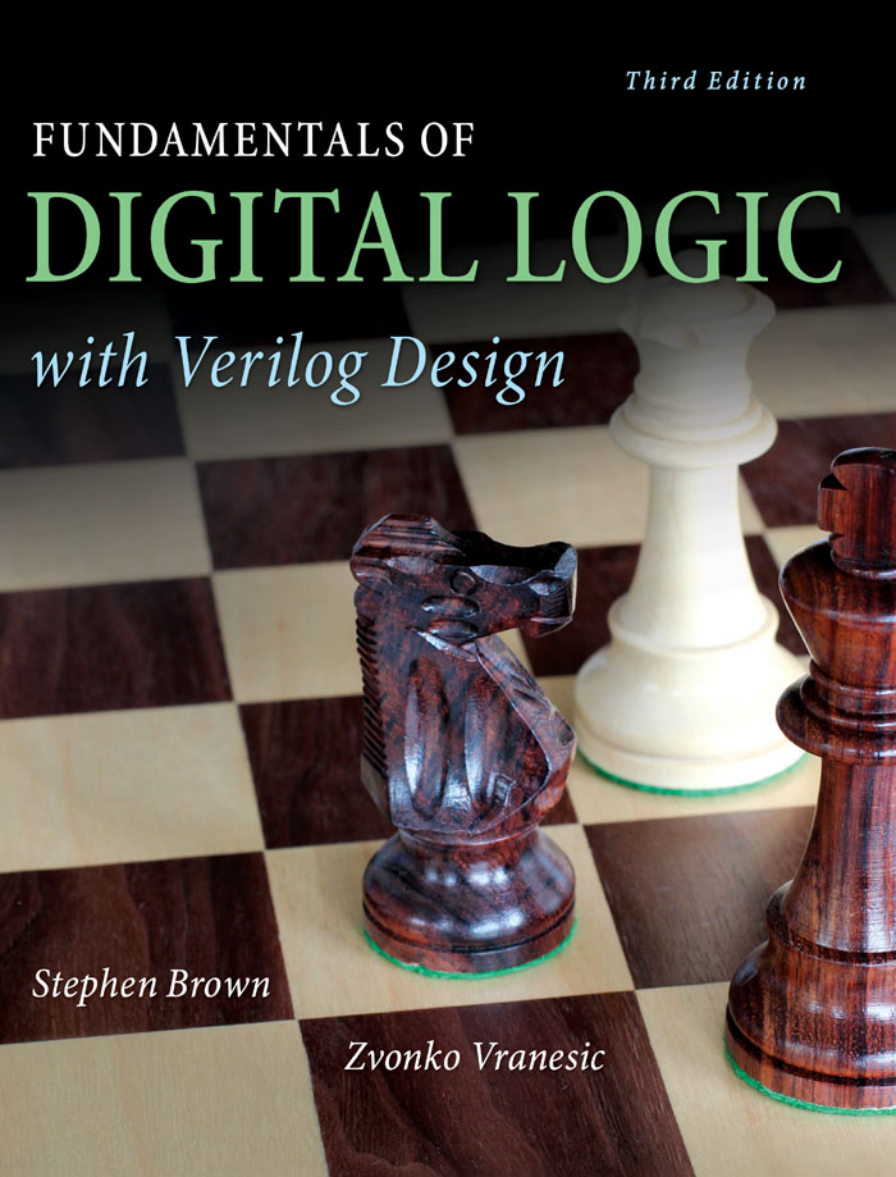
Third Edition
FUNDAMENTALS OF
DIGITAL LOGIC
with Verilog Design
Stephen Brown
Zvonko Vranesic

FUNDAMENTALS OF DIGITAL LOGIC WITH VERILOG DESIGN

THIRD EDITION

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University of Toronto

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To Susan and Anne

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About the Authors

Stephen Brown received his B.A.Sc. degree in Electrical Engineering from the University of New Brunswick, Canada, and the M.A.Sc. and Ph.D. degrees in Electrical Engineering from the University of Toronto. He joined the University of Toronto faculty in 1992, where he is now a Professor in the Department of Electrical & Computer Engineering. He is also the Director of the worldwide University Program at Altera Corporation.

His research interests include field-programmable VLSI technology and computer architecture. He won the Canadian Natural Sciences and Engineering Research Council's 1992 Doctoral Prize for the best Ph.D. thesis in Canada and has published more than 100 scientific research papers.

He has won five awards for excellence in teaching electrical engineering, computer engineering, and computer science courses. He is a coauthor of two other books: *Fundamentals of Digital Logic with VHDL Design*, 3rd ed. and *Field-Programmable Gate Arrays*.

Zvonko Vranesic received his B.A.Sc., M.A.Sc., and Ph.D. degrees, all in Electrical Engineering, from the University of Toronto. From 1963–1965 he worked as a design engineer with the Northern Electric Co. Ltd. in Bramalea, Ontario. In 1968 he joined the University of Toronto, where he is now a Professor Emeritus in the Departments of Electrical & Computer Engineering and Computer Science. During the 1978–1979 academic year, he was a Senior Visitor at the University of Cambridge, England, and during 1984–1985 he was at the University of Paris, 6. From 1995 to 2000 he served as Chair of the Division of Engineering Science at the University of Toronto. He is also involved in research and development at the Altera Toronto Technology Center.

His current research interests include computer architecture and field-programmable VLSI technology.

He is a coauthor of four other books: *Computer Organization and Embedded Systems*, 6th ed.; *Fundamentals of Digital Logic with VHDL Design*, 3rd ed.; *Microcomputer Structures*; and *Field-Programmable Gate Arrays*. In 1990, he received the Wighton Fellowship for "innovative and distinctive contributions to undergraduate laboratory instruction." In 2004, he received the Faculty Teaching Award from the Faculty of Applied Science and Engineering at the University of Toronto.

He has represented Canada in numerous chess competitions. He holds the title of International Master.

PREFACE

This book is intended for an introductory course in digital logic design, which is a basic course in most electrical and computer engineering programs. A successful designer of digital logic circuits needs a good understanding of basic concepts and a firm grasp of the modern design approach that relies on computer-aided design (CAD) tools.

The main goals of the book are (1) to teach students the fundamental concepts in classical manual digital design and (2) illustrate clearly the way in which digital circuits are designed today, using CAD tools. Even though modern designers no longer use manual techniques, except in rare circumstances, our motivation for teaching such techniques is to give students an intuitive feeling for how digital circuits operate. Also, the manual techniques provide an illustration of the types of manipulations performed by CAD tools, giving students an appreciation of the benefits provided by design automation. Throughout the book, basic concepts are introduced by way of examples that involve simple circuit designs, which we perform using both manual techniques and modern CAD-tool-based methods. Having established the basic concepts, more complex examples are then provided, using the CAD tools. Thus our emphasis is on modern design methodology to illustrate how digital design is carried out in practice today.

TECHNOLOGY

The book discusses modern digital circuit implementation technologies. The emphasis is on programmable logic devices (PLDs), which is the most appropriate technology for use in a textbook for two reasons. First, PLDs are widely used in practice and are suitable for almost all types of digital circuit designs. In fact, students are more likely to be involved in PLD-based designs at some point in their careers than in any other technology. Second, circuits are implemented in PLDs by end-user programming. Therefore, students can be provided with an opportunity, in a laboratory setting, to implement the book's design examples in actual chips. Students can also simulate the behavior of their designed circuits on their own computers. We use the two most popular types of PLDs for targeting of designs: complex programmable logic devices (CPLDs) and field-programmable gate arrays (FPGAs).

