



## The let-in Construct

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### Raiding the Bandits' Nest



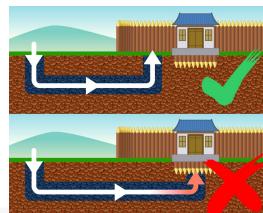
2

### Raiding the Bandits' Nest



3

### Digging Tunnels



4



Diggable Spots									
1	2	3	4	5	6	7	8	9	
1	✓	✓	✓	✓	✓	✓	✓	✓	
2	✗	✗	✗	✗	✗	✗	✗	✗	
3	✓	✓	✓	✓	✓	✓	✓	✓	
4	✗	✗	✗	✗	✗	✗	✗	✗	
5	✓	✓	✓	✓	✓	✓	✓	✓	
6	✓	✓	✓	✓	✓	✓	✓	✓	
7	✗	✗	✗	✗	✗	✗	✗	✗	
8	✗	✗	✗	✗	✗	✗	✗	✗	
9	✓	✓	✓	✓	✓	✓	✓	✓	

5

The Manhattan Distance									
1	2	3	4	5	6	7	8	9	
1	0	1	2	3	4	5	6	7	
2	1	0	1	2	3	4	5	6	
3	2	1	0	1	2	3	4	5	
4	3	2	1	0	1	2	3	4	
5	4	3	2	1	0	1	2	3	
6	5	4	3	2	1	0	1	2	
7	6	5	4	3	2	1	0	1	
8	7	6	5	4	3	2	1	0	
9	8	7	6	5	4	3	2	1	

6

The Manhattan Distance									
1	2	3	4	5	6	7	8	9	
1	0	1	2	3	4	5	6	7	
2	1	0	1	2	3	4	5	6	
3	2	1	0	1	2	3	4	5	
4	3	2	1	0	1	2	3	4	
5	4	3	2	1	0	1	2	3	
6	5	4	3	2	1	0	1	2	
7	6	5	4	3	2	1	0	1	
8	7	6	5	4	3	2	1	0	
9	8	7	6	5	4	3	2	1	

7

All Huts Covered									
1	2	3	4	5	6	7	8	9	
1	0	1	2	3	4	5	6	7	
2	1	0	1	2	3	4	5	6	
3	2	1	0	1	2	3	4	5	
4	3	2	1	0	1	2	3	4	
5	4	3	2	1	0	1	2	3	
6	5	4	3	2	1	0	1	2	
7	6	5	4	3	2	1	0	1	
8	7	6	5	4	3	2	1	0	
9	8	7	6	5	4	3	2	1	

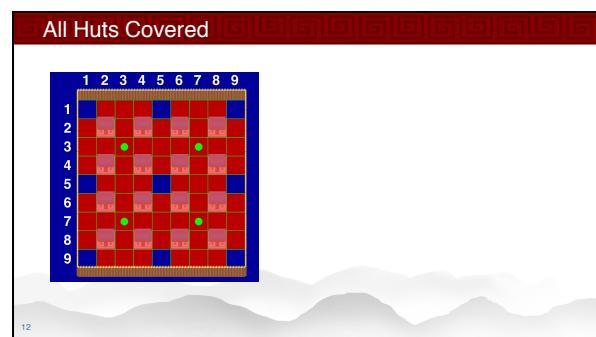
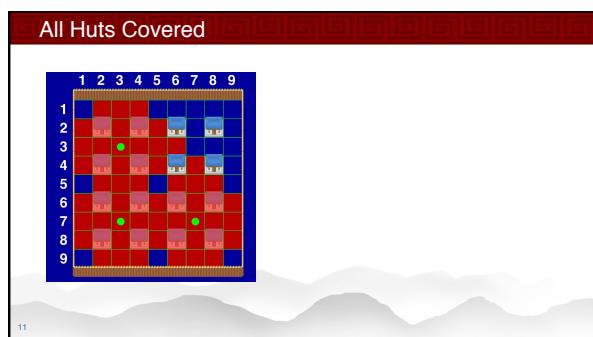
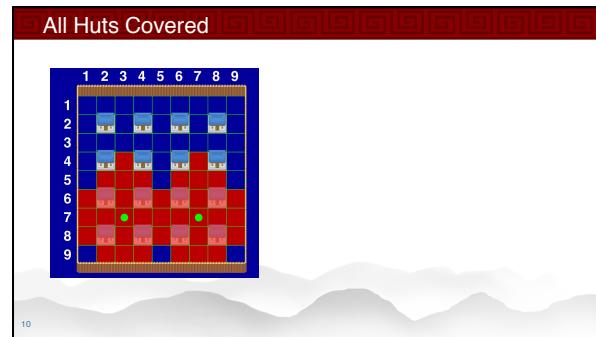
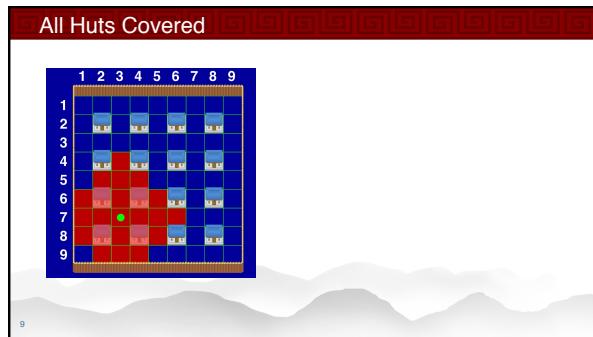
8



