Literature Review of NARs

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1 Introduction

Zhu et al.'s 'Network Vector Autoregression' paper introduced the 'NAR' model, and has been widely studied ever since. We provide a literature review of papers that have cited it, along with X. Zhu et al.'s 'Grouped Network Vector Autoregression' and H. Yin et al.'s 'A General Modeling Framework for Network Autoregressive Processes'. As they are highly cited papers, we focus on the ones that are most relevant for theoretical results and supply chain forecasting.

2 Zhu et al.'s 'Network Vector Autoregression' (2017)

This paper introduces a new statistical model called Network Vector Autoregression, and tests it on social network data. The NAR model assumes each node's response at a given time point is a linear combination of its previous value (momentum effect), the average of its connected neighbors (network effect), node-specific covariates (nodal effect), and an independent noise term.

One of the key features of this model is that the number of parameters does not increase with network size, unlike traditional vector autoregressive models (where it grows as $O(N^2)$), where N is the number of nodes.

Conditions for strict stationarity of NAR models are derived and asymptotic properties of the ordinary least squares estimator are established.

3 X. Zhu et al.'s 'Network Quantile Autoregression' (2019)

The paper proposes a Network Quantile Autoregression (NQAR) model to characterize dynamic quantile behavior in large networks. The NQAR model relates a node's response to its connected nodes and node-specific characteristics in a quantile autoregression process.

It develops theory for estimating the NQAR model parameters and establishes their asymptotic properties under assumptions on the network structure. It considers asymptotics under increasing time sample size, with fixed network dimension N, and both increasing network dimension and time.

4 Models that introduce a group structure

4.1 X. Zhu et al.'s 'Grouped Network Vector Autoregression' (2020)

X. Zhu et al's paper introduces the Group Network Autoregressive (GNAR) model, which takes into account different dynamic patterns across groups of nodes. GNAR does not assume homogeneous parameters across all nodes, so different groups can have different autoregressive coefficients (and thus the number of parameters grows with the number of groups).

It requires more complex estimation procedures like EM algorithm to identify group structure and estimate group-specific parameters, but it is more suitable for networks with heterogeneous node behaviors that cluster into groups.

4.2 E. Y. Chen et al.'s Community Network Auto-Regression for High-Dimensional Time Series (2020)

Similarly, in E. Y. Chen et al's Community Network Auto-Regression for High-Dimensional Time Series (2020), the CNAR model is introduced, which also allows heterogeneous network effects across different network communities.

It incorporates a community structure based on the stochastic block model and allows for unknown cross-sectional dependence through a latent factor structure in the errors. They also derive theoretical properties, showing consistency and asymptotic normality of the estimators.

4.3 D. Huang et al.'s Two-mode network autoregressive model for large-scale networks (2020)

The paper proposes a two-mode network autoregressive (TNAR) model for analyzing large-scale two-mode networks, designed for networks where nodes are classified into two distinct types, with edges only existing between nodes of different types (e.g. customers and merchants in an online platform), capturing cross-mode effects.

4.4 X. Zhu et al.'s Simultaneous Estimation and Group Identification for Network Vector Autoregressive Model with Heterogeneous Nodes (2023)

Similarly, this paper proposes a network vector autoregressive model with a latent group structure (GNAR). A key feature is that model parameters and network node memberships can be simultaneously estimated, which allows the number of latent groups to be over-specified while still achieving consistent estimation.

5 H. Yin et al.'s 'A General Modeling Framework for Network Autoregressive Processes' (2021)

This paper introduces a general modeling framework for NAR processes, in which non-Gaussian errors can be accommodated with spatial-autoregressive and factor-based covariance structures, and is applicable to both fixed and growing network sizes.

It develops a relaxed sufficient condition for stationarity of NAR processes, and establishes procedures for inference for an increasing number of model parameters, with OLS, GLS and ridge regularized versions. It also provides asymptotic normal distributions for the estimators and addresses the impact of misspecifying the network structure parameter estimation.

6 M. Barigozzi et al.'s Factor Network Autoregression (2022)

This paper introduces a factor network autoregressive (FNAR) model for analyzing time series data with complex network structures, incorporating multiple types of connections between nodes (a "multilayer network"). It uses a principal component approach based on tensors to summarize the multilayer network into a smaller number of "network factors". They use dimension reduction techniques such as assuming homogeneous network effects across nodes and extracting network factors

7 M. Armillotta, et al. 'Count Network Autoregression' (2023)

The paper extends previous work on network autoregression models for continuous data, to suit count-valued network time series data. It introduces Poisson Network Autoregression (PNAR) models in both linear and log-linear forms, allowing for count data with network structure. The model accommodate dependence over time and across network nodes, using a copula-based approach for the joint distribution.

8 X. Guo et al.'s Negative binomial community network vector autoregression for multivariate integer-valued time series (2024)

The models in 'Count Network Autoregression' may suffer from the risk of model misspecification due to their simple form that does not properly address the heterogeneous network effect. In Guo et al., they propose the model Negative Binomial Community Network Vector Autoregression (NB-CNVAR) for modeling multivariate integer-valued time series data with network structure, which uses different network coefficients across different communities to characterize the heterogeneous network effect. Theoretical properties are established, including consistency and asymptotic normality when the network size and time tends to infinity.

9 M. Armillotta et al.'s Nonlinear Network Autoregression (2023)

The paper studies general nonlinear models for integer and continuous valued data, extending previous work on linear Network Autoregressive (NAR) models to allow for nonlinear effects, using a smooth link function. They develop stability conditions when the network dimension is increasing, and also quasi maximum likelihood inference procedures.

9.1 M. Armillotta et al.'s Testing Linearity for Network Autoregressive Models (2023)

Related is this earlier paper by the same author, where they develop statistical methods for testing linearity in network autoregressive models, such as the quasi-score linearity test, and introduce general nonlinear frameworks for network autoregressive models with continuous and count data.

10 B. Jiang et al.'s 'Autoregressive Networks' (2023)

In this paper, while they do not consider forecasting applications, they derive a finite sample condition for the perfect recovery of community structure in network autoregressive models. They develop a new spectral clustering algorithm and provided theoretical guarantees for its performance under finite sample sizes.

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