We emphasize the use of a hardware description language in specifying the logic circuits, because the HDL-based approach is the most efficient design method to use in practice. We describe in detail the IEEE Standard Verilog HDL language and use it extensively in examples.

SCOPE OF THE BOOK

This edition of the book has been extensively restructured. All of the material that should be covered in a one-semester course is now included in Chapters 1 to 6. More advanced material is presented in Chapters 7 to 11.

Chapter 1 provides a general introduction to the process of designing digital systems. It discusses the key steps in the design process and explains how CAD tools can be used to automate many of the required tasks. It also introduces the representation of digital information.

Chapter 2 introduces the logic circuits. It shows how Boolean algebra is used to represent such circuits. It introduces the concepts of logic circuit synthesis and optimization, and shows how logic gates are used to implement simple circuits. It also gives the reader a first glimpse at Verilog, as an example of a hardware description language that may be used to specify the logic circuits.

Chapter 3 concentrates on circuits that perform arithmetic operations. It discusses numbers and shows how they can be manipulated using logic circuits. This chapter illustrates how Verilog can be used to specify the desired functionality and how CAD tools provide a mechanism for developing the required circuits.

Chapter 4 presents combinational circuits that are used as building blocks. It includes the encoder, decoder, and multiplexer circuits. These circuits are very convenient for illustrating the application of many Verilog constructs, giving the reader an opportunity to discover more advanced features of Verilog.

Storage elements are introduced in Chapter 5. The use of flip-flops to realize regular structures, such as shift registers and counters, is discussed. Verilog-specified designs of these structures are included.

Chapter 6 gives a detailed presentation of synchronous sequential circuits (finite state machines). It explains the behavior of these circuits and develops practical design techniques for both manual and automated design.

Chapter 7 is a discussion of a number of practical issues that arise in the design of real systems. It highlights problems often encountered in practice and indicates how they can be overcome. Examples of larger circuits illustrate a hierarchical approach in designing digital systems. Complete Verilog code for these circuits is presented.

Chapter 8 deals with more advanced techniques for optimized implementation of logic functions. It presents algorithmic techniques for optimization. It also explains how logic functions can be specified using a cubical representation as well as using binary decision diagrams.

Asynchronous sequential circuits are discussed in Chapter 9. While this treatment is not exhaustive, it provides a good indication of the main characteristics of such circuits. Even though the asynchronous circuits are not used extensively in practice, they provide an excellent vehicle for gaining a deeper understanding of the operation of digital circuits in general. They illustrate the consequences of propagation delays and race conditions that may be inherent in the structure of a circuit.

Chapter 10 presents a complete CAD flow that the designer experiences when designing, implementing, and testing a digital circuit.

Chapter 11 introduces the topic of testing. A designer of logic circuits has to be aware of the need to test circuits and should be conversant with at least the most basic aspects of testing.

Appendix A provides a complete summary of Verilog features. Although use of Verilog is integrated throughout the book, this appendix provides a convenient reference that the reader can consult from time to time when writing Verilog code.

The electronic aspects of digital circuits are presented in Appendix B. This appendix shows how the basic gates are built using transistors and presents various factors that affect circuit performance. The emphasis is on the latest technologies, with particular focus on CMOS technology and programmable logic devices.

WHAT CAN BE COVERED IN A COURSE

Much of the material in the book can be covered in 2 one-quarter courses. A good coverage of the most important material can be achieved in a single one-semester, or even a one-quarter course. This is possible only if the instructor does not spend too much time teaching the intricacies of Verilog and CAD tools. To make this approach possible, we organized the Verilog material in a modular style that is conducive to self-study. Our experience in teaching different classes of students at the University of Toronto shows that the instructor may spend only three to four lecture hours on Verilog, describing how the code should be structured, including the use of design hierarchy, using scalar and vector variables, and on the style of code needed to specify sequential circuits. The Verilog examples given in the book are largely self-explanatory, and students can understand them easily.

The book is also suitable for a course in logic design that does not include exposure to Verilog. However, some knowledge of Verilog, even at a rudimentary level, is beneficial to the students, and it is a great preparation for a job as a design engineer.

One-Semester Course

The following material should be covered in lectures:

- Chapter 1—all sections.
- Chapter 2—all sections.
- Chapter 3—Sections 3.1 to 3.5.
- Chapter 4—all sections.
- Chapter 5—all sections.
- Chapter 6—all sections.

One-Quarter Course

In a one-quarter course the following material can be covered:

- Chapter 1—all sections.
- Chapter 2—all sections.

- Chapter 3—Sections 3.1 to 3.3 and Section 3.5.
- Chapter 4—all sections.
- Chapter 5—all sections.
- Chapter 6—Sections 6.1 to 6.4.

VERILOG

Verilog is a complex language, which some instructors feel is too hard for beginning students to grasp. We fully appreciate this issue and have attempted to solve it. It is not necessary to introduce the entire Verilog language. In the book we present the important Verilog constructs that are useful for the design and synthesis of logic circuits. Many other language constructs, such as those that have meaning only when using the language for simulation purposes, are omitted. The Verilog material is introduced gradually, with more advanced features being presented only at points where their use can be demonstrated in the design of relevant circuits.

The book includes more than 120 examples of Verilog code. These examples illustrate how Verilog is used to describe a wide range of logic circuits, from those that contain only a few gates to those that represent digital systems such as a simple processor.

All of the examples of Verilog code presented in the book are provided on the Authors' website at

www.eecg.toronto.edu/~brown/Verilog_3e

SOLVED PROBLEMS

The chapters include examples of solved problems. They show how typical homework problems may be solved.

HOMEWORK PROBLEMS

More than 400 homework problems are provided in the book. Answers to selected problems are given at the back of the book. Solutions to all problems are available to instructors in the *Solutions Manual* that accompanies the book.

POWERPOINT SLIDES AND SOLUTIONS MANUAL

PowerPoint slides that contain all of the figures in the book are available on the Authors' website. Instructors can request access to these slides, as well as access to the Solutions Manual for the book, at:

www.mhhe.com/brownvranesic

CAD Tools

Modern digital systems are quite large. They contain complex logic circuits that would be difficult to design without using good CAD tools. Our treatment of Verilog should enable the reader to develop Verilog code that specifies logic circuits of varying degrees of complexity. To gain proper appreciation of the design process, it is highly beneficial to implement the designs using commercially-available CAD tools. Some excellent CAD tools are available free of charge. For example, the Altera Corporation has its Quartus II CAD software, which is widely used for implementing designs in programmable logic devices such as FPGAs. The Web Edition of the Quartus II software can be downloaded from Altera's website and used free of charge, without the need to obtain a license. In previous editions of this book a set of tutorials for using the Quartus II software was provided in the appendices. Those tutorials can now be found on the Authors' website. Another set of useful tutorials about Quartus II can be found on Altera's University Program website, which is located at www.altera.com/education/univ.

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Stephen Brown and Zvonko Vranesic

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chapter

1

INTRODUCTION

CHAPTER OBJECTIVES

In this chapter you will be introduced to:

- Digital hardware components
- An overview of the design process
- Binary numbers
- Digital representation of information

This book is about logic circuits—the circuits from which computers are built. Proper understanding of logic circuits is vital for today's electrical and computer engineers. These circuits are the key ingredient of computers and are also used in many other applications. They are found in commonly-used products like music and video players, electronic games, digital watches, cameras, televisions, printers, and many household appliances, as well as in large systems, such as telephone networks, Internet equipment, television broadcast equipment, industrial control units, and medical instruments. In short, logic circuits are an important part of almost all modern products.

