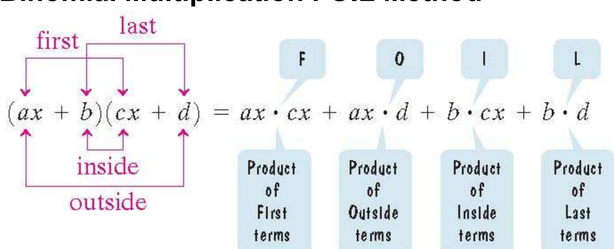
4. Quadratic Equations

1. Multiply binomials using the FOIL method.
2. Factor trinomials.
3. Solve quadratic equations by factoring.
4. Solve quadratic equations using the quadratic formula.
5. Solve problems modeled by quadratic equations.

4.1: Multiplying Two Binomials

- **Binomial:** An algebraic expression containing two terms in which each exponent that appears on the variable is a whole number.
  - **Examples:** x + 3, x + 4, 3x + 4, 5x – 3
- **Binomial Multiplication FOIL Method**



Example: FOIL Multiplication Method

Multiply: $(x + 3)(x + 4)$

Solution:

F: First terms $= x \cdot x = x^2$

O: Outside terms $= x \cdot 4 = 4x$

I: Inside terms $= 3 \cdot x = 3x$

L: Last terms $= 3 \cdot 4 = 12$

$(x + 3)(x + 4) = x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$ combine like terms

Standard Trinomial Form: ax^2+bx+c

Multiply Algebraic Expressions

$-4x^2(2x^2-5x-6)$ **Prob 4.1.7**

$(7x-3)(6x-5)$ **Prob 4.1.23**

$(3x^2-9)(2x^2+4)$ **Prob 4.1.31**

4.2: Factoring Trinomial where a =1

- *Trinomial:* A simplified algebraic expression that contains three terms in which all variables have whole number exponents.

Standard Trinomial Form: ax^2+bx+c

- We can use the FOIL method to multiply two binomials to obtain a trinomial:

Factored Form F O I L Trinomial Form

$(x-3)(x+7) = x^2 + 7x - 3x - 21 = x^2 + 4x - 21$

- *Factoring* an algebraic expression containing the sum or difference of terms means finding an equivalent expression that is a product.

Factor the Trinomial Expressions

x^2+9x+8 Prob 4.2.13

$x^2-7x+12$ Prob 4.2.19

x^2+x-30 Prob 4.2.31

x^2-x-9 Prob 4.2.33

$3x^4-9x^3-30x^2$ Prob 4.2.55

Strategy for Factoring $x^2 + bx + c$

- **Factor:** $x^2 + 6x + 8$

Step 1: Factor out anything common to all factors (nothing).

Step 2: Enter x as the first term of each factor.

$x^2 + 6x + 8 = (x \quad)(x \quad)$

Step 3: List all pairs of factors of constant c.
If c is positive then factors have same signs.
If c is negative then factors have opposite signs.

Factors of 8 are: 1, 8 –1, –8 2, 4 –2, –4

Step 4: Try various combinations of these c factors. The correct factorization is the one where the sum of factors is equal to b term.

$(x+2)(x+4)$ 2 + 4 = 6

Step 5: Check your factoring by multiplying the binomials to determine if they factor correctly to be the original trinomial.

$(x+2)(x+4) = x^2 + 4x + 2x + 8 = x^2 + 6x + 8$

Factoring $ax^2 + bx + c$ using AC Method

Factor: $3x^2 - 20x + 28$ ax^2+bx+c

Step 1: Factor out anything common to all terms (nothing).

Step 2: Determine product of *ac* terms which will be factored in step 3.

$3x^2 - 20x + 28$ $a \cdot c = 3 \cdot 28 = 84$

Step 3: List all pairs of factors of product *ac*.
If *ac* is positive then factors have same signs. *b* determines sign
If *ac* is negative then factors have opposite signs.

Factors of 84 are: –1, –84 –4, –21 –6, –14 –7, –12

Step 4: Try various combinations of these *ac* factors. The correct factorization is the one where the sum of factors is equal to *b* term.

Step 5: Split the middle term into two terms. –6 – 14 = –20

$3x^2 - 20x + 28 = 3x^2 - 6x - 14x + 28$

Step 6: Determine if first terms have something in common and factor out

$3x(x-2) - 14x + 28 = 3x(x-2) - 14(x-2) = (3x-14)(x-2)$

Step 7: Try factoring out same binomial from last two terms

Step 8: Apply Distributive law to create product of two binomials

Factor the Trinomial Expressions

$3x^2 - 16x + 5$ Prob 4.2.39

$6x^2 + x - 1$ Prob 4.2.47

$4x^2 - 4x - 15$ Prob 4.2.51

$12x^3 + 6x^2 - 18x$ Prob 4.2.1

 9

Solving Quadratic Equations by Factoring

- A *Quadratic Equation* in x is an equation that can be written in the form:

  $$ax^2 + bx + c = 0$$

  where a,b, and c are real numbers, with $a \neq 0$.

