1.3 Rational Numbers

- 1. Define the rational numbers
- 2. Reduce rational numbers
- 3. Convert between mixed numbers and improper fractions
- 4. Express rational numbers as decimals.
- 5. Express decimals in the form a/b
- 6. Multiply and divide rational numbers
- 7. Add and subtract rational numbers
- 8. Use the order of operations agreement with rational numbers
- 9. Solve problems involving rational numbers

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Defining the Rational Numbers

- The set of rational numbers is the set of all numbers which can be expressed in the form where a and b are integers and b is not equal to 0.
- ❖ The integer a is called the numerator
- **The integer** b is called the *denominator* b
- **❖** Examples of rational numbers: ½, -½, ⅓, 5, 0
- Equivalent Rational Numbers
 - ◆ To reduce a rational number to its lowest terms
 - divide numerator and denominator by their greatest common divisor $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}$

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Reducing a Rational Number

Reduce $\frac{130}{455}$ to lowest terms.



- Solution: Begin by finding the 5 13 Greatest Common Factor of 130 and 455.
- **❖** Thus, $130 = 2 \cdot 5 \cdot 13$, and $455 = 5 \cdot 7 \cdot 13$
- ❖ Divide the numerator and the denominator of the given rational number by 5 ⋅ 13 or 65

$$\frac{130}{455} = \frac{2 \cdot 5 \cdot 13}{5 \cdot 7 \cdot 13} = \frac{2}{7}$$

or

$$\frac{130}{455} = \frac{130 \div 65}{455 \div 65} = \frac{2}{3}$$

- * There are no common divisors of 2 and 7 other than 1.
- * Thus, the rational number 2/7 is in its lowest terms.

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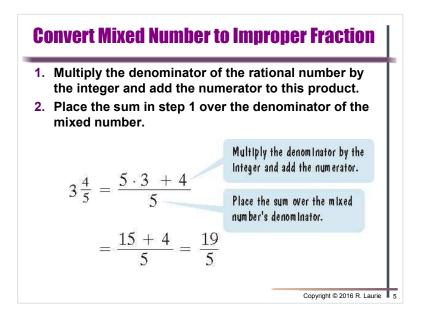
Mixed Numbers, Improper Fractions, and Decimal

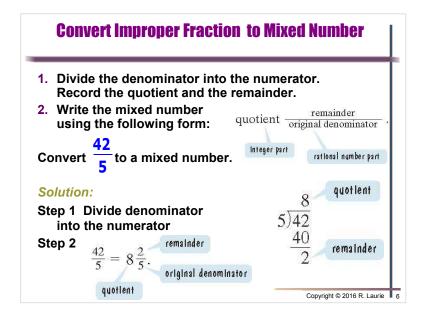
A mixed number consists of the sum of an integer and a rational number, expressed without the use of an addition sign.

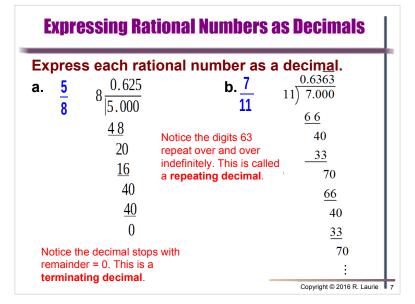
Example:

$$3\,\frac{4}{5}\,.\qquad \begin{array}{l} \text{The Integer Is 3 and the rational} \\ \text{number Is}\,\frac{4}{5},\,3\frac{4}{5}\,\,\text{means}\,3+\frac{4}{5}\,. \end{array}$$

- *An *improper fraction* is a rational number whose numerator is greater than denominator
- Any rational number can be expressed as a decimal number by dividing the denominator into the numerator 3.8







Expressing Decimals as a Fraction

Express terminating decimal as a quotient of integers:

- a. 0.7
- b. 0.49
- c. 0.048

Solution:

- a. $0.7 = \frac{7}{10}$ because the 7 is in the tenths position.
- b. $0.49 = \frac{100}{100}$ because the digit on the right, 9, is in the hundredths position.

c.
$$0.048 = \frac{48}{1000} = \frac{48 \div 8}{1000 \div 8} = \frac{6}{125}$$

because the digit on the right, 8, is thousandths position and can be reduced to lowest terms

Multiplying Rational Numbers

- The product of two rational numbers is the product of their numerators divided by the
- product of their denominators. $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$ $\left(-\frac{2}{3}\right)\left(-\frac{9}{4}\right) = \frac{(-2)(-9)}{3\cdot 4} = \frac{18}{12} = \frac{3\cdot 6}{2\cdot 6} = \frac{3}{2}$ or $1\frac{1}{2}$

Multiply across. Simplify to lowest terms.

❖ Pre-simplify Example

$$\left(-\frac{2}{3}\right)\left(-\frac{9}{4}\right) = \frac{3}{2} \quad \text{or} \quad 1\frac{1}{2}$$

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Dividing Rational Numbers

- *The quotient of two rational numbers is a product of the first number and the reciprocal of the second number
- Flip last number and multiply by first number
- **If** $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$

$$-\frac{3}{5} \div \frac{7}{11} = -\frac{3}{5} \cdot \frac{11}{7} = -\frac{3 \cdot 11}{5 \cdot 7} = -\frac{33}{35}$$

by using the reciprocal.

across.

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Add and Subtract Rational Numbers

The sum or difference of two rational numbers with identical denominators is the sum or difference of numerators over common denominator.

