Mathematical foundations of Computer Science.

Exercise 1:

Let p and q be the propositions
p: I bought a lottery ticket this week.
q: I won the million pound jackpot.
Express each of the following propositions as plain English sentences.

a) $\neg p$
b) $p \vee q$
c) $p \rightarrow q$
d) $p \wedge q$
e) $\neg p \rightarrow \neg q$
f) $\neg p \wedge \neg q$
g) $\neg q \vee \neg q$
h) $\neg p \vee (p \wedge q)$

Exercise 2:

Let p, q and r be the following propositions.
p: Grizzly bears have been seen in the area.
q: Hiking is safe on the trail.
r: berries are ripe along the trail.
Write these propositions using p, q and r and logical connectives.

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.
b) Grizzly bears have not been seen in the area and hiking n the trail is safe, but berries along the trail are ripe.
c) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.
d) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.
e) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

Exercise 3:

1) Determine whether each of these conditional statements is true or false.

a) if $1 + 1 = 2$ then $2 + 2 = 5$
b) if $1 + 1 = 3$ then $2 + 2 = 4$
c) if $1 + 1 = 3$ then $2 + 2 = 5$
d) if monkeys can fly, then $1 + 1 = 3$

Exercise 4:

1) How many rows will appear in each of the truth tables for the propositions below?

a) $p \rightarrow \neg q$
b) $(p \vee \neg r) \wedge (q \vee \neg s)$
c) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$
d) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

2) Construct the truth table for each of the following compound propositions.

a) $p \to \neg q$

b) $(p \vee \neg r) \wedge (q \vee \neg s)$

c) $p \wedge \neg p$

d) $p \vee \neg p$

e) $(p \vee \neg q) \to q$

f) $(p \vee q) \to (p \wedge q)$

g) $(p \to q) \to (q \to p)$

h) $(p \to q) \wedge (q \to p)$

i) $p \wedge \neg p$