The material in this book will introduce the reader to the many issues involved in the design of logic circuits. It explains the key ideas with simple examples and shows how complex circuits can be derived from elementary ones. We cover the classical theory used in the design of logic circuits because it provides the reader with an intuitive understanding of the nature of such circuits. But, throughout the book, we also illustrate the modern way of designing logic circuits using sophisticated *computer aided design (CAD)* software tools. The CAD methodology adopted in the book is based on the industry-standard design language called the *Verilog hardware description language*. Design with Verilog is first introduced in Chapter 2, and usage of Verilog and CAD tools is an integral part of each chapter in the book.

Logic circuits are implemented electronically, using transistors on an integrated circuit chip. Commonly available chips that use modern technology may contain more than a billion transistors, as in the case of some computer processors. The basic building blocks for such circuits are easy to understand, but there is nothing simple about a circuit that contains billions of transistors. The complexity that comes with large circuits can be handled successfully only by using highly-organized design techniques. We introduce these techniques in this chapter, but first we briefly describe the hardware technology used to build logic circuits.

1.1 DIGITAL HARDWARE

Logic circuits are used to build computer hardware, as well as many other types of products. All such products are broadly classified as *digital hardware*. The reason that the name *digital* is used will be explained in Section 1.5—it derives from the way in which information is represented in computers, as electronic signals that correspond to digits of information.

The technology used to build digital hardware has evolved dramatically over the past few decades. Until the 1960s logic circuits were constructed with bulky components, such as transistors and resistors that came as individual parts. The advent of integrated circuits made it possible to place a number of transistors, and thus an entire circuit, on a single chip. In the beginning these circuits had only a few transistors, but as the technology improved they became more complex. Integrated circuit chips are manufactured on a silicon wafer, such as the one shown in Figure 1.1. The wafer is cut to produce the individual chips, which are then placed inside a special type of chip package. By 1970 it was possible to implement all circuitry needed to realize a microprocessor on a single chip. Although early microprocessors had modest computing capability by today's standards, they opened the door for the information processing revolution by providing the means for implementation of affordable personal computers.

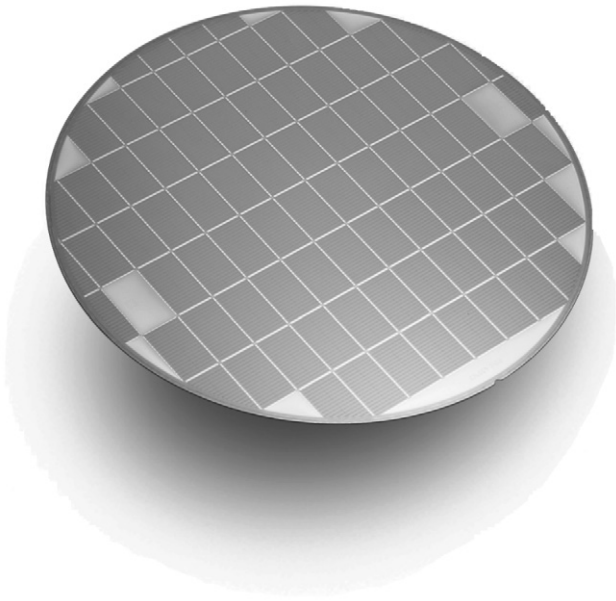

Figure 1.1 A silicon wafer (courtesy of Altera Corp.).

About 30 years ago Gordon Moore, chairman of Intel Corporation, observed that integrated circuit technology was progressing at an astounding rate, approximately doubling the number of transistors that could be placed on a chip every two years. This phenomenon, informally known as *Moore's law*, continues to the present day. Thus in the early 1990s microprocessors could be manufactured with a few million transistors, and by the late 1990s it became possible to fabricate chips that had tens of millions of transistors. Presently, chips can be manufactured containing billions of transistors.

Moore's law is expected to continue to hold true for a number of years. A consortium of integrated circuit associations produces a forecast of how the technology is expected to evolve. Known as the *International Technology Roadmap for Semiconductors (ITRS)* [1], this forecast discusses many aspects of technology, including the maximum number of transistors that can be manufactured on a single chip. A sample of data from the ITRS is given in Figure 1.2. It shows that chips with about 10 million transistors could be successfully manufactured in 1995, and this number has steadily increased, leading to today's chips with over a billion transistors. The roadmap predicts that chips with as many as 100 billion transistors will be possible by the year 2022. There is no doubt that this technology will have a huge impact on all aspects of people's lives.

The designer of digital hardware may be faced with designing logic circuits that can be implemented on a single chip or designing circuits that involve a number of chips placed on a *printed circuit board (PCB)*. Frequently, some of the logic circuits can be realized

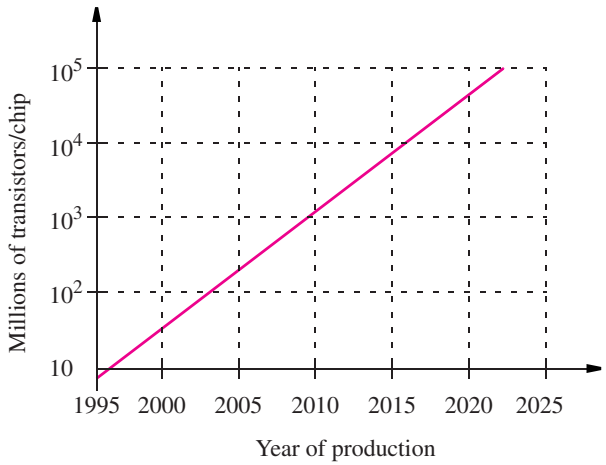

Figure 1.2 An estimate of the maximum number of transistors per chip over time.

in existing chips that are readily available. This situation simplifies the design task and shortens the time needed to develop the final product. Before we discuss the design process in detail, we should introduce the different types of integrated circuit chips that may be used.

There exists a large variety of chips that implement various functions that are useful in the design of digital hardware. The chips range from simple ones with low functionality to extremely complex chips. For example, a digital hardware product may require a microprocessor to perform some arithmetic operations, memory chips to provide storage capability, and interface chips that allow easy connection to input and output devices. Such chips are available from various vendors.

For many digital hardware products, it is also necessary to design and build some logic circuits from scratch. For implementing these circuits, three main types of chips may be used: standard chips, programmable logic devices, and custom chips. These are discussed next.

1.1.1 STANDARD CHIPS

Numerous chips are available that realize some commonly-used logic circuits. We will refer to these as *standard chips*, because they usually conform to an agreed-upon standard in terms of functionality and physical configuration. Each standard chip contains a small amount of circuitry (usually involving fewer than 100 transistors) and performs a simple function. To build a logic circuit, the designer chooses the chips that perform whatever functions are needed and then defines how these chips should be interconnected to realize a larger logic circuit.

Standard chips were popular for building logic circuits until the early 1980s. However, as integrated circuit technology improved, it became inefficient to use valuable space on PCBs for chips with low functionality. Another drawback of standard chips is that the functionality of each chip is fixed and cannot be changed.

1.1.2 PROGRAMMABLE LOGIC DEVICES

In contrast to standard chips that have fixed functionality, it is possible to construct chips that contain circuitry which can be configured by the user to implement a wide range of different logic circuits. These chips have a very general structure and include a collection of *programmable switches* that allow the internal circuitry in the chip to be configured in many different ways. The designer can implement whatever functions are required for a particular application by setting the programmable switches as needed. The switches are programmed by the end user, rather than when the chip is manufactured. Such chips are known as *programmable logic devices (PLDs)*.

