Homework 2

106307030 財管四 廖偉博

Q1

[20pts] a. 生成一筆資料:

 $X_i = a + \varepsilon, i = 1, \ldots, 20$

- a 為0~10 任意數字。 $arepsilon \sim N(0,2)$
- 注意: X必須在0~11內。

Ans:

詳見後方程式碼

```
[1] 10.2993431 0.2530285 7.8253714 1.0328434 8.0877121 8.6233766 2.7870334 5.7396099 2.0653691 [10] 2.2152110 7.4146825 9.1865351 7.1735514 0.9105971 0.7907242 9.0423911 9.7440828 9.8078056 [19] 9.4195485 2.3070893
```

[20pts] b. Cauchy(θ, 1) 的密度函數,取log後一次微分如下,請寫出此function

$$f(heta) = -2\sum_{i=1}^n rac{ heta-x_i}{\{1+(heta-x_i)^2\}}$$

Ans:詳見後方程式碼

[10pts] c. 代入a生成的資料至b的function,並令heta=0.3。

Ans:

詳見後方程式碼

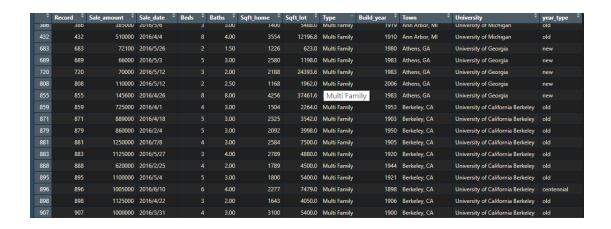
```
> #1.c
> theta = 0.3
> cauchy(theta, y)
[1] 8.656367
```

Q2

建立[哈佛大學]地區房屋價格的迴歸預測模型,找出是什麼因子影響不同房型的房價。
[10pts] a. 根據 Build_year,建立一個新類別變數 year_type,1899 年以前的房子為" centennial",1900~1959 年為"old",1960 年以上為"new"。

Ans:

詳見後方程式碼

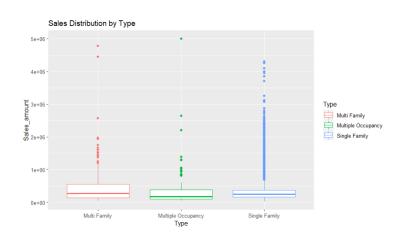


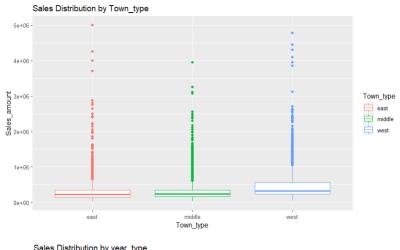
[40pts] b. 決定好你的最佳配適模型後,總結你的發現並根據解釋變數預測房屋價格。 Ans:

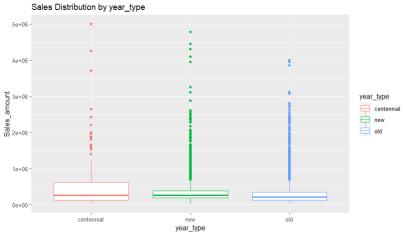
1.先確定你要預測的對象 y 以及可能放進去的多個 x 變數



發現有一個明顯的離群值,將其刪除







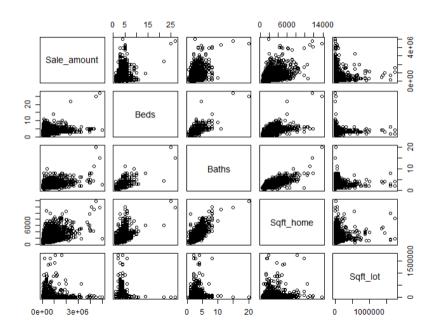
2.處理資料的遺漏值

因為此 dataset 沒有遺漏值,所以跳過此步驟。

3.可以先檢視連續型 x 變數之間的相關性,以及連續型 x 對 y 變數的相關性,看是否要在這步就篩選變數,或做變數變換

=>我先檢測連續型變數與 Sale_amount 間的關係,由下圖可看出 Sqft_lot 與 Sale_amount 的相關性相當低,因此先不把 Sqft_lot 放入模型。

