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Mathematics
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Consider the polynomial $p(x) = x^3 + x^2 + x + 1$. In each of the following situations, decide whether it is irreducible or not. If not, factor it as much as you can.

1. $p(x) \in \mathbb{Z}[x]$.
2. $p(x) \in \mathbb{Z}_2[x]$.
3. $p(x) \in \mathbb{Z}_3[x]$.
4. $p(x) \in \mathbb{R}[x]$.
5. $p(x) \in \mathbb{C}[x]$.

Solution.

1. In $\mathbb{Z}$, observe that $x = -1$ is a root of $p(x)$, so that $x + 1$ is a factor of $p(x)$. Thus it follows that
$$p(x) = (x + 1)(x^2 + 1).$$
2. In $\mathbb{Z}_2[x]$, $x = -1$ is a root of $x^2 + 1$. Thus we have that $p(x) = (x + 1)^3$.
3. In $\mathbb{Z}_3[x]$, $x^2 + 1$ has no roots. Thus
$$p(x) = (x + 1)(x^2 + 1).$$
4. As is the case with 1 and 3, we have that
$$p(x) = (x + 1)(x^2 + 1)$$
in $\mathbb{R}[x]$.
5. In $\mathbb{C}[x]$, it follows that
$$p(x) = (x + 1)(x - i)(x + i).$$