PLDs are available in a wide range of sizes, and can be used to implement very large logic circuits. The most commonly-used type of PLD is known as a *field-programmable gate array (FPGA)*. The largest FPGAs contain billions of transistors [2, 3], and support the implementation of complex digital systems. An FPGA consists of a large number of small logic circuit elements, which can be connected together by using programmable switches in the FPGA. Because of their high capacity, and their capability to be tailored to meet the requirements of a specific application, FPGAs are widely used today.

1.1.3 CUSTOM-DESIGNED CHIPS

FPGAs are available as off-the-shelf components that can be purchased from different suppliers. Because they are programmable, they can be used to implement most logic circuits found in digital hardware. However, they also have a drawback in that the programmable switches consume valuable chip area and limit the speed of operation of implemented circuits. Thus in some cases FPGAs may not meet the desired performance or cost objectives. In such situations it is possible to design a chip from scratch; namely, the logic circuitry that must be included on the chip is designed first and then the chip is manufactured by a company that has the fabrication facilities. This approach is known as *custom* or *semi-custom design*, and such chips are often called *application-specific integrated circuits (ASICs)*.

The main advantage of a custom chip is that its design can be optimized for a specific task; hence it usually leads to better performance. It is possible to include a larger amount of logic circuitry in a custom chip than would be possible in other types of chips. The cost of producing such chips is high, but if they are used in a product that is sold in large quantities, then the cost per chip, amortized over the total number of chips fabricated, may be lower than the total cost of off-the-shelf chips that would be needed to implement the same function(s). Moreover, if a single chip can be used instead of multiple chips to achieve the same goal, then a smaller area is needed on a PCB that houses the chips in the final product. This results in a further reduction in cost.

A disadvantage of the custom-design approach is that manufacturing a custom chip often takes a considerable amount of time, on the order of months. In contrast, if an FPGA can be used instead, then the chips are programmed by the end user and no manufacturing delays are involved.

1.2 THE DESIGN PROCESS

The availability of computer-based tools has greatly influenced the design process in a wide variety of environments. For example, designing an automobile is similar in the general approach to designing a furnace or a computer. Certain steps in the development cycle must be performed if the final product is to meet the specified objectives.

The flowchart in Figure 1.3 depicts a typical development process. We assume that the process is to develop a product that meets certain expectations. The most obvious requirements are that the product must function properly, that it must meet an expected level of performance, and that its cost should not exceed a given target.

The process begins with the definition of product specifications. The essential features of the product are identified, and an acceptable method of evaluating the implemented features in the final product is established. The specifications must be tight enough to ensure that the developed product will meet the general expectations, but should not be unnecessarily constraining (that is, the specifications should not prevent design choices that may lead to unforeseen advantages).

From a complete set of specifications, it is necessary to define the general structure of an initial design of the product. This step is difficult to automate. It is usually performed by a human designer because there is no clear-cut strategy for developing a product's overall structure—it requires considerable design experience and intuition.

After the general structure is established, CAD tools are used to work out the details. Many types of CAD tools are available, ranging from those that help with the design of individual parts of the system to those that allow the entire system's structure to be represented in a computer. When the initial design is finished, the results must be verified against the original specifications. Traditionally, before the advent of CAD tools, this step involved constructing a physical model of the designed product, usually including just the key parts. Today it is seldom necessary to build a physical model. CAD tools enable designers to simulate the behavior of incredibly complex products, and such simulations are used to determine whether the obtained design meets the required specifications. If errors are found, then appropriate changes are made and the verification of the new design is repeated through simulation. Although some design flaws may escape detection via simulation, usually all but the most subtle problems are discovered in this way.

When the simulation indicates that the design is correct, a complete physical prototype of the product is constructed. The prototype is thoroughly tested for conformance with the specifications. Any errors revealed in the testing must be fixed. The errors may be minor, and often they can be eliminated by making small corrections directly on the prototype of the product. In case of large errors, it is necessary to redesign the product and repeat the steps explained above. When the prototype passes all the tests, then the product is deemed to be successfully designed and it can go into production.

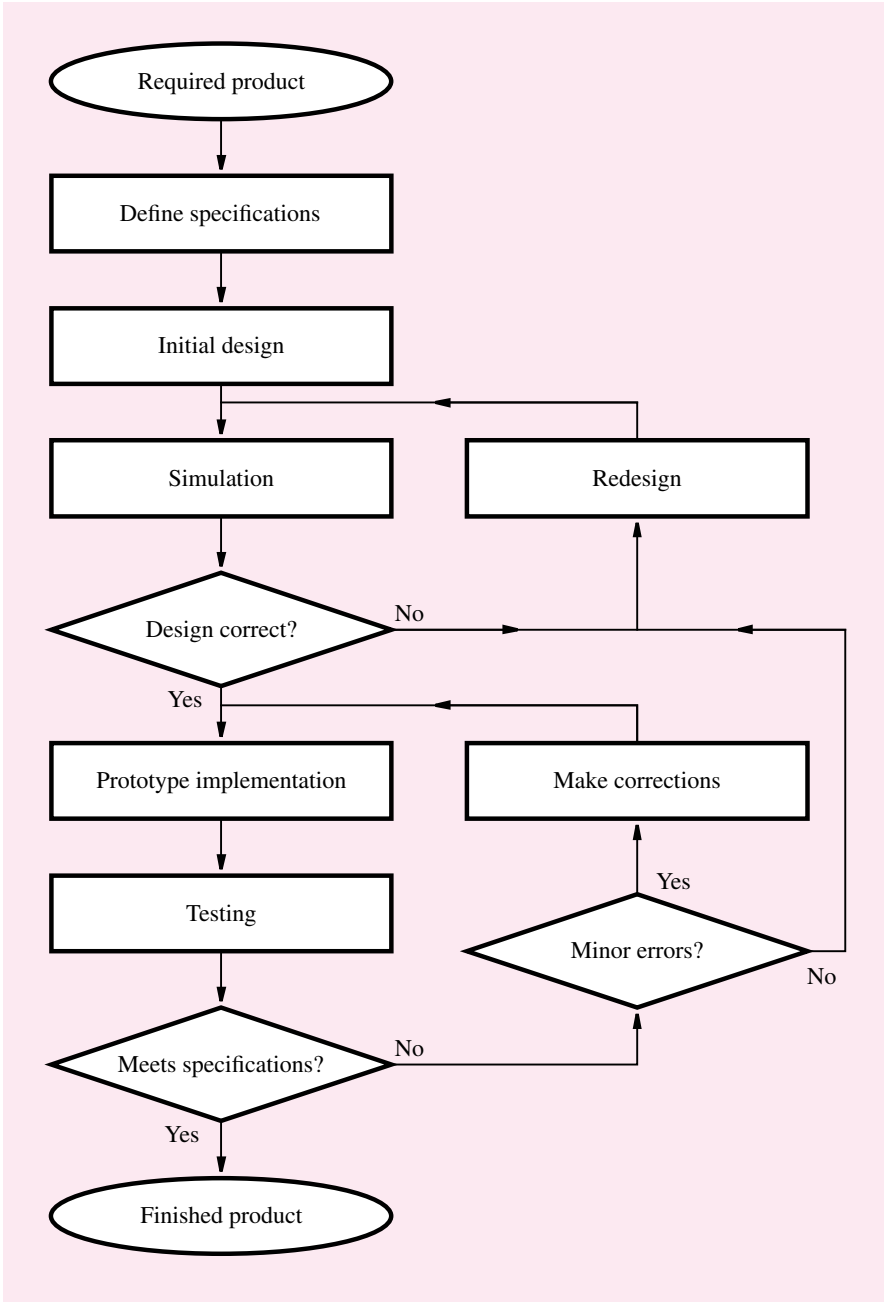

Figure 1.3 The development process.

