3.1: Algebraic Translations of English Phrases

Addition	Subtraction	Multiplication	Division
sum more than increased by plus added to Total of	minus decreased by subtracted from difference between less than fewer than	times product of percent of a number multiplied by twice fraction of	divided by quotient reciprocal
sum of 6 and 7 3 more then 8 9 increased by total 11 and 15 2 plus 4 7 is added to 12	11 minus 7 9 less 5 3 less than 6 difference 9 and 2 subtract 2 from 30	12 times 7 product 11 and 6 10 multiplied by 3 twice 5 20% of 30	Quotient 9 and 2 12 divided by 3

3.2: Strategy for Solving Word Problems

Step 1 Read the problem carefully several times until you can state in your own words what is given and what the problem is looking for. Let x (or any variable) represent one of the unknown quantities in the problem.

Step 2 If necessary, write expressions for any other unknown quantities in the problem in terms of x.

Step 3 Write an equation in x that models the verbal conditions of the problem.

Step 4 Solve the equation and answer the problem's question.

Step 5 Check the solution *in the original wording* of the problem, not in the equation obtained from the words.

Example 1: Cost of Operation

❖A Printer that costs \$60 requires replacement cartridges that sell for \$23. Over the life of the printer the cost of the printer and replacement cartridges is \$865. How many cartridges were used over the life of the printer?

Step 1: Let x represent one of the unknown quantities.
$x =$ quantity of cartridges used

Step 2: Represent the other unknown quantities in terms of x.
There are no other unknown quantities, so we can skip this step.

Step 3: Write equation in x that models the conditions.
Cost of Printer Operation: $60+23x=865$

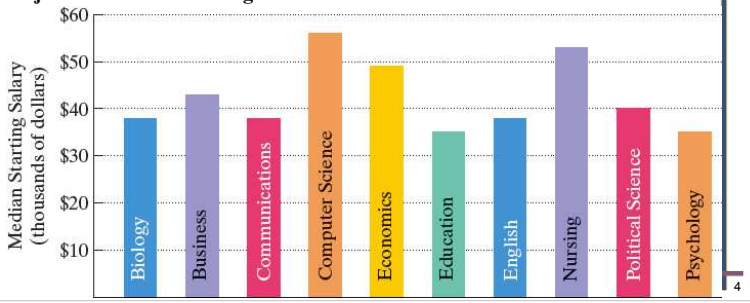
Step 4: Solve the equation and answer question.
$60+23x=865 \rightarrow \quad 60-60+23x=865-60 \rightarrow \quad 23x=805 \rightarrow \quad x=35$

Step 5: Check the proposed solution in the wording of the problem.
$\$60+\$23\cdot 35=\$865$

So the quantity of cartridges used over the life of the printer is 35.

Example 2: Education Pays Off

Ten most popular college majors with median starting salaries for recent college graduates. The median starting salary of a business major exceeds that of a psychology major by \$8 thousand. The median starting salary of an English major exceeds that of a psychology major by \$3 thousand. Combined, their median starting salaries are \$116 thousand. Determine the median starting salaries of psychology majors, business majors, and English majors with bachelor's degrees.



Example 2: Solution

Step 1: Let x represent one of the unknown quantities.

Let x = median starting salary, in thousands $ for psychology majors

Step 2: Represent the other unknown quantities in terms of x.

$x + 8$ = median starting salary, in thousands $ for business majors.

$x + 3$ = median starting salary, in thousands $ for English majors.

Step 3: Write equation in x that models the conditions.

$x + (x + 8) + (x + 3) = 116$

Step 4: Solve the equation and answer question. X = 35

- starting salary of psychology majors: x = $35K
- starting salary of business majors: x + 8 = 35 + 8 = $43K
- starting salary of English majors: x + 3 = 35 + 3 = $38K.

Step 5: Check the proposed solution in the wording of the problem. The solution checks.

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Example 3: Chose Telephone Plans

❖ You are choosing between two telephone plans. Plan A has a monthly fee of $20.00 with a charge of $0.05 per call. Plan B has a monthly fee of $5.00 with a charge of $0.10 per call. After how many calls will costs of the two plans be the same?

Step 1: Let x represent one of the unknown quantities.

x = quantity of calls

Step 2: Represent the other unknown quantities in terms of x.

There are no other unknown quantities, so we can skip this step.

Step 3: Write equation in x that models the conditions.

$\text{Plan A Cost} = 20 + 0.05x \qquad \text{Plan B Cost} = 5 + 0.10x$

Step 4: Solve the equation and answer question.

$20 - 5 = 0.10x - 0.05x \rightarrow 15 = 0.05x \rightarrow x = 300$

Step 5: Check the proposed solution in the wording of the problem.