散佈圖:



相關係數:

```
        Sale_amount
        Beds
        Baths
        Sqft_home
        Sqft_lot

        Sale_amount
        1.0000000
        0.31039615
        0.4541136
        0.5174500
        0.11630919

        Beds
        0.3103961
        1.00000000
        0.5872812
        0.6039610
        0.03065125

        Baths
        0.4541136
        0.58728121
        1.0000000
        0.7592398
        0.12205319

        Sqft_home
        0.5174500
        0.60396101
        0.7592398
        1.0000000
        0.18627041

        Sqft_lot
        0.1163092
        0.03065125
        0.1220532
        0.1862704
        1.00000000
```

- 4.類別型的 x 變數需要先轉成 dummy variable
- =>我將房型、年份類型和所在地區轉換為 dummy variable

(1) 房型

TypeMulti TypeMultiple Cocupancy TypeMulti TypeMultiple Cocupancy TypeMultip

(2)年份類型

1 0 1 1 2 2 0 1 1 3 3 0 1 1 4 0 1 1 5 0 1 1 1 1 1 5 0 1 1 1 1 1 5 1 1 1 1	^	Typecentennial	‡	Typenew	\$
3 0 1 4 0 1 5 0 1 6 0 1 7 0 1 8 0 1 10 0 1 11 0 0 11 0 0 11 12 0 1 12 0 1 13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	1				1
4 0 1 5 0 1 6 0 1 7 0 1 8 0 1 10 0 1 11 0 0 11 0 0 12 0 1 13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	2				1
5 0 1 6 0 1 7 0 1 8 0 1 9 0 1 10 0 1 11 0 0 12 0 1 13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	3				1
6 0 1 7 0 1 8 0 1 9 0 1 10 0 1 11 0 0 12 0 1 13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	4				1
7 0 1 8 0 1 9 0 1 10 0 1 11 0 0 11 0 0 12 0 1 13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	5				1
8 0 1 9 0 1 10 0 1 11 0 0 12 13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	6				1
9 0 1 10 0 1 11 0 0 11 12 0 1 13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	7				1
10 0 1 11 0 0 0 12 0 1 13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	8				1
11 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	9				1
12 0 1 13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	10				1
13 0 1 14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	11				0
14 0 1 15 0 1 16 0 1 17 0 1 18 0 1	12				1
15 0 1 16 0 1 17 0 1 18 0 1	13				1
16 0 1 17 0 1 18 0 1 19 0 1	14				1
17 0 1 18 0 1 19 0 1	15				1
18 0 1 19 0 1	16				1
19 0 1	17				1
	18				1
20 0 1	19				1
	20				1

(3)所在地區

_	Typeeast	‡	Typemiddle	‡
		0		
		0		
3		0		
4		0		
5		0		
6		0		
7		0		
8		0		
9		0		
10		0		
11		0		
12		0		
13		0		
14		0		
15		0		
16		0		
17		0		
18		0		
19		0		
20		0		

5.跑迴歸模型,檢視模型各項指標、是否有符合常態假設、做離群值的偵測,幫助我 們篩選變數及樣本或其他處理

利用 AIC backward 法

```
Call:
lm(formula = Sale_amount ~ Beds + Baths + Sqft_home + Typeeast +
    Typemiddle + Typecentennial + Typenew, data = train)
Coefficients:
    (Intercept)
191851.7
                                  Beds
                                                     Baths
                                                                      Sqft_home
                                                                                            Typeeast
                            -10310.7
                                                                           135.8
                                                                                           -238609.5
                                                   88378.2
     Typemiddle Typecentennial
                                                   Typenew
      -230376.6
                             221058.2
                                                -170041.1
```

利用 AIC forward 法

```
Ca11:
lm(formula = Sale_amount ~ Sqft_home + Typenew + Baths + Typemiddle +
    Typeeast + Typecentennial + Beds, data = train)
Coefficients:
    (Intercept)
191851.7
                                                                                        Typemiddle
                          Sqft_home
                                                                         Baths
                                                  Typenew
                                135.8
                                                -170041.1
                                                                       88378.2
                                                                                          -230376.6
       Typeeast Typecentennial
                                                      Beds
      -238609.5
                            221058.2
                                                 -10310.7
```