1.3 STRUCTURE OF A COMPUTER

To understand the role that logic circuits play in digital systems, consider the structure of a typical computer, as illustrated in Figure 1.4*a*. The computer case houses a number of printed circuit boards (PCBs), a power supply, and (not shown in the figure) storage units, like a hard disk and DVD or CD-ROM drives. Each unit is plugged into a main PCB, called the *motherboard*. As indicated on the bottom of the figure, the motherboard holds several integrated circuit chips, and it provides slots for connecting other PCBs, such as audio, video, and network boards.

Figure 1.4*b* illustrates the structure of an integrated circuit chip. The chip comprises a number of subcircuits, which are interconnected to build the complete circuit. Examples of subcircuits are those that perform arithmetic operations, store data, or control the flow of data. Each of these subcircuits is a logic circuit. As shown in the middle of the figure, a logic circuit comprises a network of connected *logic gates*. Each logic gate performs a very simple function, and more complex operations are realized by connecting gates together. Logic gates are built with transistors, which in turn are implemented by fabricating various layers of material on a silicon chip.

This book is primarily concerned with the center portion of Figure 1.4*b*—the design of logic circuits. We explain how to design circuits that perform important functions, such as adding, subtracting, or multiplying numbers, counting, storing data, and controlling the processing of information. We show how the behavior of such circuits is specified, how the circuits are designed for minimum cost or maximum speed of operation, and how the circuits can be tested to ensure correct operation. We also briefly explain how transistors operate, and how they are built on silicon chips.

1.4 LOGIC CIRCUIT DESIGN IN THIS BOOK

In this book we use a modern design approach based on the Verilog hardware description language and CAD tools to illustrate many aspects of logic circuit design. We selected this technology because it is widely used in industry and because it enables the readers to implement their designs in FPGA chips, as discussed below. This technology is particularly well-suited for educational purposes because many readers have access to facilities for using CAD tools and programming FPGA devices.

To gain practical experience and a deeper understanding of logic circuits, we advise the reader to implement the examples in this book using CAD software. Most of the major vendors of CAD systems provide their software at no cost to university students for educational use. Some examples are Altera, Cadence, Mentor Graphics, Synopsys, and Xilinx. The CAD systems offered by any of these companies can be used equally well with this book. Two CAD systems that are particularly well-suited for use with this book are the Quartus II software from Altera and the ISE software from Xilinx. Both of these CAD systems support all phases of the design cycle for logic circuits and are powerful and easy to use. The reader is encouraged to visit the website for these companies, where

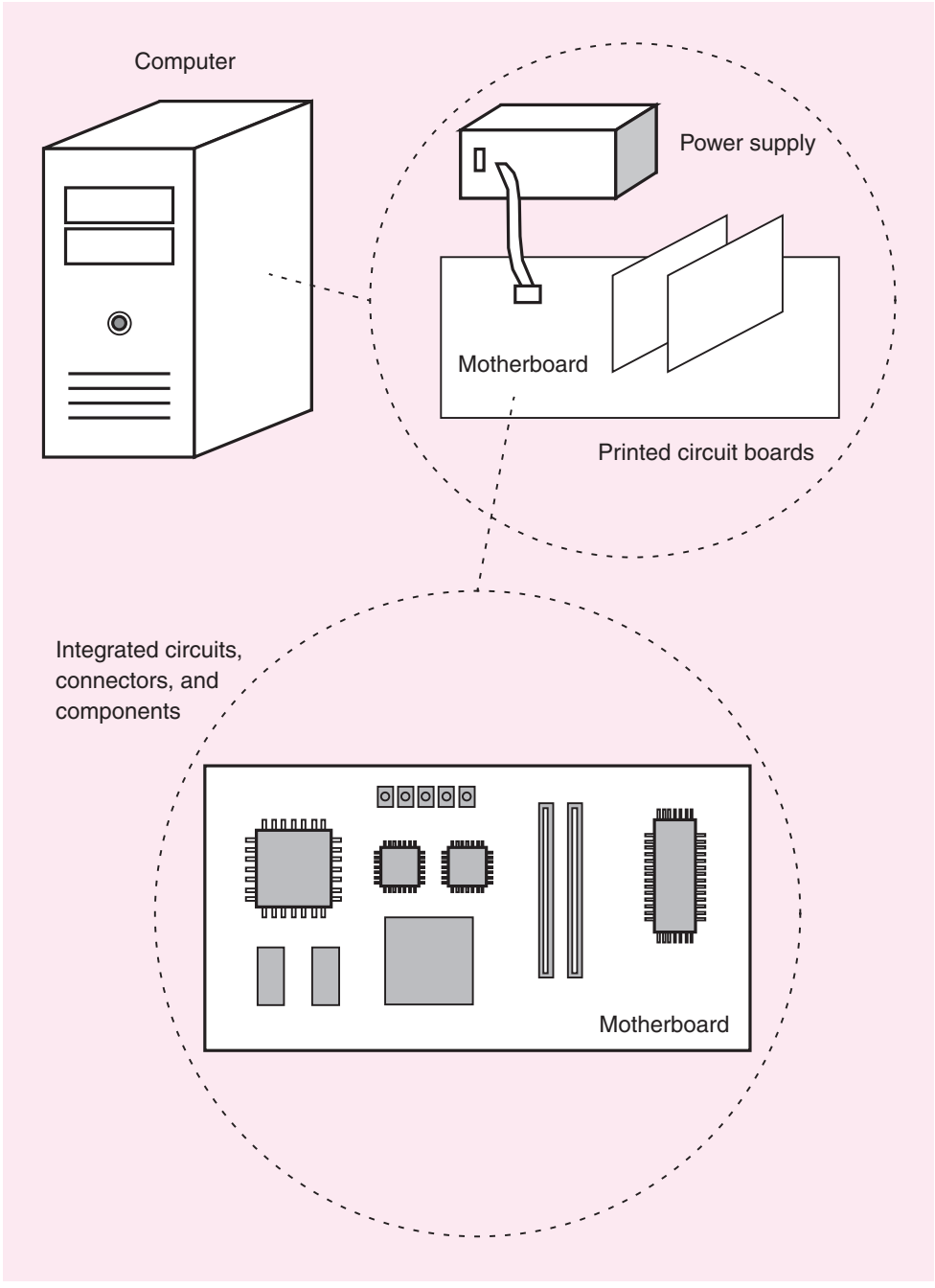

Figure 1.4 A digital hardware system (Part *a*).

