Sample Assignment 1

Okko Makkonen*
September 13, 2022

These are some fundamental and elementary theorems within the mathematics done in ancient Greece with proofs in modern language.

Problem 1

Claim. The square root of 2 is irrational.

Proof. Assume that there exists $a, b \in \mathbb{Z}$ with $\gcd(a, b) = 1$ such that $\frac{a^2}{b^2} = 2$. Then $a^2 = 2b^2$, so a is even. Thus, $a = 2c$, and $2c^2 = b^2$, which means that b is even. This contradicts our assumption that $\gcd(a, b) = 1$, so $\sqrt{2} \notin \mathbb{Q}$. □

Problem 2

Claim. There are infinitely many primes.[1]

Proof. Assume that there are finitely many primes, say $p_1, \ldots, p_N$. Consider the number $q = p_1 \cdot \ldots \cdot p_N + 1$. It is clear that $p_i \nmid q$, so by the fundamental theorem of arithmetic there must be another prime factor of q that is not on the list $p_1, \ldots, p_N$. This contradictics our assumption of having a finite number of primes. □

*Aalto University

[1]This was first shown by Euclid.