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## Cost of Digging

	1	2	3	4	5	6	7	8	9
1	\$9	\$9	\$9	\$9	\$9	\$9	\$9	\$9	\$9
2	\$8	\$8	\$8	\$8	\$8	\$8	\$8	\$8	\$9
3	\$7	\$7	\$7	\$7	\$7	\$7	\$7	\$8	\$9
4	\$6	\$6	\$6	\$6	\$6	\$7	\$7	\$8	\$9
5	\$5	\$5	\$5	\$5	\$5	\$6	\$7	\$8	\$9
6	\$4	\$4	\$4	\$5	\$5	\$7	\$7	\$8	\$9
7	\$3	\$3	\$3	\$4	\$5	\$6	\$7	\$8	\$9
8	\$2	\$3	\$3	\$5	\$5	\$7	\$7	\$8	\$9
9	\$1	\$2	\$3	\$4	\$5	\$6	\$7	\$8	\$9

13

## Cost of Digging

	1	2	3	4	5	6	7	8	9
1	\$9	\$9	\$9	\$9	\$9	\$9	\$9	\$9	\$9
2	\$8	\$8	\$8	\$8	\$8	\$8	\$8	\$8	\$9
3	\$7	\$7	\$7	\$7	\$7	\$7	\$7	\$8	\$9
4	\$6	\$6	\$6	\$6	\$6	\$7	\$7	\$8	\$9
5	\$5	\$5	\$5	\$5	\$5	\$6	\$7	\$8	\$9
6	\$4	\$4	\$4	\$5	\$5	\$7	\$7	\$8	\$9
7	\$3	\$3	\$3	\$4	\$5	\$6	\$7	\$8	\$9
8	\$2	\$3	\$3	\$5	\$5	\$7	\$7	\$8	\$9
9	\$1	\$2	\$3	\$4	\$5	\$6	\$7	\$8	\$9

14



## Data and Variables (bandits.mzn)

### ⌘ Data

```
int: size; set of int: SIZE = 1..size;  
int: nPts; set of int: POINT = 1..nPts;  
int: mDist;  
  
array[SIZE,SIZE] of int: cost;
```

### ⌘ Variables

```
array[POINT] of var SIZE: ptR;  
array[POINT] of var SIZE: ptC;
```

15

## Constraints (bandits.mzn)

### ⌘ All tunnel exits different

```
alldifferent(  
  [(ptR[i]-1)*size + ptC[i] |  
   i in POINT]  
) ;
```

### ⌘ Kind of tedious and difficult to understand.

16



## The let-in construct

- ⌘ The let-in construct allows new variables and parameters to be introduced and used **locally** at “any point” in the model

- ⌘ Format

```
let {<type>:<varname> [ = <expr>] ;  
      ...  
      <type>:<varname> [ = <expr>] [;]} in  
<expr>
```

- ⌘ Parameters introduced **must** have a defining expression, but variables do not
- ⌘ New vars/pars **visible only** in remainder of let-in expression

17

## Constraints (bandits.mzn)

- ⌘ All tunnel exits different

```
let {array [POINT] of var 1..size*size:  
      points = [(ptR[i]-1)*size + ptC[i] |  
                 i in POINT]} in  
      alldifferent(points);
```

- ⌘ Make the definition more modular and easier to understand.

