Throughout assume $g, h \in G$, an abelian group, and that the order of g is 300. Do the following with reasons.

1. The order of g^{720}.
2. The smallest n such that S_n has an element of the same order as g.
3. The number of subgroups of $\langle g \rangle$ of order 30.
4. The number of elements of $\langle g \rangle$ of order 30.
5. Given that h is of order 400, the largest possible order of an element in G (as far as you know).

Bonus. An element of that largest order as in 5.

Solution.

1. $\left| g^{720} = \dfrac{300}{\gcd(300, 720)} \right| = 5.$
2. $300 = 4 \cdot 3 \cdot 25$, so that $n = 4 + 3 + 25 = 32$.
3. There is only 1 subgroup of $\langle g \rangle$ of order 30.
4. The number of elements of $\langle g \rangle$ of order 30 is

$$\phi(30) = \phi(2 \cdot 3 \cdot 5) = \phi(2)\phi(3)\phi(5) = 8.$$

5. The largest possible order is $\operatorname{lcm}(400, 300) = 1200$.

Bonus. Since $\gcd(|g^{240}|, |h|) = 1$ it follows that the $|g^{240}h| = |g^{240}| \cdot |h| = 1200$.