姓名：

学院：

学号：

上海科技大学

2024-2025 学年第 1 学期本科生期中考试卷

开课单位：信息学院

授课教师：邵子瑜 & 文鼎柱

考试科目：《面向信息科学的概率论与数理统计》

课程代码：SI 140A

考试时间：2024 年 11 月 12 日 10 点 15 分 – 11 点 55 分。

考试成绩录入表：

题目	1	2	3	4	5	6	7	8	总分
计分									
复核									

评卷人签名：　　　　　　复核人签名：

日期：　　　　　　　　　日期：

编写说明：

1. 要求评卷人和复核人不能是同一人。
2. 试卷内页和答题纸编排格式由各学院和出题教师根据实际需要自定，每页须按顺序标注页码（除封面外），要求排版清晰、美观，便于在页面左侧装订。为方便印刷归档，建议使用 A4 双面印刷（学校有印刷一体机提供）。
3. 主考教师编写试卷时尽可能保证试题科学、准确、合理，如考试过程中发现试题有误，主考教师需负责现场解释，此类情况学校将作为教学评估记录的一部分。

Probability & Statistics for EECS
Fall 2024
Midterm
2024/11/12
Time Limit: 100 Minutes

Name (Print): ____________________

Advisor Name ____________________

This exam contains 10 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

Try to answer as many problems as you can. The following rules apply:

- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total:	100	

1. (20 points) How many number of distinct **positive** integer-valued vectors $(x_1, x_2, \dots, x_5)$ satisfying the equation and inequalities

$$x_1 + x_2 + \cdots + x_5 = 2024$$
$$x_1 > 1, x_2 \geq 9, x_3 > 9, x_4 \geq 7, x_5 > 8.$$

2. (20 points) You toss a fair coin three times:
   (a) (10 points) What is the probability that you observe exactly one head?
   (b) (10 points) Given that you have observed at least one heads, what is the probability that you observe at least two heads?

3. (10 points) A frog starts at 0 on the number line and makes a sequence of jumps to the right. In any one jump, independent of previous jumps, this frog leaps a positive integer distance m with probability $\frac{1}{2^m}$. Find the probability that this frog will eventually land at 20241112 on the number line.

4. (10 points) Bob's database of friends contains n entries, but due to a software bug, the addresses correspond to the names in a totally random fashion. Bob writes a holiday card to each of his friends and sends it to the (software-corrupted) address. Let X denote the number of friends of him who will get the correct card. Given $0 \leq k \leq n$, Find $E\left[\binom{X}{k}\right]$.

5. (10 points) An urn contains w white balls and b black balls, which are randomly drawn one by one without replacement. Let X denote the number of black balls drawn before drawing r $(1 \leq r \leq w)$ white balls.

   (a) (5 points) Find the PMF of X.

   (b) (5 points) Find $E(X)$.

6. (10 points) Suppose there are n types of toys, which you are collecting one by one, with the goal of getting a complete set. When collecting toys, the toy types are random (as is sometimes the case, for example, with toys included in cereal boxes or included with kids' meals from a fast food restaurant). Assume that each time you collect a toy, it is equally likely to be any of the n types. Let N denote the number of toys needed until you have a complete set. Show that for any $c > 0$,

$$P(N > \lceil n \log n + c \cdot n \rceil) \leq e^{-c}.$$

where $\lceil x \rceil$ denotes the minimal integer that is not less than x.

7. (10 points) Given three i.i.d random variables X, Y, Z, satisfying distribution FS(p). Find $P(X < Y < Z)$.

8. (10 points) Suppose a fair coin with probability $1/2$ for heads is tossed repeatedly, and we obtain a sequence of H and T (H denotes Head and T denotes Tail). Let N denote the number of toss to observe the first occurrence of the pattern "HHH".

   (a) (5 points) Find $E(N)$.

   (b) (5 points) Find $\mathrm{Var}(N)$.

	Y discrete	Y continuous
X discrete	$P(Y=y\|X=x)=\frac{P(X=x\|Y=y)P(Y=y)}{P(X=x)}$	$f_Y(y\|X=x)=\frac{P(X=x\|Y=y)f_Y(y)}{P(X=x)}$
X continuous	$P(Y=y\|X=x)=\frac{f_X(x\|Y=y)P(Y=y)}{f_X(x)}$	$f_{Y\|X}(y\|x)=\frac{f_{X\|Y}(x\|y)f_Y(y)}{f_X(x)}$

	Y discrete	Y continuous
X discrete	$P(X=x)=\sum_y P(X=x\|Y=y)P(Y=y)$	$P(X=x)=\int_{-\infty}^{\infty} P(X=x\|Y=y)f_Y(y)dy$
X continuous	$f_X(x)=\sum_y f_X(x\|Y=y)P(Y=y)$	$f_X(x)=\int_{-\infty}^{\infty} f_{X\|Y}(x\|y)f_Y(y)dy$

Figure 1: Bayes' Rule & LOTP.

Table of distributions

Name	Param.	PMF or PDF	Mean	Variance
Bernoulli	p	$P(X=1)=p, P(X=0)=q$	p	pq
Binomial	n,p	$\binom{n}{k}p^k q^{n-k}$, for $k \in \{0,1,\dots,n\}$	np	npq
FS	p	pq^{k-1}, for $k \in \{1,2,\dots\}$	$1/p$	q/p^2
Geom	p	pq^k, for $k \in \{0,1,2,\dots\}$	q/p	q/p^2
NBinom	r,p	$\binom{r+n-1}{r-1}p^r q^n, n \in \{0,1,2,\dots\}$	rq/p	rq/p^2
HGeom	w,b,n	$\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$, for $k \in \{0,1,\dots,n\}$	$\mu=\frac{nw}{w+b}$	$\left(\frac{w+b-n}{w+b-1}\right)n\frac{\mu}{n}\left(1-\frac{\mu}{n}\right)$
Poisson	λ	$\frac{e^{-\lambda}\lambda^k}{k!}$, for $k \in \{0,1,2,\dots\}$	λ	λ
Uniform	$a<b$	$\frac{1}{b-a}$, for $x \in (a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal	μ,σ^2	$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}$	μ	σ^2
Log-Normal	μ,σ^2	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-(\log x-\mu)^2/(2\sigma^2)}, x>0$	$\theta=e^{\mu+\sigma^2/2}$	$\theta^2(e^{\sigma^2}-1)$
Expo	λ	$\lambda e^{-\lambda x}$, for $x>0$	$1/\lambda$	$1/\lambda^2$
Gamma	a,λ	$\Gamma(a)^{-1}(\lambda x)^a e^{-\lambda x}x^{-1}$, for $x>0$	a/λ	a/λ^2
Beta	a,b	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$, for $0<x<1$	$\mu=\frac{a}{a+b}$	$\frac{\mu(1-\mu)}{a+b+1}$
Chi-Square	n	$\frac{1}{2^{n/2}\Gamma(n/2)}x^{n/2-1}e^{-x/2}$, for $x>0$	n	$2n$
Student-t	n	$\frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)}(1+x^2/n)^{-(n+1)/2}$	0 if $n>1$	$\frac{n}{n-2}$ if $n>2$