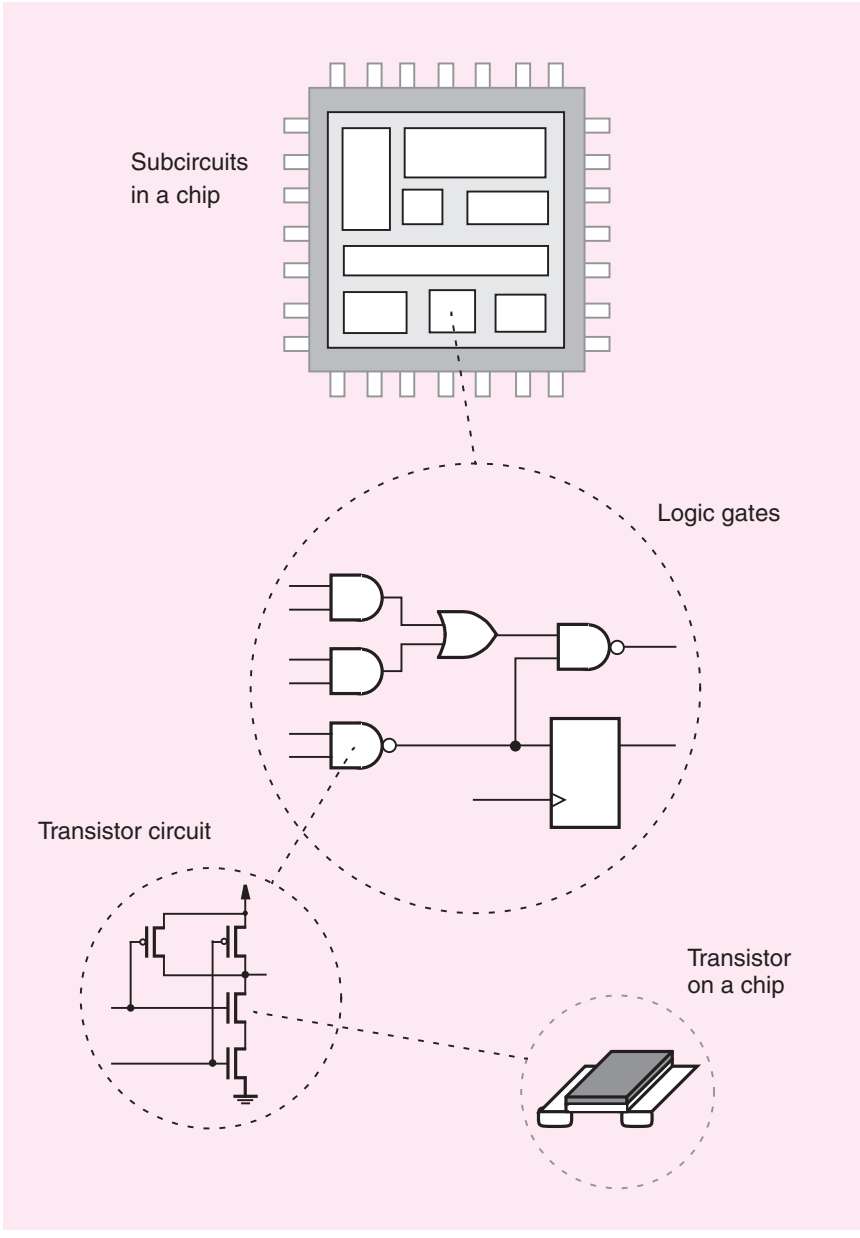

Figure 1.4 A digital hardware system (Part *b*).

the software tools and tutorials that explain their use can be downloaded and installed onto any personal computer.

To facilitate experimentation with logic circuits, some FPGA manufacturers provide special PCBs that include one or more FPGA chips and an interface to a personal computer.

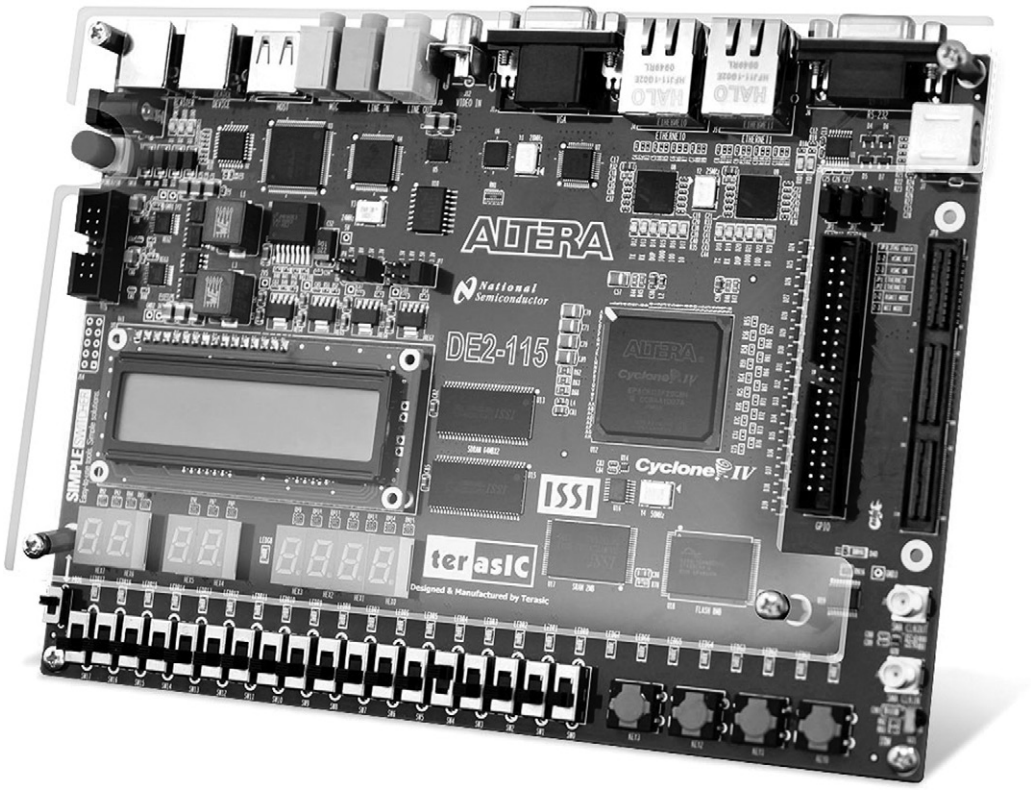

Figure 1.5 An FPGA board.

Once a logic circuit has been designed using the CAD tools, the circuit can be *programmed* into an FPGA on the board. Inputs can then be applied to the FPGA by way of switches and other devices, and the generated outputs can be examined. An example of such a board is depicted in Figure 1.5. This type of board is an excellent vehicle for learning about logic circuits, because it provides a collection of simple input and output devices. Many illustrative experiments can be carried out by designing and implementing logic circuits using the FPGA chip on the board.

1.5 DIGITAL REPRESENTATION OF INFORMATION

In Section 1.1 we mentioned that information is represented in logic circuits as electronic signals. Each of these signals can be thought of as representing one *digit* of information. To make the design of logic circuits easier, each digit is allowed to take on only two possible values, usually denoted as 0 and 1. These logic values are implemented as voltage levels in a circuit; the value 0 is usually represented as 0 V (ground), and the value 1 is the voltage

level of the circuit's power supply. As we discuss in Appendix B, typical power-supply voltages in logic circuits range from 1 V DC to 5 V DC.

In general, all information in logic circuits is represented as combinations of 0 and 1 digits. Before beginning our discussion of logic circuits in Chapter 2, it will be helpful to examine how numbers, alphanumeric data (text), and other information can be represented using the digits 0 and 1.

1.5.1 BINARY NUMBERS

In the familiar decimal system, a number consists of digits that have 10 possible values, from 0 to 9, and each digit represents a multiple of a power of 10. For example, the number 8547 represents $8 \times 10^3 + 5 \times 10^2 + 4 \times 10^1 + 7 \times 10^0$. We do not normally write the powers of 10 with the number, because they are implied by the positions of the digits. In general, a decimal integer is expressed by an n-tuple comprising n decimal digits

$$D = d_{n-1}d_{n-2} \cdots d_1 d_0$$

which represents the value

$$V(D) = d_{n-1} \times 10^{n-1} + d_{n-2} \times 10^{n-2} + \cdots + d_1 \times 10^1 + d_0 \times 10^0$$

This is referred to as the *positional number representation*.