If $\frac{a}{b}$ and $\frac{c}{b}$ are rational numbers, then

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

Examples:

$$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$$

$$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$$
 $\frac{11}{12} - \frac{5}{12} = \frac{11-5}{12} = \frac{6}{12} = \frac{1 \cdot 6}{2 \cdot 6} = \frac{1}{2}$

$$-5\frac{1}{4} - \left(-2\frac{3}{4}\right) = -\frac{21}{4} - \left(-\frac{11}{4}\right) = -\frac{21}{4} + \frac{11}{4} = \frac{-21 + 11}{4} = \frac{-10}{4} = -\frac{5}{2} \quad \text{or} \quad -2\frac{1}{2}$$

Add and Subtract Rational Numbers

- The sum or difference of two rational numbers with different denominators, we use the Least Common Multiple of their denominators to rewrite the rational numbers.
- * The Least Common Multiple of their denominators is called the Least Common Denominator or LCD.

$$\frac{3}{4} + \frac{1}{6} = \frac{3}{4} \frac{3}{3} + \frac{1}{6} \frac{2}{2}$$

$$= \frac{9}{12} + \frac{2}{12} \text{ Not}$$

We multiply the first rational number by 3/3 and the second one by 2/2 to

Add numerators and put this sum over the

Exercise: Simplify using PEMDAS

$$\left| \frac{-\frac{1}{2}}{-\frac{1}{2}} \right|^2 - \left| \frac{7}{10} - \frac{8}{15} \right|^2 (-18)$$

$$\frac{\frac{19}{5}}{3 + \frac{1}{6}} - \frac{7}{8} \div \frac{3}{2}$$
 Prob 1.3.99

1.4 The Irrational Numbers

- Define the irrational numbers.
- Simplify square roots.
- Perform operations with square roots.
- Rationalize the denominator.

The set of *irrational numbers* is the set of numbers whose decimal representations are neither terminating nor repeating.

$$\pi \approx 3.1415926535897932384626433832795...$$
 $\sqrt{2} \approx 1.414213562373095...$
 $\sqrt{27} \approx 5.196152422706632...$

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Square Roots

- *The principal square root of a nonnegative number n, written \sqrt{n} , is the positive number that when multiplied by itself gives n.
- **❖** For example, $\sqrt{36}$ =6 because 6 ⋅ 6 = 36.
- Notice that $\sqrt{36}$ is a rational number because 6 is a terminating decimal.
- *Not all square roots are irrational.
- *For example, here are a few perfect squares:

$ \bullet 0 = 0^2 $	$\sqrt{0}=0$ The square root of a	
◆ 1 = 1 ²	$\sqrt{1}$ =1 perfect square is a	
♦ 4 = 2 ²	$\sqrt{4}$ =2 rational number	
◆ 9 = 3 ²	$\sqrt{9}=3$	D 1 -

The Product Rule For Square Roots

❖ If a and b represent non-negative numbers

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$
 and $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

- * The square root of a product is the product of the square roots.
- Simplify, if possible:

$$\sqrt{75} = \sqrt{25} \cdot 3$$

$$= \sqrt{25} \cdot \sqrt{3}$$

$$= 5\sqrt{3}$$

$$\sqrt{500} = \sqrt{100} \cdot 5$$

$$= \sqrt{100} \cdot \sqrt{5}$$

$$= 10\sqrt{5}$$
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Adding and Subtracting Square Roots

- The number that multiplies a square root is called the square root's coefficient.
- ❖ Square roots with the same radicand can be added or subtracted by adding or subtracting their coefficients:

$$a\sqrt{c} + b\sqrt{c} = (a+b)\sqrt{c}$$
 $a\sqrt{c} - b\sqrt{c} = (a-b)\sqrt{c}$

$$a\sqrt{c} - b\sqrt{c} = (a - b)\sqrt{c}$$

Sum of coefficients times

Difference of coefficients times the common square root

$$7\sqrt{2}+5\sqrt{2}=(7+5)\sqrt{2}=12\sqrt{2}$$

$$2\sqrt{5}-6\sqrt{5}=(2-6)\sqrt{5}=-4\sqrt{5}$$

Product Rule Exercises

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$
 and $\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$

$$\sqrt{2}\cdot\sqrt{5} = \sqrt{2\cdot5} = \sqrt{10}$$

$$\sqrt{7}\cdot\sqrt{7} = \sqrt{49} = 7$$

$$\sqrt{6} \cdot \sqrt{12} = \sqrt{6 \cdot 12} = \sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$$

$$3\sqrt{5} \cdot (2\sqrt{5} + 3\sqrt{15})$$
 Prob 1.4.31

$$12\sqrt{12}+3\sqrt{27}-4\sqrt{75}$$
 Prob 1.4.39

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Dividing Square Roots = Quotient Rule

If a and b represent nonnegative real numbers and $b \neq 0$, then

 $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

The quotient of two square roots is the square root of the quotient.

$$\frac{\sqrt{90}}{\sqrt{2}} = \sqrt{\frac{90}{2}} = \sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

$$\frac{\sqrt{75}}{\sqrt{3}} = \sqrt{\frac{75}{3}} = \sqrt{25} = 5$$

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Time is Relative

- **❖Planet of the Apes (1968)**
 - **◆**Einstein's Special Relativity Equation
 - $ightharpoonup R_a =$ Relative Age Astronaut

 - ♦ v = Velocity
 - $\diamond c$ = Speed of light

$$_{a}=R_{f}\sqrt{1-\left(\frac{v}{c}\right)^{2}}$$