利用 BIC backward 法

```
Call:
lm(formula = Sale_amount ~ Beds + Baths + Sqft_home + Typeeast +
    Typemiddle + Typecentennial + Typenew, data = train)
Coefficients:
   (Intercept)
                          Beds
                                         Baths
                                                      Sqft_home
                                                                       Typeeast
      191851.7
                      -10310.7
                                       88378.2
                                                          135.8
                                                                      -238609.5
    Typemiddle Typecentennial
                                       Typenew
     -230376.6
                      221058.2
                                     -170041.1
```

皆得出同樣的模型

因此設定模型一為:Sale_amount =191851.7 – 10310.7Beds + 88378.2Baths + 135.8Sqft_home – 238609.5Typeeast – 230376.6Typemiddle + 221058.2Typecentennial – 170041.1Typenew

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
                         1.270e+04
                                    15.109 < 2e-16 ***
               1.919e+05
(Intercept)
                                     -2.659 0.00786 **
Beds
               -1.031e+04
                          3.878e+03
Baths
               8.838e+04 5.174e+03
                                     17.082
                                             < 2e-16 ***
                          4.769e+00
Sqft_home
               1.358e+02
                                    28.463
                                             < 2e-16 ***
Typeeast
               -2.386e+05
                          8.701e+03 -27.422
                                             < 2e-16 ***
                                            < 2e-16 ***
Typemiddle
               -2.304e+05 8.467e+03 -27.208
Typecentennial 2.211e+05 2.386e+04
                                     9.266
                                            < 2e-16 ***
              -1.700e+05 7.402e+03 -22.974 < 2e-16 ***
Typenew
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 277800 on 7992 degrees of freedom
Multiple R-squared: 0.3853,
                               Adjusted R-squared: 0.3848
F-statistic: 715.7 on 7 and 7992 DF, p-value: < 2.2e-16
```

再觀察步驟三的散佈圖,發現 Beds 和 Sqft_home 有線性關係,因此再加入 Beds 和 Sqft_home 的交互作用項觀察

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
                         1.664e+04
                                    17.169 < 2e-16 ***
               2.857e+05
(Intercept)
Beds
               -3.836e+04 5.036e+03
                                    -7.616 2.91e-14 ***
               8.333e+04 5.183e+03 16.079 < 2e-16 ***
Baths
               1.105e+02 5.573e+00 19.820 < 2e-16 ***
Sqft_home
                         8.675e+03 -27.017 < 2e-16 ***
              -2.344e+05
Typeeast
              -2.271e+05
                         8.437e+03 -26.913
                                            < 2e-16 ***
Typemiddle
                                            < 2e-16 ***
Typecentennial 2.275e+05 2.376e+04
                                      9.577
              -1.613e+05 7.436e+03 -21.688 < 2e-16 ***
Typenew
Beds:Sqft_home 7.509e+00 8.661e-01
                                      8.670 < 2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 276500 on 7991 degrees of freedom
Multiple R-squared: 0.3911, Adjusted R-squared: 0.3905
F-statistic: 641.5 on 8 and 7991 DF, p-value: < 2.2e-16
```

發現加入 Beds 和 Sqft_home 的交互作用項厚的模型的 R-squared 確實有上升,且做 ANOVA 分析發現複雜的模型比簡單的模型較好,因此加入此交互作用項成為模型二。

模型二:Sale_amount = 285700 - 38360Beds + 83330Baths + 110.5Sqft_home + 7.509Beds:Sqft_home - 234400Typeeast – 227100 Typemiddle + 227500Typecentennial - 161300Typenew

