Sample Assignment

Okko Makkonen[1]
September 10, 2022

Problem 1

Claim. The square root of 2 is irrational.

Proof. Assume that there exists $a, b \in \mathbb{Z}$ with $\gcd(a, b) = 1$ such that $\frac{a^2}{b^2} = 2$. Then $a^2 = 2b^2$, so a is even. Thus, $a = 2c$, and $2c^2 = b^2$, which means that b is even. This contradicts our assumption that $\gcd(a, b) = 1$, so $\sqrt{2} \notin \mathbb{Q}$. $\square$

Problem 2

Claim. There are infinitely many primes.

Proof. Assume that there are finitely many primes, say $p_1, \ldots, p_N$. Consider the number $q = p_1 \cdot \cdots \cdot p_N + 1$. It is clear that $p_i \nmid q$, so by the fundamental theorem of arithmetic there must be another prime factor of q that is not on the list $p_1, \ldots, p_N$. This contradictics our assumption of having a finite number of primes. $\square$

[1] Aalto University