Normal and Bending Stresses in Simple Cross Sections

Theory

- closed polygon
- counterclockwise
- must not intersect itself

Symbols

y0, z0	arbitrary (reference) axis system
y0C, z0C	coordinates of section centroid
y, z	axes parallel to y0, z0, origin in section centroid
ybar, zbar	principal axis system, ybar major, origin in centroid
iy0, iz0, iyz0	moments of inertia in reference system
iy, iz, iyz	moments of inertia in centroid axis system
i1, i2	principal moments of inertia, i1 >= i2
Iybar, Izbar	principal moments of inertia, Iybar > Izbar
phi1	angle between y and ybar, direction of major principal axis
phi2	direction of minor principal axis
A	area of cross section
r1, r2	radii of gyration

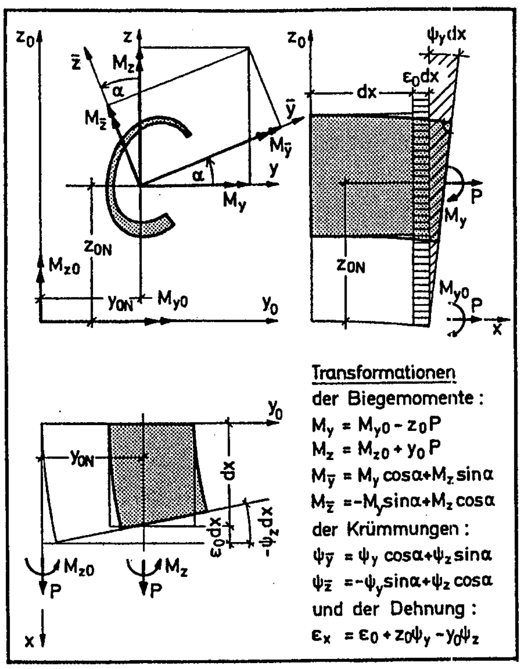

Bild 3.1/1. Elementare Biegetheorie des Stabes. Schnittlasten und Verformungen am Stabelement. Koordinatentransformation

Properties in Reference System

Area

$$A = \int_\Omega \mathrm{d}y_0 \, \mathrm{d}z_0$$

Centroid

$$y_{0C} = \frac{1}{A} \int_\Omega y_0 \, \mathrm{d}y_0 \, \mathrm{d}z_0$$
$$z_{0C} = \frac{1}{A} \int_\Omega z_0 \, \mathrm{d}y_0 \, \mathrm{d}z_0$$

Moments of Interia

$$I_{y0} = \int_\Omega z_0^2 \quad \mathrm{d}y_0 \, \mathrm{d}z_0$$
$$I_{z0} = \int_\Omega y_0^2 \quad \mathrm{d}y_0 \, \mathrm{d}z_0$$
$$I_{yz0} = \int_\Omega y_0 \, z_0 \quad \mathrm{d}y_0 \, \mathrm{d}z_0$$

Properties in Centroid System

$$I_y = I_{y0} - z_{0C}^2 A$$
$$I_z = I_{z0} - y_{0C}^2 A$$
$$I_{yz} = I_{yz0} - y_{0C} z_{0C} A$$

Principal Axes System

Principal moments of inertia. $I_{\bar{y}}$ is the larger one.

$$I_{\bar{y}} = \frac{1}{2}\left[(I_y + I_z) + \sqrt{(I_y - I_z)^2 + 4 I_{yz}^2}\right]$$
$$I_{\bar{z}} = \frac{1}{2}\left[(I_y + I_z) - \sqrt{(I_y - I_z)^2 + 4 I_{yz}^2}\right]$$

Radii of gyration

$$r_{\bar{y}} = \sqrt{\frac{I_{\bar{y}}}{A}}$$
$$r_{\bar{z}} = \sqrt{\frac{I_{\bar{z}}}{A}}$$

Direction of Principal Axes

$$\tan 2\phi = \frac{2 I_{yz}}{I_z - I_y}$$

Here add some remarks how to find which direction belongs to major axis and which to the minor.

Applied Loads

Normal force N and bending moments M_{yP}, M_{zP} applied at some load application point P. P is given in reference system: y_{0P}, z_{0P}.

Moments with respect to centroid:

$$M_y = M_{yP} - (z_{0C} - z_{0P}) N$$
$$M_z = M_{zP} + (y_{0C} - y_{0P}) N$$

Moments with respect to principal axes:

$$M_{\bar{y}} = M_y \cos\phi + M_z \sin\phi$$
$$M_{\bar{z}} = -M_y \sin\phi + M_z \cos\phi$$

Stresses

$$\sigma(\bar{y},\bar{z}) = \frac{N}{A} + \bar{z}\frac{M_{\bar{y}}}{I_{\bar{y}}} - \bar{y}\frac{M_{\bar{z}}}{I_{\bar{z}}}$$

For that, we need to transform point coordinates into principal axis system. Given some point V (y0V, z0V), what are coordinates $\bar{y}_V$?

Point in centroiid system y,z:

$$y_V = y_{0V} - y_{0C}$$
$$z_V = z_{0V} - z_{0C}$$

Point in principal axes system:

$$\bar{y}_V = y_V \cos\phi + z_V \sin\phi$$
$$\bar{z}_V = -y_V \sin\phi + z_V \cos\phi$$