再觀察步驟三的散佈圖,發現 Beds 和 Baths 也有線性關係,因此再加入 Beds 和 Baths 的交互作用項觀察

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)
               2.849e+05 1.729e+04
                                   16.484
                                           < 2e-16 ***
              -3.814e+04 5.225e+03 -7.299 3.17e-13 ***
Beds
Baths
               8.190e+04 1.044e+04
                                     7.842 5.00e-15 ***
Sqft_home
               1.124e+02 1.340e+01
                                     8.387
                                           < 2e-16 ***
              -2.343e+05 8.677e+03 -27.006
                                           < 2e-16 ***
Typeeast
              -2.270e+05 8.438e+03 -26.907
                                           < 2e-16 ***
Typemiddle
Typecentennial 2.275e+05 2.376e+04
                                     9.575 < 2e-16 ***
                                           < 2e-16 ***
              -1.613e+05 7.437e+03 -21.687
Typenew
Beds:Sqft_home 7.012e+00 3.257e+00
                                     2.153
                                            0.0314 *
              3.696e+02 2.331e+03
                                     0.159 0.8741
Beds:Baths
Signif. codes: 0 '*** 0.001 '** 0.01 '* 0.05 '.' 0.1 ' 1
Residual standard error: 276500 on 7990 degrees of freedom
Multiple R-squared: 0.3911,
                             Adjusted R-squared: 0.3904
F-statistic: 570.1 on 9 and 7990 DF, p-value: < 2.2e-16
```

發現加入 Beds 和 Sqft_home 的交互作用項厚的模型的 R-squared 沒有上升,且做 ANOVA 分析發現簡單的模型比複雜的模型較好,不加入此交互作用項。

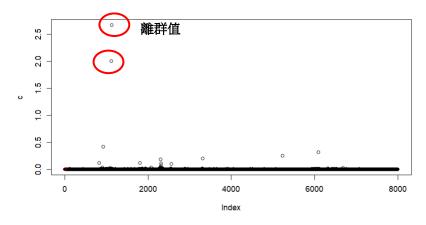
再觀察步驟三的散佈圖,發現 Baths 和 Sqft_home 也有線性關係,因此再加入 Baths 和 Sqft_home 的交互作用項觀察

```
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
                                       17.035
                                                < 2e-16 ***
(Intercept)
                 2.831e+05
                            1.662e+04
Beds
                -1.335e+04
                            6.833e+03
                                        -1.953
                                                 0.0508
                                         7.112 1.24e-12 ***
Baths
                 5.366e+04
                            7.544e+03
Sqft_home
                 1.010e+02
                            5.833e+00
                                       17.310
                                                < 2e-16
                -2.347e+05
                            8.660e+03 -27.104
                                                < 2e-16 ***
Typeeast
                            8.423e+03 -26.865
Typemiddle
                                                < 2e-16 ***
                -2.263e+05
                 2.295e+05
                            2.372e+04
                                         9.677
                                                < 2e-16
Typecentennial
                -1.563e+05
                            7.480e+03 -20.896
                                                < 2e-16
Typenew
                            1.718e+00
                -5.151e-01
                                       -0.300
                                                 0.7643
Beds:Sqft_home
Baths:Sqft_home 1.155e+01 2.138e+00
                                         5.405 6.68e-08 ***
                0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Signif. codes:
Residual standard error: 276000 on 7990 degrees of freedom
Multiple R-squared: 0.3933,
                               Adjusted R-squared: 0.3926
F-statistic: 575.5 on 9 and 7990 DF, p-value: < 2.2e-16
```

發現加入 Baths 和 Sqft_home 的交互作用項厚的模型的 R-squared 確實有上升,且做 ANOVA 分析發現複雜的模型比簡單的模型較好,因此加入此交互作用項成為模型三。

模型三: Sale_amount = 283100 - 13350Beds + 53660Baths + 101Sqft_home - 0.5151Beds:Sqft_home + 11.55Baths:Sqft_home -234700Typeeast - 226300 Typemiddle + 229500Typecentennial - 156300Typenew

檢查是否有離群值:



發現有兩筆離群值,將其刪除

最後,將模型一和模型三預測出的結果所算出的 RMSE 去做比較,發現模型三的 AIC、BIC、RMSE 確實都有下降。