Because the digits have 10 possible values and each digit is weighted as a power of 10, we say that decimal numbers are *base*-10 numbers. Decimal numbers are familiar, convenient, and easy to understand. However, since digital circuits represent information using only the values 0 and 1, it is not practical to have digits that can assume ten values. In these circuits it is more appropriate to use the binary, or *base*-2, system which has only the digits 0 and 1. Each binary digit is called a *bit*. In the binary number system, the same positional number representation is used so that

$$B = b_{n-1}b_{n-2} \cdots b_1 b_0$$

represents an integer that has the value

$$\begin{aligned} V(B) &= b_{n-1} \times 2^{n-1} + b_{n-2} \times 2^{n-2} + \cdots + b_1 \times 2^1 + b_0 \times 2^0 \\ &= \sum_{i=0}^{n-1} b_i \times 2^i \end{aligned} \qquad \textbf{[1.1]}$$

For example, the binary number 1101 represents the value

$$V = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

Because a particular digit pattern has different meanings for different bases, we will indicate the base as a subscript when there is potential for confusion. Thus to specify that 1101 is a base-2 number, we will write $(1101)_2$. Evaluating the preceding expression for V gives $V = 8 + 4 + 1 = 13$. Hence

$$(1101)_2 = (13)_{10}$$

Table 1.1 Numbers in decimal and binary.

Decimal representation	Binary representation
00	0000
01	0001
02	0010
03	0011
04	0100
05	0101
06	0110
07	0111
08	1000
09	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

The range of integers that can be represented by a binary number depends on the number of bits used. Table 1.1 lists the first 15 positive integers and shows their binary representations using four bits. An example of a larger number is $(10110111)_2 = (183)_{10}$. In general, using n bits allows representation of positive integers in the range 0 to $2^n - 1$.

In a binary number the right-most bit is usually referred to as the *least-significant bit (LSB)*. The left-most bit, which has the highest power of 2 associated with it, is called the *most-significant bit (MSB)*. In digital systems it is often convenient to consider several bits together as a group. A group of four bits is called a *nibble*, and a group of eight bits is called a *byte*.

1.5.2 CONVERSION BETWEEN DECIMAL AND BINARY SYSTEMS

A binary number is converted into a decimal number simply by applying Equation 1.1 and evaluating it using decimal arithmetic. Converting a decimal number into a binary number is not quite as straightforward, because we need to construct the number by using powers of 2. For example, the number $(17)_{10}$ is $2^4 + 2^0 = (10001)_2$, and the number $(50)_{10}$ is $2^5 + 2^4 + 2^1 = (110010)_2$. In general, the conversion can be performed by successively dividing the decimal number by 2 as follows. Suppose that a decimal number $D = d_{k-1} \cdots d_1 d_0$, with a value V, is to be converted into a binary number $B = b_{n-1} \cdots b_2 b_1 b_0$. Then, we can write V in the form

$$V = b_{n-1} \times 2^{n-1} + \cdots + b_2 \times 2^2 + b_1 \times 2^1 + b_0$$

Convert $(857)_{10}$

			Remainder	
$857 \div 2$	=	428	1	LSB
$428 \div 2$	=	214	0	
$214 \div 2$	=	107	0	
$107 \div 2$	=	53	1	
$53 \div 2$	=	26	1	
$26 \div 2$	=	13	0	
$13 \div 2$	=	6	1	
$6 \div 2$	=	3	0	
$3 \div 2$	=	1	1	
$1 \div 2$	=	0	1	MSB

Result is $(1101011001)_2$

Figure 1.6 Conversion from decimal to binary.

If we now divide V by 2, the result is

$$\frac{V}{2} = b_{n-1} \times 2^{n-2} + \cdots + b_2 \times 2^1 + b_1 + \frac{b_0}{2}$$

The quotient of this integer division is $b_{n-1} \times 2^{n-2} + \cdots + b_2 \times 2 + b_1$, and the remainder is b_0. If the remainder is 0, then $b_0 = 0$; if it is 1, then $b_0 = 1$. Observe that the quotient is just another binary number, which comprises $n - 1$ bits, rather than n bits. Dividing this number by 2 yields the remainder b_1. The new quotient is

$$b_{n-1} \times 2^{n-3} + \cdots + b_2$$

Continuing the process of dividing the new quotient by 2, and determining one bit in each step, will produce all bits of the binary number. The process continues until the quotient becomes 0. Figure 1.6 illustrates the conversion process, using the example $(857)_{10} = (1101011001)_2$. Note that the least-significant bit (LSB) is generated first and the most-significant bit (MSB) is generated last.

So far, we have considered only the representation of positive integers. In Chapter 3 we will complete the discussion of number representation by explaining how negative numbers are handled and how fixed-point and floating-point numbers may be represented. We will also explain how arithmetic operations are performed in computers.

1.5.3 ASCII CHARACTER CODE

Alphanumeric information, such as letters and numbers typed on a computer keyboard, is represented as codes consisting of 0 and 1 digits. The most common code used for this type of information is known as the *ASCII code*, which stands for the American Standard Code for Information Interchange. The code specified by this standard is presented in Table 1.2.

Table 1.2 The seven-bit ASCII code.

Bit positions	Bit positions 654							
3210	000	001	010	011	100	101	110	111
0000	NUL	DLE	SPACE	0	@	P	´	p
0001	SOH	DC1	!	1	A	Q	a	q
0010	STX	DC2	”	2	B	R	b	r
0011	ETX	DC3	#	3	C	S	c	s
0100	EOT	DC4	$	4	D	T	d	t
0101	ENQ	NAK	%	5	E	U	e	u
0110	ACK	SYN	&	6	F	V	f	v
0111	BEL	ETB	’	7	G	W	g	w
1000	BS	CAN	(	8	H	X	h	x
1001	HT	EM	)	9	I	Y	i	y
1010	LF	SUB	*	:	J	Z	j	z
1011	VT	ESC	+	;	K	[	k	{
1100	FF	FS	,	<	L	\	1	|
1101	CR	GS	-	=	M	]	m	}
1110	SO	RS	.	>	N	^	n	~
1111	SI	US	/	?	O	—	o	DEL

NUL	Null/Idle	SI	Shift in
SOH	Start of header	DLE	Data link escape
STX	Start of text	DC1-DC4	Device control
ETX	End of text	NAK	Negative acknowledgement
EOT	End of transmission	SYN	Synchronous idle
ENQ	Enquiry	ETB	End of transmitted block
ACQ	Acknowledgement	CAN	Cancel (error in data)
BEL	Audible signal	EM	End of medium
BS	Back space	SUB	Special sequence
HT	Horizontal tab	ESC	Escape
LF	Line feed	FS	File separator
VT	Vertical tab	GS	Group separator
FF	Form feed	RS	Record separator
CR	Carriage return	US	Unit separator
SO	Shift out	DEL	Delete/Idle

Bit positions of code format = | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

The ASCII code uses seven-bit patterns to denote 128 different characters. Ten of the characters are decimal digits 0 to 9. As the table shows, the high-order bits have the same pattern, $b_6b_5b_4 = 011$, for all 10 digits. Each digit is identified by the low-order four bits, b_{3-0}, using the binary patterns for these digits. Capital and lowercase letters are encoded in a way that makes sorting of textual information easy. The codes for A to Z are in ascending numerical sequence, which means that the task of sorting letters (or words) can be accomplished by a simple arithmetic comparison of the codes that represent the letters.

In addition to codes that represent characters and letters, the ASCII code includes punctuation marks such as ! and ?, commonly used symbols such as & and %, and a collection of control characters. The control characters are those needed in computer systems to handle and transfer data among various devices. For example, the carriage return character, which is abbreviated as CR in the table, indicates that the carriage, or cursor position, of an output device, such as a printer or display, should return to the left-most column.