$\text{Plan A} = 20 + 0.05(300) = \$35 \qquad \text{Plan B} = 5 + 0.10(300) = \$35$

So the two plans cost the same after 300 calls per month

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3.3: Ratios and Solving Proportions

❖ ***Ratios*** compares quantities by Division

- 2 cups of milk per serving
- 0.5 cups of sugar per 3 cups of flour

❖ ***Proportions*** are an equation defined by two ratios $\dfrac{3 \text{ eggs}}{10 \text{ pancakes}} = \dfrac{x}{6 \text{ pancakes}}$

❖ Cross Multiplying used for proportions

if $\dfrac{a}{b} = \dfrac{c}{d}$, then $ad = bc$ where $b \neq 0$ and $d \neq 0$

$3 \text{ eggs} \cdot 6 \text{ pancakes} = x \cdot 10 \text{ pancakes}$

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Proportion Exercise Solutions

if $\dfrac{a}{b} = \dfrac{c}{d}$, then $ad = bc$ where $b \neq 0$ and $d \neq 0$

$\dfrac{63}{x} = \dfrac{7}{5} \rightarrow 63 \cdot 5 = 7x \rightarrow x = \dfrac{63 \cdot 5}{7} = 45$

$\dfrac{22}{60 - x} = \dfrac{2}{x} \rightarrow 22x = 2(60 - x)$

$22x = 120 - 2x \rightarrow x = \dfrac{120}{24} = 5$

$\dfrac{3 \text{ eggs}}{10 \text{ pancakes}} = \dfrac{x}{6 \text{ pancakes}} \qquad x = \dfrac{3 \text{ eggs} \cdot 6 \text{ pancakes}}{10 \text{ pancakes}}$

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Estimating Wildlife Population

- **Wildlife biologists catch, tag, and the release 135 buffalo back into a wildlife refuge. One month later they observe a sample of 140 buffalo, 30 of which are tagged. Assuming the ratio of tagged buffalo in the sample remains the same. How many buffalo in refuge?**

$$\frac{135 \text{ tagged}}{x} = \frac{30 \text{ tagged}}{140 \text{ population}}$$

$$135 \text{ tagged} \cdot 140 \text{ population} = x \cdot 30 \text{ tagged}$$

$$x = \frac{135 \text{ tagged} \cdot 140 \text{ population}}{30 \text{ tagged}} = 630 \text{ population buffalo}$$

3.4: Percents

- ***Percents*** **are the result of expressing numbers as a part of 100.**
- **The word *percent* means per hundred % .**

Example: Express $\frac{5}{8}$ as a percent.

Solution:

Step 1. Divide the numerator by the denominator.

$5 \div 8 = 0.625$

Step 2. Multiply the quotient by 100.

$0.625 \times 100 = 62.5$

Step 3. Add a percent sign.

62.5%

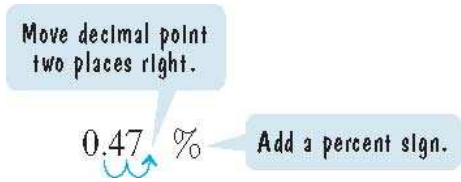
Expressing a Decimal as a Percent

To express a decimal as a percent:

1. **Move the decimal point two places to the right.**
2. **Attach a percent sign.**

Express: **Express 0.47 as a percent.**

Solution:



Thus, 0.47 = 47%.

Expressing a Percent as a Decimal

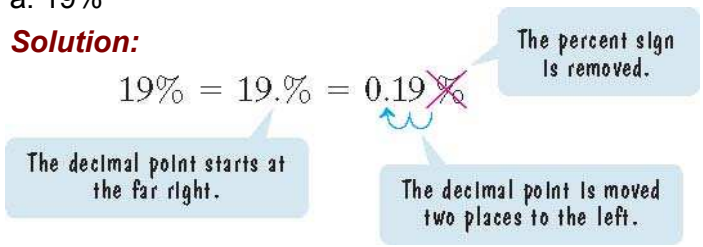
To express a percent as a decimal number:

1. **Move the decimal point two places to the left.**
2. **Remove the percent sign.**

Example: Express each percent as a decimal:

a. 19%

Solution:



Percent, Sales Tax, & Discounts

Many applications involving percent are based on the following formula:

$$A = P \cdot B$$

A is P Percent of B

Note: "is" implies equality and "of" implies multiplication.

We use this formula to determine *sales tax* collected by states, counties, cities on sales items to customers.

Sales tax amount = tax rate × item's cost

Example: Percent and Sales Tax

Suppose that the local sales tax rate is 7.5% and you purchase a bicycle for $894.

a. How much tax is paid?

b. What is the bicycle's total cost?

Solution:

a. Sales tax amount = tax rate × item's cost

7.5% × $894 = 0.075 × $894 = $67.05

The tax paid is $67.05.

b. Total Cost = $894.00 + $67.05 = $961.05

The bicycle's total cost is $961.05.

Percent and Discount Sales Price

- **Businesses reduce prices, or *discount*, to attract customers and to reduce inventory.**
- **The *discount rate* is a percent of original price.**

Discount amount = discount rate × original price

Example:

A computer with an original price of $1460 is on sale at 15% off.

a. What is the discount amount?

b. What is the computer's sale price?

Solution:

a. Discount amount = discount rate × original price = 0.15 × 1460 = $219

b. A computer's sale price is the original price, $1460, minus the discount amount, $219.

Sale price = $1460 – $219 = $1241

The computer's discount amount is $219 and sale price is $1241.