```
> AIC(model,k = 2)
[1] 223265.4
> AIC(model_3,k = 2)
[1] 223165.3
```

```
> BIC(model)
[1] 223328.3
> BIC(model_3)
[1] 223242.2
```

```
> RMSE1
[1] 10498731
> RMSE2
[1] 10256357
```

6.利用測試集驗證預測的結果,如果不存在測試集,可以在步驟 4 後切出一部分的觀察值當作測試集

我將前 8000 筆資料設為訓練集,其餘的資料設定為測試集

TypeMulti ⁵ Family	TypeMultiple Occupancy	† Typeeast	† Typemiddle	‡ Т <u>у</u>	ypecentennial ‡	Турепеж	predict2
1	1	0	1	0	0	1	-11641.624
C)	0	1	0	0	1	-6003.110
C)	0	1	0	0	1	1543.321
C)	0	1	0	0	1	4039.370
C)	0	1	0	0	1	6870.214
C)	0	1	0	0	1	6870.214
C)	0	1	0	0	1	9533.660
C)	0	1	0	0	1	12197.106
C)	0	1	0	0	1	13861.760
C)	0	1	0	0	1	16858.137
C)	0	1	0	0	1	17565.981
C)	0	1	0	0	1	18189.860
0)	0	1	0	0	1	18411.814

得出預測結果。

```
程式碼:
#1.a
y = c()
while(length(y) < 20) {
 a = sample(0:10, 1)
 e = rnorm(1, 0, sqrt(2))
 if(a+e > 0 && a+e <11){
  y = append(y, a+e)
 }
}
У
#1.b
cauchy <- function(theta, x){</pre>
 n <- length(x)
 y <- matrix(0,n)
 for(i in c(0:n-1)){
 y[i] \leftarrow (theta - x[i]) / (1 + (theta - x[i])^2)
 }
 return(-2 * sum(y))
}
#1.c
theta = 0.3
cauchy(theta, y)
#2.a
install.packages("tidyverse")
library(tidyverse)
year_type <- ifelse (houseprice$Build_year<=1899, "centennial",</pre>
```

```
ifelse(houseprice$Build_year>=1960, "new", "old"))
print(year_type)
houseprice$year_type <- c(year_type)</pre>
#2.b
library(broom)
attach(houseprice)
require(ggplot2)
#觀察房型的箱型圖
ggplot(data = houseprice) + geom_boxplot(aes( x= Type, y= Sale_amount, colour = Type)) +
labs( x = 'Type',
    y = 'Sales amount',
    title = 'Sales Distribution by Type')
#發現一個明顯的離群值,將其刪除
houseprice <- houseprice %>% filter(Sale_amount < 7500000)
attach(houseprice)
ggplot(data = houseprice) + geom_boxplot(aes( x= Type, y= Sale_amount, colour = Type)) +
labs( x = 'Type',
    y = 'Sales_amount',
    title = 'Sales Distribution by Type')
#將城市依照西區、中部、東區做分類
Town_type <- ifelse (houseprice$Town %in% c("Tacoma, WA", "Corvallis, OR", "Eugene,
OR", "San Luis Obispo, CA",
                       "Claremont, CA", "Berkeley, CA", "Logan, UT", "Bozeman, MT",
                       "Flagstaff, AZ", "Tempe, AZ"), "west",
          ifelse (houseprice$Town %in% c("Boulder, CO","Fort Collins, CO","Fargo, ND",
```

```
"Grand Forks, ND", "Manhattan, KS", "Lincoln, NE",
                           "Lawrence, KS", "College Station, TX", "Minneapolis, MN",
                           "Iowa City, IA", "Ames, IA", "Columbia, MO", "Fayetteville, AR",
                           "Madison, WI", "Champaign-Urbana, IL", "Bloomington,
IL"), "middle", "east"))
print(Town_type)
houseprice$Town_type <- c(Town_type)
#觀察房子地區的箱型圖
ggplot(data = houseprice) + geom_boxplot(aes( x= Town_type, y= Sale_amount, colour =
Town_type)) +
labs(x = 'Town_type',
    y = 'Sales_amount',
    title = 'Sales Distribution by Town_type')
#觀察房子年份的箱型圖
ggplot(data = houseprice) + geom_boxplot(aes( x= year_type, y= Sale_amount, colour =
year_type)) +
labs( x = 'year_type',
    y = 'Sales_amount',
    title = 'Sales Distribution by year_type')
#將 Type 轉為 dummy variables,並試著做迴歸分析
library(dummies)
houseprice$Type = as.factor(as.character(houseprice$Type))
type_df <- data.frame(Type = houseprice$Type)</pre>
type dummies <- dummy.data.frame(type df)
type_dummies <- type_dummies[,-c(3)]
fit.1 = Im(Sale_amount~.,data = type_dummies)
```