  Standard Quadratic Equation Form: $ax^2+bx+c=0$
- *Zero-Product Principle*

  If the product of two factors is zero, then one (or both) of the factors must have a value of zero.

  If $A \cdot B = 0$, then $A = 0$ or $B = 0$

  Step 1: Rewrite Equation in Standard Quadratic Form

  Step 2: Factor into product of two binomials = 0

  Step 3: Determine x values that will make binomials = 0

  $$x^2 - 7x + 12 = (x-3)(x-4) = 0$$

  $$x - 3 = 0 \mid x - 4 = 0 \rightarrow x = \{3, 4\}$$

  Step 4: Check Solutions in original equation

 10

Example: Solving Quadratic Equation by Factoring

Solve: $x^2 - 2x = 35$

Step 1: Move all the terms to one side by subtracting 35 from both sides:

$x^2 - 2x - 35 = 0$

Step 2: Factor.

$(x - 7)(x + 5) = 0$

Steps 3: Set each factor equal to zero and solve each resulting equation:

$x - 7 = 0$ or $x + 5 = 0$

$x = 7$ $\quad x = -5$

Step 4: Check the solutions in the original equation:

$x^2 - 2x = 35$

Check 7	Check –5
$7^2 - 2 \cdot 7 = 35$	$(-5)^2 - 2(-5) = 35$
$49 - 14 = 35$	$25 + 10 = 35$
$35 = 35$	$35 = 35$

Both solutions are true.

If a solution is false in original equation, it is an *extraneous solution.*

 11

Factor the Trinomial Expressions

$(2x+3)(3x-1)=0$ Prob 4.2.63

$x^3 - 3x^2 + 2x = 0$ Prob 4.2.75

$3x^2 - 5x - 2 = 0$ Prob 4.2.79

 12

The Quadratic Formula

The solutions of the quadratic equation in the form $ax^2 + bx + c = 0$, with $a \neq 0$, can be determine using the *quadratic formula:*

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve: $2x^2 + 9x - 5 = 0$

Solution: $a = 2$, $b = 9$, $c = -5$

$$x_1, x_2 = \frac{-9 \pm \sqrt{9^2 - 4 \cdot 2 \cdot (-5)}}{2 \cdot 2} = \frac{-9 \pm \sqrt{81 - (-40)}}{4} = \frac{-9 \pm \sqrt{121}}{4} = \frac{-9 \pm 11}{4}$$

$$x_1, x_2 = \frac{-9 \pm 11}{4}$$

$$x_1 = \frac{-9 + 11}{4} = \frac{1}{2}$$

$$x_2 = \frac{-9 - 11}{4} = \frac{-9 - 11}{4} = \frac{-20}{4} = -5$$

 13

Example: Solving a Quadratic Equation Using the Quadratic Formula

Solve: $2x^2 = 4x + 1$

Solution: **Move all the terms to the right:**

$2x^2 - 4x - 1 = 0$ $\quad a = 2, b = -4$ **and** $c = -1$

Use the quadratic formula:

$$x_1, x_2 = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-1)}}{2(2)} = \frac{4 \pm \sqrt{16 - (-8)}}{4} = \frac{4 \pm \sqrt{24}}{4}$$

$$x_1 = \frac{4 + 2\sqrt{6}}{4} = \frac{2 + \sqrt{6}}{2}$$

$$x_2 = \frac{4 - 2\sqrt{6}}{4} = \frac{2 - \sqrt{6}}{2}$$

Note: If the expression under the square root simplifies to a negative number, then the quadratic equation has no real solutions.

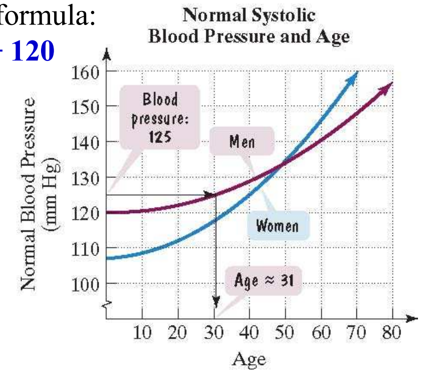
 14

Example: Blood Pressure and Age

A person's normal systolic blood pressure measured in millimeters of mercury depends on his or her age – for men, according to the formula:

$P = 0.006A^2 - 0.02A + 120$

Find the age, to the nearest year, of a man whose systolic blood pressure is 125 mm Hg.



 15

Solution: Blood Pressure and Age

From the formula, $125 = 0.006A^2 - 0.02A + 120$

Rewriting the equation: $0.006A^2 - 0.02A - 5 = 0$

we get the values $a = 0.006$, $b = -0.02$ and $c = -5$

$$A = \frac{-(-0.02) \pm \sqrt{(-0.02)^2 - 4(0.006)(-5)}}{2(0.006)}$$

$$= \frac{0.02 \pm \sqrt{0.1204}}{0.012}$$

$$A \approx \frac{0.02 - 0.347}{0.012} \approx -27 \text{(reject)}$$

$$A \approx \frac{0.02 + 0.347}{0.012} \approx 31$$

Therefore, 31 is the approximate age for a man with a normal systolic blood pressure of 125 mm Hg.

 16