$$w = (X^T X)^{-1} X^T y$$

$$= \left(\begin{pmatrix} x_1 & x_2 & \cdots & x_N \\ 1 & 1 & \cdots & 1 \end{pmatrix} \underset{2 \times N}{} \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_N \end{pmatrix} \right)^{-1} \begin{pmatrix} x_1 & x_2 & \cdots & x_N \\ 1 & 1 & \cdots & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

$$= \begin{pmatrix} \sum_{k=1}^{N} x_k & \sum_{k=1}^{N} x_k^2 \\ N & \sum_{k=1}^{N} x_k \end{pmatrix}^{-1} \overset{N \times 2}{} \begin{pmatrix} x_1 & x_2 & \cdots & x_N \\ 1 & 1 & \cdots & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

$$= \frac{1}{N^2 \bar{x} \bar{x} - N^2 \overline{x^2}} \begin{pmatrix} \sum_{k=1}^{N} x_k & -\sum_{k=1}^{N} x_k^2 \\ -N & \sum_{k=1}^{N} x_k \end{pmatrix} \underset{2 \times 2}{} \begin{pmatrix} x_1 & x_2 & \cdots & x_N \\ 1 & 1 & \cdots & 1 \end{pmatrix} \underset{2 \times N}{} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

$$= \frac{1}{N^2 \bar{x} \bar{x} - N^2 \overline{x^2}} \begin{pmatrix} x_1 N \bar{x} - N \overline{x^2} & x_2 \bar{x} - N \overline{x^2} & \cdots & x_N \bar{x} - N \overline{x^2} \\ -N x_1 + \underset{N \bar{x}}{\underbrace{\sum_{k=1}^{N} x_k}} & -N x_2 + \underset{N \bar{x}}{\underbrace{\sum_{k=1}^{N} x_k}} & \cdots & -N x_N + \underset{N \bar{x}}{\underbrace{\sum_{k=1}^{N} x_k}} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}$$

$$= \frac{1}{N^2 \bar{x} \bar{x} - N^2 \overline{x^2}} \begin{pmatrix} N \bar{x} \sum_{k=1}^{N} x_k y_k - N \overline{x^2} \sum_{k=1}^{N} y_k \\ -N \sum_{k=1}^{N} x_k y_k + \sum_{k=1}^{N} y_k N \bar{x} \end{pmatrix}$$

$$= \frac{1}{N^2 \bar{x} \bar{x} - N^2 \overline{x^2}} \begin{pmatrix} N^2 \bar{x} \overline{xy} - N^2 \overline{x^2} \bar{y} \\ -N^2 \overline{xy} + N^2 \bar{x} \bar{y} \end{pmatrix}$$

$$= \begin{pmatrix} b \\ a \end{pmatrix} \qquad \text{よって } a、b \text{ はそれぞれ以下のように表せる.}$$

$$a = \frac{\bar{x} \bar{y} - \overline{xy}}{\bar{x}^2 - \overline{x^2}} = \frac{\overline{xy} - \bar{x} \bar{y}}{\overline{x^2} - (\bar{x})^2}$$

$$b = \frac{\bar{x}\,\overline{xy} - \overline{x^2}\,\bar{y}}{(\bar{x})^2 - (\overline{x^2})}$$

$$= \frac{\overline{x^2}\,\bar{y} - \bar{x}\,\overline{xy}}{\overline{x^2} - (\bar{x})^2}$$

$$= -\bar{x}\,\frac{\overline{xy} - \bar{x}\,\bar{y}}{\overline{x^2} - (\bar{x})^2} + \frac{\overline{x^2}\,\bar{y} - (\bar{x})^2\,\bar{y}}{\overline{x^2} - (\bar{x})^2}$$

$$= -a\bar{x} + \bar{y}$$

————————————— #