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MATHEMATICS
CAL STATE LONG BEACH

1. Let f be the function defined by $f(x) = x^3 - 49x^2 + 623x - 2015$, and let $g(x) = f(x+5)$. Compute the sum of the roots of g.

   Solution. Using the Rational Root Theorem, we get the following factorization

   $$f(x) = (x-5)(x-13)(x-31),$$

   so that the roots of f are 5, 13, and 31. Now observe that c is a root of f if and only if $c-5$ is a root of g; thus the sum of the roots of g is $(5-5)+(13-5)+(31-5) = 34$.

2. Lay six long strings in parallel. Two people (each of whom cannot see what the other is doing) work at opposite ends of the bundle of strings. Each randomly pairs off the strings at his or her end and glues together the ends of each pair. Then stretch the strings out. What is the probability that the strings now form a single large loop?

   Solution.

3. Compute the product $\cos\left(\frac{\pi}{5}\right)\cos\left(\frac{2\pi}{5}\right)\cos\left(\frac{3\pi}{5}\right)\cos\left(\frac{4\pi}{5}\right)$.

   Solution.