7.3-4: Probability Theory

1. Find the probability that an event will not occur
2. Find the probability of one event or a second event occurring
3. Find the probability of one event and a second event occurring.
4. Compute conditional probabilities.

7.3: Probability of an Event Not Occurring

- *Complement* of *E*: If we know $P(E)$, the probability of an event *E*, we can determine the probability that the event will not occur, denoted by $P(\text{not } E)$
- The probability that an event *E* will not occur is equal to 1 minus the probability that it will occur.

$$P(\text{not } E) = 1 - P(E)$$

- The probability that an event *E* will occur is equal to 1 minus the probability that it will not occur

$$P(E) = 1 - P(\text{not } E)$$

- Using over-line notation, if $\overline{E}$ is the complement of *E*, then

$$P(\overline{E}) = 1 - P(E) \text{ and } P(E) = 1 - P(\overline{E})$$

Example: Probability of an Event Not Occurring

If you are dealt one card from a standard 52-card deck, find the probability that you are not dealt a queen.

Solution:

Because $P(\text{not } E) = 1 - P(E)$ then

$$P(\text{not a queen}) = 1 - P(\text{queen})$$
$$= 1 - \frac{1}{13}$$
$$= \frac{13}{13} - \frac{1}{13}$$
$$= \frac{12}{13}$$

OR Probabilities with Mutually Exclusive Events

- Mutually Exclusive Events: Events A and B are mutually exclusive if it is impossible for them to occur simultaneously.
- OR Probabilities with Mutually Exclusive Events:
  - If A and B are mutually exclusive events, then

    P(A or B) = P(A) + P(B)

A B

- Example: If one card is randomly selected from a deck of cards, what is the probability of selecting a king or a queen

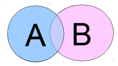
K Q

Solution:

$$P(\text{king or queen}) = P(\text{king}) + P(\text{queen}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

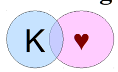
OR Probabilities that are NOT Mutually Exclusive

- If A and B are not mutually exclusive events then the probability that A or B will occur is determined by adding their individual probabilities and then subtracting the probability that A and B occur simultaneously.



$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- Example: If one card is randomly selected from a deck of 52 cards, what is the probability of selecting a King or a Heart card.



Solution:

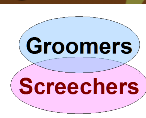
P (king or heart) = P(k) + P(♥) – P(k and ♥)

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

OR Probability are Events Mutually Exclusive?

In a group of 25 baboons, 18 enjoy grooming their neighbors, 16 enjoy screeching wildly, while 10 enjoy doing both. If one baboon is selected at random, find the probability that it enjoys grooming its neighbors or screeching wildly.

Solution:



Since 10 of the baboons enjoy both grooming their neighbors and screeching wildly, these events are not mutually exclusive.

P (Grm or Scr) = P(Grm) + P(Scr) – P(Grm and Scr)

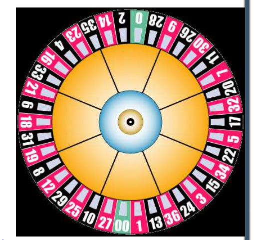
$$= \frac{18}{25} + \frac{16}{25} - \frac{10}{25} = \frac{18+16-10}{25} = \frac{24}{25}$$

AND Probabilities with Independent Events

- *Independent Events:* Two events are independent events if the occurrence of either of them has no effect on the probability of the other.
- AND Probabilities with Independent Events

  If A and B are independent events, then

  $P(A \text{ and } B) = P(A) \cdot P(B)$



Example: A U.S. roulette wheel has 38 numbered slots (1 through 36, 0, and 00). 18 are black, 18 are red, and 2 are green. The ball can land on any slot with equal probability. What is the probability of red occurring on 2 consecutive plays?

$$P(\text{Red and Red}) = P(\text{Red}) \cdot P(\text{Red}) = \frac{18}{38} \cdot \frac{18}{38} = \frac{81}{361} = 0.224 = 22.4\,\%$$

Children are Independent Events

- If two or more events are independent, we can find the probability of them all occurring by multiplying their probabilities.
- What is the probability of having six girls in a row?

The probability of a baby girl is ½, so the probability of six girls in a row is ½ used as a factor six times.

Solution:

$$P(\text{six girls}) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^6 = \frac{1}{64} = 0.015625 = 1.5625\,\%$$

Hurricanes and Probabilities

- If the probability that South Florida will be hit by a hurricane in any single year is 5/19:
  1) What is the probability that South Florida will be hit by a hurricane in three consecutive years?
  2) What is the probability that South Florida will not be hit by a hurricane in the next three years?
  3) What is the probability that South Florida will be hit by a hurricane at least once in next three years?

$$P(\text{Hurricane for 3 consecutive years}) = \frac{5}{19}\cdot\frac{5}{19}\cdot\frac{5}{19} = \left(\frac{5}{19}\right)^3 = \frac{125}{6859} = 0.0182 = 1.82\,\%$$

$$P(\text{No Hurricane in Next 3 years}) = \left(1-\frac{5}{19}\right)^3 = \left(\frac{14}{19}\right)^3 = 0.400 = 40.0\,\%$$

$$P(\text{At least one Hurricane in Next 3 years}) = (1-0.400) = 0.600 = 60.0\,\%$$

