TA Lecture 06 - Midterm Answers

April 17-18

School of Information Science and Technology,
ShanghaiTech University

上海科技大学
ShanghaiTech University

Outline

Midterm Answers

Problem 1

(10 points) How many number of distinct positive integer-valued vectors $(x_1, x_2, \ldots, x_5)$ satisfying the equation and inequalities

$$x_1 + x_2 + \cdots + x_5 = 48$$
$$x_1 > 3, x_2 \geq 7, x_3 > 4, x_4 \geq 6, x_5 > 4.$$

Problem 1 Solution

Problem 2

(10 points) A system composed of 5 homogeneous devices is shown in the following figure. It is said to be functional when there exists at least one end-to-end path that devices on such path are all functional. For such a system, if each device, which is independent of all other devices, functions with probability $p = 1/2$, then what is the probability that the system functions? Such a probability is also called the system reliability.

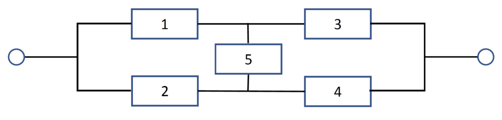

Figure 1: An illustration of the system composed of 5 homogeneous devices.

Problem 2 Solution

Problem 3

(10 points) A frog starts at 0 on the number line and makes a sequence of jumps to the right. In any one jump, independent of previous jumps, this frog leaps a positive integer distance m with probability $\frac{1}{2^m}$. Find the probability that this frog will eventually land at 1997 on the number line.

Problem 3 Solution

Problem 4

(10 points) A six-sided fair dice is rolled four times independently. What is more likely: a sum of 15 or a sum of 16? (You need to compute the corresponding probability for each case).

Problem 4 Solution

Problem 5

(10 points) Consider a coin that comes up heads with probability p and tails with probability $1 - p$. Let q_n be the probability that after n independent tosses, there have been an odd number of heads. Find q_n.

Problem 5 Solution

Problem 6

(10 points) An urn contains w white balls and b black balls, which are randomly drawn one by one without replacement. Let X denote the number of black balls drawn before drawing r $(1 \leq r \leq w)$ white balls.

(a) (5 points) Find the PMF of X.

(b) (5 points) Find $E(X)$.

Problem 6 Solution

Problem 7

(20 points) Bob's database of friends contains n entries, but due to a software bug, the addresses correspond to the names in a totally random fashion. Bob writes a holiday card to each of his friends and sends it to the (software-corrupted) address. Let X denote the number of friends of him who will get the correct card.

1. (5 points) Find $E(X)$.
2. (5 points) Find $\text{Var}(X)$.
3. (5 points) Find the PMF of X.
4. (5 points) When $n \to \infty$, show that the distribution of X converges to a Poisson distribution.

Problem 7 Solution

Problem 8

(20 points) Consider the following 7-door version of the Monty Hall problem. There are 7 doors, behind one of which there is a car (which you want), and behind the rest of which there are goats (which you don't want). Initially, all possibilities are equally likely for where the car is. You choose a door. Monty Hall then opens 3 goat doors, and offers you the option of switching to any of the remaining 3 doors.

Problem 8 Continued

(a) (10 points) Assume that Monty Hall knows which door has the car, will always open 3 goat doors and offer the option of switching, and that Monty chooses with equal probabilities from all his choices of which goat doors to open. Should you switch? What is your probability of success if you switch to one of the remaining 3 doors?

(b) Generalize the above to a Monty Hall problem where there are $n \geq 3$ doors, of which Monty opens m goat doors, with $1 \leq m \leq n-2$.

Problem 8 Solution