The ASCII code is used to encode information that is handled as text. It is not convenient for representation of numbers that are used as operands in arithmetic operations. For this purpose, it is best to convert ASCII-encoded numbers into a binary representation that we discussed before.

The ASCII standard uses seven bits to encode a character. In computer systems a more natural size is eight bits, or one byte. There are two common ways of fitting an ASCII-encoded character into a byte. One is to set the eighth bit, b_7, to 0. Another is to use this bit to indicate the *parity* of the other seven bits, which means showing whether the number of 1s in the seven-bit code is even or odd. We discuss parity in Chapter 4.

1.5.4 DIGITAL AND ANALOG INFORMATION

Binary numbers can be used to represent many types of information. For example, they can represent music that is stored in a personal music player. Figure 1.7 illustrates a music player, which contains an electronic memory for storing music files. A music file comprises a sequence of binary numbers that represent tones. To convert these binary numbers into sound, a *digital-to-analog (D/A) converter* circuit is used. It converts digital values into corresponding voltage levels, which create an analog voltage signal that drives the speakers inside the headphones. The binary values stored in the music player are referred to as *digital* information, whereas the voltage signal that drives the speakers is *analog* information.

1.6 THEORY AND PRACTICE

Modern design of logic circuits depends heavily on CAD tools, but the discipline of logic design evolved long before CAD tools were invented. This chronology is quite obvious because the very first computers were built with logic circuits, and there certainly were no computers available on which to design them!

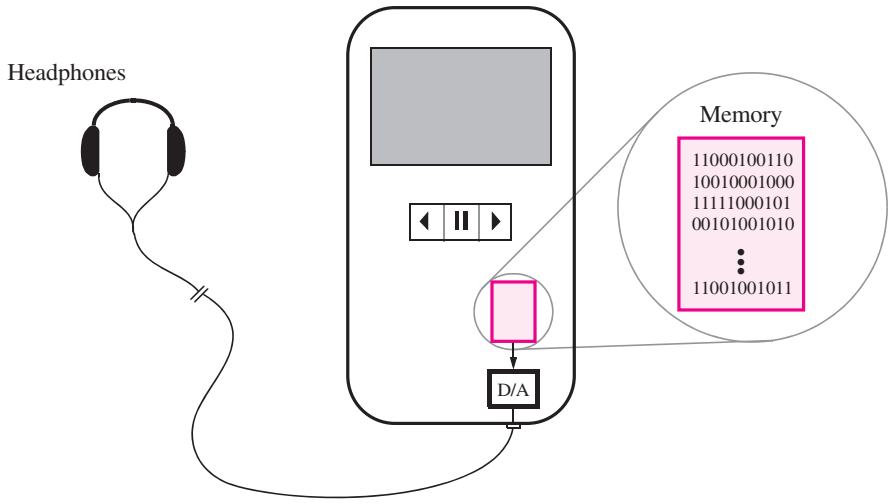

Figure 1.7 Using digital technology to represent music.

Numerous manual design techniques have been developed to deal with logic circuits. Boolean algebra, which we will introduce in Chapter 2, was adopted as a mathematical means for representing such circuits. An enormous amount of "theory" was developed showing how certain design issues may be treated. To be successful, a designer had to apply this knowledge in practice.

CAD tools not only made it possible to design incredibly complex circuits but also made the design work much simpler in general. They perform many tasks automatically, which may suggest that today's designer need not understand the theoretical concepts used in the tasks performed by CAD tools. An obvious question would then be, Why should one study the theory that is no longer needed for manual design? Why not simply learn how to use the CAD tools?

There are three big reasons for learning the relevant theory. First, although the CAD tools perform the automatic tasks of optimizing a logic circuit to meet particular design objectives, the designer has to give the original description of the logic circuit. If the designer specifies a circuit that has inherently bad properties, then the final circuit will also be of poor quality. Second, the algebraic rules and theorems for design and manipulation of logic circuits are directly implemented in today's CAD tools. It is not possible for a user of the tools to understand what the tools do without grasping the underlying theory. Third, CAD tools offer many optional processing steps that a user can invoke when working on a design. The designer chooses which options to use by examining the resulting circuit produced by the CAD tools and deciding whether it meets the required objectives. The only way that the designer can know whether or not to apply a particular option in a given situation is to know what the CAD tools will do if that option is invoked—again, this implies that the designer must be familiar with the underlying theory. We discuss the logic circuit theory extensively in this book, because it is not possible to become an effective logic circuit designer without understanding the fundamental concepts.

There is another good reason to learn some logic circuit theory even if it were not required for CAD tools. Simply put, it is interesting and intellectually challenging. In the modern world filled with sophisticated automatic machinery, it is tempting to rely on tools as a substitute for thinking. However, in logic circuit design, as in any type of design process, computer-based tools are not a substitute for human intuition and innovation. Computer-based tools can produce good digital hardware designs only when employed by a designer who thoroughly understands the nature of logic circuits.

PROBLEMS

Answers to problems marked by an asterisk are given at the back of the book.

*1.1 Convert the following decimal numbers into binary, using the method shown in Figure 1.6.
(a) $(20)_{10}$
(b) $(100)_{10}$
(c) $(129)_{10}$
(d) $(260)_{10}$
(e) $(10240)_{10}$

1.2 Convert the following decimal numbers into binary, using the method shown in Figure 1.6.
(a) $(30)_{10}$
(b) $(110)_{10}$
(c) $(259)_{10}$
(d) $(500)_{10}$
(e) $(20480)_{10}$

1.3 Convert the following decimal numbers into binary, using the method shown in Figure 1.6.
(a) $(1000)_{10}$
(b) $(10000)_{10}$
(c) $(100000)_{10}$
(c) $(1000000)_{10}$

*1.4 In Figure 1.6 we show how to convert a decimal number into binary by successively dividing by 2. Another way to derive the answer is to constuct the number by using powers of 2. For example, if we wish to convert the number $(23)_{10}$, then the largest power of 2 that is not larger than 23 is $2^4 = 16$. Hence, the binary number will have five bits and the most-significant bit is $b_4 = 1$. We then perform the subtraction $23 - 16 = 7$. Now, the largest power of 2 that is not larger than 7 is $2^2 = 4$. Hence, $b_3 = 0$ (because $2^3 = 8$ is larger than 7) and $b_2 = 1$. Continuing this process gives

$$\begin{aligned} 23 &= 16 + 4 + 2 + 1 \\ &= 2^4 + 2^2 + 2^1 + 2^0 \\ &= 10000 + 00100 + 00010 + 00001 \\ &= 10111 \end{aligned}$$

Using this method, convert the following decimal numbers into binary.
(a) $(17)_{10}$
(b) $(33)_{10}$
(c) $(67)_{10}$
(d) $(130)_{10}$
(e) $(2560)_{10}$
(f) $(51200)_{10}$

1.5 Repeat Problem 3 using the method described in Problem 4.

***1.6** Convert the following binary numbers into decimal.
(a) $(1001)_2$
(b) $(11100)_2$
(c) $(111111)_2$
(d) $(101010101010)_2$

1.7 Convert the following binary numbers into decimal.
(a) $(110010)_2$
(b) $(1100100)_2$
(c) $(11001000)_2$
(d) $(110010000)_2$

***1.8** What is the minimum number of bits needed to represent the following decimal numbers in binary?
(a) $(270)_{10}$
(b) $(520)_{10}$
(c) $(780)_{10}$
(d) $(1029)_{10}$

1.9 Repeat Problem 8 for the following decimal numbers:
(a) $(111)_{10}$
(b) $(333)_{10}$
(c) $(555)_{10}$
(d) $(1111)_{10}$

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