Percent and Price Markup

- **Businesses *mark-up* the cost of items so they can make a profit and pay for their operating expenses.**
- **The *mark-up rate* is a percent of the cost of item.**

Price of item = mark-up rate × cost of item

Example:

A mark-up rate of 50% is applied to a shirt that costs the store $40. What is the selling price of the shirt after markup?

Solution:

Selling Price = $40 + 50% × $40 = $40 + $20 = $60

Percent Increase and Decrease

If a quantity changes, its *percent increase* or its *percent decrease* can be found as follows:

1. Find the fraction for the percent increase or decrease:

$$\%\ \text{Increase} = \frac{\text{New Amount} - \text{Original Amount}}{\text{Original Amount}}$$

$$\%\ \text{Decrease} = \frac{\text{Original Amount} - \text{New Amount}}{\text{Original Amount}}$$

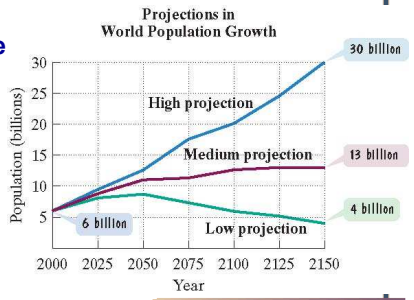
2. Find the percent increase or decrease by expressing the fraction in step 1 as a percent.

Example: Percent Increase and Decrease

In 2000, world population was approximately 6 billion. The data is from United Nations Family Planning Program and are based on optimistic or pessimistic expectations for successful control of human population growth.

a. Find the percent increase in world population from 2000 to 2150 using the high projection data.

b. Find the percent decrease in world population from 2000 to 2150 using the low projection data.



Solution: Percent Increase and Decrease

a. Use the data shown on the blue, high-projection, graph.

$$\text{Percent increase} = \frac{\text{amount of increase}}{\text{original amount}}$$

$$= \frac{30-6}{6} = \frac{24}{6} = 4 = 400\%$$

Projected percent increase in world population is 400%

b. Use the data shown on the green, low-projection, graph.

$$\text{Percent decrease} = \frac{\text{amount of decrease}}{\text{original amount}}$$

$$= \frac{6-4}{6} = \frac{2}{6} = \frac{1}{3} = 33\frac{1}{3}$$

Projected percent decrease in world population is 33⅓%

3.5: Linear Inequalities

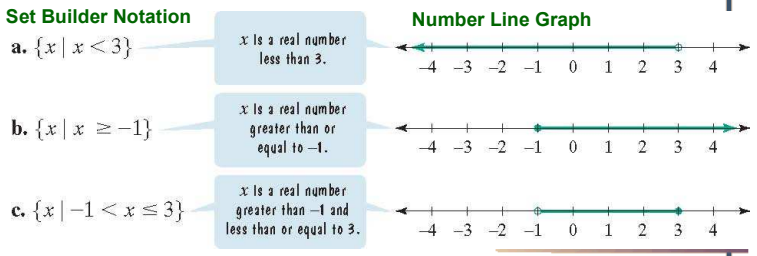
- A *linear inequality* :

  $ax + b \le c$

  where inequality symbols can be <, >, ≤, or ≥.
- *Solving an inequality* is the process of finding the set of numbers that make an inequality a true statement.
  - *A solution set* is the set of all numbers that satisfy the inequality.
- The procedure for solving linear inequalities is the same as the procedure for solving linear equations, with one important exception:
  - When multiplying or dividing both sides of the inequality by a negative number, reverse the direction of the inequality symbol, changing the sense of the inequality.

Inequality: Set Builder and Graph

- The solution set for an inequality
  - Set Builder Notation
  - Number Line Graph
    - Solid circle represents ≥ or ≤
    - Empty circle represents > or <



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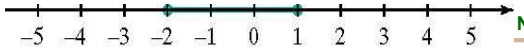
Example: Solving a Linear Inequality

- Solve and graph the solution set: $-3 < 2x + 1 \le 3$
- Solution: We will get x by itself on the left side.

$$-3 < 2x + 1 \le 3$$
$$-3 - 1 < 2x + 1 - 1 \le 3 - 1$$
$$-4 < 2x \le 2$$
$$\frac{-4}{2} < \frac{2x}{2} \le \frac{2}{2}$$
$$-2 < x \le 1$$

Set Builder Notation

- The solution set is $\{x \mid -2 < x \le 1\}$. The graph is:.



Number Line Graph

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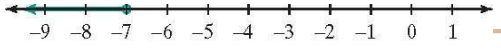
Example: Solving a Linear Inequality

- Solve and graph the solution set: $6x - 12 > 8x + 2$
- Solution: We will get x by itself on the left side.

$$6x - 12 > 8x + 2$$
$$6x - 8x - 12 > 8x - 8x + 2$$
$$-2x - 12 > 2$$
$$-2x - 12 + 12 > 2 + 12$$
$$-2x > 14$$
$$\frac{-2x}{-2} < \frac{14}{-2}$$

Set Builder Notation

- The solution set is $\{x \mid x < -7\}$. The graph is: $x < -7$



Number Line Graph

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