6.6 Prove that if V is a real inner-product space, then

$$\langle u, v\rangle = \frac{||u+v||^2 - ||u-v||^2}{4}$$

for all $u.v \in V$.

Proof.

6.10 On $\mathcal{P}_2(\mathbb{R})$, consider the inner product given by

$$\langle p, q\rangle = \int_0^1 p(x)q(x)dx.$$

Apply the Gram-Schmidt procedure to the basis $(1, x, x^2)$ to produce an orthonormal basis of $\mathcal{P}_2(\mathbb{R})$.

Solution.

6.13 Suppose $(e_1, \ldots, e_m)$ is an orthonormal list of vectors in V. Let $v \in V$. Prove that

$$||v||^2 = |\langle v, e_1\rangle|^2 + \cdots + |\langle v, e_m\rangle|^2$$

if and only if $v \in \operatorname{span}(e_1, \ldots, e_m)$.

Proof.

6.17 Prove that if $P \in \mathcal{L}(V)$ is such that $P^2 = P$ and every vector in null P is orthogonal to every vector in range P, then P is an orthogonal projection.

Proof.

6.29 Suppose $T \in \mathcal{L}(\mathcal{V})$ and U is a subspace of V. Prove that U is invariant under T if and only if $U^\perp$ is invariant under $T*$.

Proof.