

5. (5 points) Let U_1, U_2 be i.i.d random variables and $U_1 \sim \text{Unif}(0,1)$. Let $V = \min(U_1, U_2)$, $Z = \max(U_1, U_2)$. Find the joint CDF of V and Z.

6. (5 points) Given n i.i.d. random variables $X_1, \dots, X_n$ satisfying distribution Unif(0, 1). Let $X_{(1)} < X_{(2)} < \dots < X_{(n)}$ be the ordering of $X_1, \dots, X_n$. Find the PDF of $X_{(n)} - X_{(1)}$.

7. (5 points) Find the expectation of the returned value of the following program code:

```
float sum=0.0;
while(sum<=1.0) {
    sum+= Unif(0,1);
    }
return sum;
```

8. (10 points) Suppose a vase contains balls numbered $1, 2, \ldots, N$. We draw n balls without replacement from the vase. Each ball is selected with equal probability, i.e., in the first draw each ball has probability $\frac{1}{N}$, in the second draw each ball has probability $\frac{1}{N-1}$, and so on. For $i = 1, 2, \ldots, n$, let X_i denote the number on the ball in the ith draw.

   (a) (5 points) Find $\mathrm{Var}(X_i)$

   (b) (5 points) Find $\mathrm{Cov}(X_i, X_j)$, $i \neq j$, $i, j \in \{1, 2, \ldots, n\}$.

9. (10 points) Let (X, Y) be bivariate normal random variable where $X \sim \mathcal{N}(0,1)$, $Y \sim \mathcal{N}(0,1)$, $\mathrm{Corr}(X,Y) = \rho$. Compute the joint probability $P(X > 0, Y > 0)$.

	Y discrete	Y continuous
X discrete	$P(Y=y\|X=x)=\frac{P(X=x\|Y=y)P(Y=y)}{P(X=x)}$	$f_Y(y\|X=x)=\frac{P(X=x\|Y=y)f_Y(y)}{P(X=x)}$
X continuous	$P(Y=y\|X=x)=\frac{f_X(x\|Y=y)P(Y=y)}{f_X(x)}$	$f_{Y\|X}(y\|x)=\frac{f_{X\|Y}(x\|y)f_Y(y)}{f_X(x)}$

	Y discrete	Y continuous
X discrete	$P(X=x)=\sum_y P(X=x\|Y=y)P(Y=y)$	$P(X=x)=\int_{-\infty}^{\infty} P(X=x\|Y=y)f_Y(y)dy$
X continuous	$f_X(x)=\sum_y f_X(x\|Y=y)P(Y=y)$	$f_X(x)=\int_{-\infty}^{\infty} f_{X\|Y}(x\|y)f_Y(y)dy$

Figure 1: Bayes' Rule & LOTP.

Table of distributions

Name	Param.	PMF or PDF	Mean	Variance
Bernoulli	p	$P(X=1)=p, P(X=0)=q$	p	pq
Binomial	n,p	$\binom{n}{k}p^k q^{n-k}$, for $k \in \{0,1,\dots,n\}$	np	npq
FS	p	pq^{k-1}, for $k \in \{1,2,\dots\}$	$1/p$	q/p^2
Geom	p	pq^k, for $k \in \{0,1,2,\dots\}$	q/p	q/p^2
NBinom	r,p	$\binom{r+n-1}{r-1}p^r q^n, n \in \{0,1,2,\dots\}$	rq/p	rq/p^2
HGeom	w,b,n	$\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$, for $k \in \{0,1,\dots,n\}$	$\mu=\frac{nw}{w+b}$	$(\frac{w+b-n}{w+b-1})n\frac{\mu}{n}(1-\frac{\mu}{n})$
Poisson	λ	$\frac{e^{-\lambda}\lambda^k}{k!}$, for $k \in \{0,1,2,\dots\}$	λ	λ
Uniform	$a<b$	$\frac{1}{b-a}$, for $x \in (a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal	μ,σ^2	$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}$	μ	σ^2
Log-Normal	μ,σ^2	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-(\log x-\mu)^2/(2\sigma^2)}, x>0$	$\theta=e^{\mu+\sigma^2/2}$	$\theta^2(e^{\sigma^2}-1)$
Expo	λ	$\lambda e^{-\lambda x}$, for $x>0$	$1/\lambda$	$1/\lambda^2$
Gamma	a,λ	$\Gamma(a)^{-1}(\lambda x)^a e^{-\lambda x}x^{-1}$, for $x>0$	a/λ	a/λ^2
Beta	a,b	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$, for $0<x<1$	$\mu=\frac{a}{a+b}$	$\frac{\mu(1-\mu)}{a+b+1}$
Chi-Square	n	$\frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2}$, for $x>0$	n	$2n$
Student-t	n	$\frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)}(1+x^2/n)^{-(n+1)/2}$	0 if $n>1$	$\frac{n}{n-2}$ if $n>2$