AND Probabilities with Dependent Events

- Dependent Events: Two events are dependent events if the occurrence of one of them has an effect on the probability of the other.
- If A and B are dependent events, then

  P(A and B) = P(A) · P(B given that A has occurred)

There are 20 chocolates in a box and five of chocolates contain cherries. What is the probability of selecting three Chocolate covered cherries in a row?

$$P(\text{3 Cherries}) = \frac{5}{20}\cdot\frac{4}{19}\cdot\frac{3}{18} = \frac{1}{114} = 0.00877 = 0.877\,\%$$

Travel Partner Decisions

- Good News: You have won a free trip to Madrid and can take two people with you, all expenses paid. Bad news: Ten of your cousins have appeared out of nowhere and are begging you to take them. You write each cousin's name on a card, place the cards in a hat, and select one name. Then you select a second name without replacing the first card. If three of your ten cousins speak Spanish, what is the probability of selecting two Spanish-speaking cousins?

$$P(\text{2 Spanish Speakers}) = \frac{3}{10}\cdot\frac{2}{9} = \frac{1}{15} = 0.0667 = 6.67\,\%$$

Getting a Flush in Poker

- You are dealt a 5 card poker hand. What is the probability of being dealt all hearts?

- What is the probability of being dealt all cards of same suit (flush)?

$$P(\text{5 Hearts}) = \frac{13}{52}\cdot\frac{12}{51}\cdot\frac{11}{50}\cdot\frac{10}{49}\cdot\frac{9}{48} = \frac{33}{66640} = 0.000495 = 0.0495\,\%$$

$$P(\text{Flush}) = \frac{33}{66640} + \frac{33}{66640} + \frac{33}{66640} + \frac{33}{66640} = \frac{132}{66640} = 0.00198 = 0.198\,\%$$

Conditional Probability

- Conditional Probability is the probability of event *B* occurring, assuming that event *A* has already occurred. Conditional probability is denoted by: *P*(*B*|*A*)
- A letter is randomly selected from the letters of the English alphabet. Find the probability of selecting a vowel, given that the outcome is a letter that precedes h.
  - Solution: We are looking for P(vowel | letter precedes h).

    This is the probability of a vowel if the sample space is restricted to the set of letters that precede h.

    S = {a, b, c, d, e, f, g}

    There are 7 possible outcomes in the sample space. We can select a vowel from this set in one of two ways:

    a or e

    P(vowel | letter precedes h) = 2/7

Conditional Probability

- A letter is randomly selected from the letters of the English alphabet. Find the probability of selecting a letter that precedes h, given that the outcome is a vowel.
  - Solution: We are looking for P(letter precedes h | vowel).

    This is the probability of a letter that precedes h if the sample space is restricted to the set of vowels.

    S = {a, e, i, o, u}

    There are 5 possible outcomes in the sample space. We can select a vowel from this set in one of two ways:

    a or e

    P(letter precedes h | vowel) = 2/5

Medical Testing Data

Mammography Data for U.S. women age 40 to 50

Mammography Screening on 100,000 U.S. Women, Ages 40 to 50			
	Breast Cancer	No Breast Cancer	Total
Positive Mammogram	720	6,944	7,664
Negative Mammogram	80	92,256	92,336
Total	800	99,200	100,000

Find the probability that a woman in this age range has a positive mammogram, given that she does not have breast cancer.

Solution:
Empirical Probability problem.
There are 6944 + 92,256 or 99,200 women without breast cancer.

$$P(\text{+mamogram} \mid \text{no breast cancer}) = \frac{6944}{99{,}200} = 0.07 = 7\%$$

Mammography Screening on 100,000 U.S. Women, Ages 40 to 50			
	Breast Cancer	No Breast Cancer	Total
Positive Mammogram	720	6,944	7,664
Negative Mammogram	80	92,256	92,336
Total	800	99,200	100,000

Find the probability that a woman in this age range:
1) Does not have breast cancer, given that she has a +mammogram.
2) Has a +mammogram, given that she has breast cancer.
3) Has breast cancer, given that she has +mammogram.
4) Has breast cancer, given that she has (–)mammogram.

Solutions: Empirical Probability problem.

$$1)\ P(\text{no breast cancer} \mid \text{+mamogram}) = \frac{6944}{7{,}664} = 0.906 = 90.6\%$$

$$2)\ P(\text{+mamogram} \mid \text{breast cancer}) = \frac{720}{800} = 0.90 = 90\%$$

$$3)\ P(\text{breast cancer} \mid \text{+mamogram}) = \frac{720}{7664} = 0.094 = 9.4\%$$

$$4)\ P(\text{breast cancer} \mid \text{(-)mamogram}) = \frac{80}{92{,}336} = 0.0008664 = 0.08664\%$$