18



## Constraints (bandits.mzn)

### Defining whether a spot is diggable

#### Only the non-building spots

```
forall(i in POINT)
  (not((ptR[i] mod 2) = 0 /\
        (ptC[i] mod 2) = 0));
```

OR

```
forall(i in POINT)
  ((ptR[i] mod 2) = 1 \/
   (ptC[i] mod 2) = 1);
```

19

## Constraints (bandits.mzn)

### All huts must be covered

```
predicate covered(var int: x, var int: y) =
  let {var POINT: i, var int: dist =
    abs(x-ptR[i]) + abs(y-ptC[i])} in
    dist <= mDist;

constraint let {
  array [1..(size div 2)] of 1..size-1:
  huts = [i*2|i in 1..(size div 2)]} in
    forall(i,j in huts) (covered(i,j));
```

### See how let-in helps modularize expressions and improves readability

20



## Objective (bandits.mzn)

- Minimizing the total cost of building the tunnels

```
var int: total_cost = sum(i in POINT)
  (cost[ptR[i],ptC[i]]);
solve minimize total_cost;
```

21

## Running the Model (bandits.mzn)

```
Row: [5, 7, 3]
Col: [2, 6, 6]
Cost: 18
-----
=====
```

22



## Another let-in Example

- ⌘ Recall the **Patrol** problem

- ⌘ Replace

```
forall(d in DAY)
    (sum(s in SOLDIER)
        (roster[s,d] = EVE) >= 1);
forall(d in DAY)
    (sum(s in SOLDIER)
        (roster[s,d] = EVE) <= u);
```

- ⌘ By

```
forall(d in DAY)
    (let {var int: on = sum(s in SOLDIER)
          (roster[s,d]=EVE)
          } in 1 <= on /\ on <= u);
```

- ⌘ Easier than array of intermediates

23

## Defining Division with Local Variable

- ⌘ Suppose your solver does not support `div`

- ⌘ Division using multiplication

- **non-negative only**

- $x \text{ div } y = z \leftrightarrow x = z * y + r, 0 \leq r < y$

- ⌘ We need a **new variable** `r` for remainder

- ⌘ Code

```
predicate divp(var int:x,var int: y,var int: z)
    = let {var 0..ub(y)-1: r} in
      x = z * y + r /\ r < y;
```

- ⌘ What happens with division by **zero**?

- ⌘ What happens with division by **negative**?

24



## Predicate Assumptions Using Assert

- ⌘ Our predicate assumed non-negativity

- let's make that assumption explicit
  - using an assertion

```
predicate divp(var int:x,var int: y,var int: z)
  = assert(lb(y) >= 0,
    "divp: y must be non-negative",
    let {var 0..ub(y)-1: r} in
      x = z * y + r /\ r < y);
```

- ⌘ Advantages

- Documents the assumption
  - Prevents misuse

25

## Reflection Functions

- ⌘ Sometime we want to know properties of variables in the model

- lb(x) a lower bound on all possible values of x
  - ub(x) an upper bound on all possible values
  - dom(x) a superset of all possible values of x
  - lb\_array(x): lower bound on all vars in array x
  - ub\_array(x): upper bound on array

- ⌘ Beware these are **not guaranteed** to be the declared bounds

```
var -4..6: x;    var -4...-2: y;
constraint x = abs(y);
lb(x) = 0 or lb(x) = -4 or lb(x) = 2 (-4...-2)
```

26



## Assertions again

- ⌘ Defensive programming requires that we **check** assumptions in predicates.

- `assert(boolexp,stringexp,exp)`

- returns `exp` if `boolexp` holds,
    - otherwise prints `stringexp` and aborts

- ⌘ For example,

```
var -4..4:x;  
var -4..4:y;  
constraint divp(x,y,3);
```

- ⌘ Results in

```
Minizinc: evaluation error:  
dvip.mzn:9:  
    in call 'divp'  
dvip.mzn:2:  
    in call 'assert'  
Assertion failed: divp: y must be non-negative
```

27

## Summary

- ⌘ Let-in expressions
  - allow introduction of parameters and variables for use locally
- ⌘ Local parameters must be defined by =
- ⌘ Local decisions need not be
- ⌘ Help modularize expressions and improve readability of models

28



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