```
#將 Town 轉為 dummy variables,並試著做迴歸分析
houseprice$Town_type = as.factor(as.character(houseprice$Town_type))
town_df <- data.frame(Type = houseprice$Town_type)</pre>
town_dummies <- dummy.data.frame(town_df)
town_dummies <- town_dummies[,-c(3)]
fit.2 = Im(Sale_amount~.,data = town_dummies)
summary(fit.2)
##將 year_type 轉為 dummy variables,並試著做迴歸分析
houseprice$year_type = as.factor(as.character(houseprice$year_type))
year_df <- data.frame(Type = houseprice$year_type)</pre>
year_dummies <- dummy.data.frame(year_df)
year_dummies <- year_dummies[,-c(3)]</pre>
fit.3 = Im(Sale_amount~.,data = year_dummies)
summary(fit.3)
#將 dummy variables 合併到原本的 houseprice 表中
houseprice_final <- cbind(houseprice, type_dummies, town_dummies, year_dummies)
houseprice_final <- houseprice_final[-c(1, 3, 8:13)]
attach(houseprice_final)
#觀察連續型 x 變數之間的關係,以及連續型 x 對 y 變數的相關性
pairs(houseprice_final[,c(1:5)])
library(corrplot)
cor=cor(houseprice_final[,c(1:5)])
cor
```

summary(fit.1)

```
#將 Sqft_lot 變數刪除
houseprice_final <- houseprice_final[-c(5)]
attach(houseprice_final)
#將前 8000 筆資料切為訓練集,其餘為測試集
train=houseprice_final[1:8000,]
test=houseprice_final[8001:10658,]
### stepwise
full <- Im(Sale_amount~.,data=train)</pre>
glance(full) %>% select(AIC,BIC)
null <-lm(Sale_amount~1,data=train)</pre>
#AIC
step(full, direction="backward")
step(null, scope=list(lower=null, upper=full), direction="forward")
#BIC
step(full, direction="backward", criterion = "BIC")
#做迴歸分析
model <- Im(Sale_amount ~ Beds + Baths + Sqft_home +
       Typeeast + Typemiddle + Typecentennial + Typenew, data = train)
summary(model)
#加入交互作用項 Beds:Sqft_home
model_1 <- Im(Sale_amount ~ Beds + Baths + Sqft_home +Beds:Sqft_home +
       Typeeast + Typemiddle + Typecentennial + Typenew, data = train)
summary(model_1)
anova(model, model_1)
```

```
#加入交互作用項 Beds:Baths
model_2 <- Im(Sale_amount ~ Beds + Baths + Sqft_home + Beds:Sqft_home +
        Beds:Baths + Typeeast + Typemiddle + Typecentennial + Typenew, data = train)
summary(model_2)
anova(model_1, model_2)
#加入交互作用項 Baths:Sqft_home
model_3 <- Im(Sale_amount ~ Beds + Baths + Sqft_home + Beds:Sqft_home +
        Baths:Sqft_home + Typeeast + Typemiddle + Typecentennial + Typenew, data =
train)
summary(model_3)
anova(model_1, model_3)
#發現有兩筆離群值,將其刪除
c= cooks.distance(model_3) #>=1, might be outlier, can remove
plot(c)
which(c>1)
houseprice_final <- houseprice_final[-c(1120,1128),]
attach(houseprice_final)
#做 AIC、BIC、RMSE 測試
train=houseprice_final[1:8000,]
test=houseprice_final[8001:10656,]
AIC(model, k = 2)
AIC(model_3, k = 2)
BIC(model)
BIC(model_3)
predict1 = predict(model,test)
predict2 = predict(model_3,test)
test1 <- cbind(test, predict1)
```

test2 <- cbind(test, predict2)

 $RMSE1 = sqrt(mean(sum((test \$Sale_amount-predict 1)^2)))$

 $RMSE2 = sqrt(mean(sum((test \$Sale_amount-predict2)^2)))$

RMSE1

RMSE2