Problem 1

BH CH2 #38(b)

Consider the following n-door version of the Monty Hall problem. There are n doors, behind one of which there is a car (which you want), and behind the rest of which there are goats (which you don't want). Initially, all possibilities are equally likely for where the car is. You choose a door. Monty Hall then opens m goat doors, and offers you the option of switching to any of the remaining $n - m - 1$ doors.
Assume that Monty Hall knows which door has the car, will always open m goat doors and offer the option of switching, and that Monty chooses with equal probabilities from all his choices of which goat doors to open. Should you switch? What is your probability of success if you switch?

Solution

Without loss of generality, we can assume the contestant picked door 1 (if she didnt pick door 1, we could simply relabel the doors, or rewrite this solution with the door numbers permuted). Let S be the event of success in getting the car, and let C_j be the event that the car is behind door j. Conditioning on which door has the car, by LOTP we have

$$P(S) = P(S|C_1) \cdot \frac{1}{n} + \cdots + P(S|C_n) \cdot \frac{1}{n}.$$

Suppose you employs the non-switching strategy. The only possibility of success is that the car is indeed behind door 1, which implies

$$P_{\text{non-switching}}(S) = 1 \cdot \frac{1}{n} + 0 \cdot \frac{1}{n} + \cdots + 0 \cdot \frac{1}{n} = \frac{1}{n}.$$

Suppose you employs the switching strategy. If the car is behind door 1, then switching will fail, so $P(\text{get car}|C_1) = 0$. Otherwise, since Monty always reveals m goat, the probability of getting a car by switching to a remaining unopened door is $\frac{1}{n-m-1}$. Thus,

$$P_{\text{switching}}(S) = 0 \cdot \frac{1}{n} + \frac{1}{n-m-1} \cdot \frac{1}{n} + \cdots + \frac{1}{n-m-1} \cdot \frac{1}{n} = \frac{n-1}{(n-m-1)n}.$$

This value is greater than $\frac{1}{n}$, so you should switch.