# 5 Chapter 5

## 5.1 Section 1

5.01 If $f(x) = |x^3|$, find $f'(x)$.

**Solution.** It is clear that $f'(x) = 3x^2$ if $x > 0$ and $f'(x) = -3x^2$ if $x < 0$. Now we have that

$$f'(0) = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0} \frac{|h^3|}{h}.$$

And since

$$\lim_{h\to 0^+} \frac{|h^3|}{h} = \lim_{h\to 0^+} \frac{h^3}{h} = 0 = \lim_{h\to 0^-} \frac{-h^3}{h} = \lim_{h\to 0^-} \frac{|h^3|}{h},$$

it follows that $f'(0) = 0$. Thus $f'(x) = |3x^2|$.

5.02 Let $f(x) = x|x|$; show that

$$f''(x) = \begin{cases} 2 & \text{if } x > 0 \\ -2 & \text{if } x < 0 \end{cases}$$

and that 0 is not in the domain of $f''(x)$.

**Proof.** From example 5.1, we know that $f'(x) = 2|x|$. It is clear what $f''(x)$ is when $x$ is nonzero, so it suffices to show that $f''(0)$ is undefined. To that end, we have that

$$f''(0) = \lim_{h\to 0} \frac{f'(0+h) - f'(0)}{h} = \lim_{h\to 0} \frac{2|h|}{h},$$

which does not exist since the one-sided limits are not equal. Thus 0 is not in the domain of $f''$.

5.03 Find $f'(x)$ if

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 2 \\ 4x - 4 & \text{if } x < 2 \end{cases}$$

**Solution.** It is clear that $f'(x) = 2x$ if $x > 2$ and that $f'(x) = 4$ if $x < 2$, so we only need to investigate if $f'(2)$ exists. To that end, we have that

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h\to 0} \frac{f(2+h) - 4}{h}. \end{aligned}$$

Since

$$\lim_{h\to 0^-} \frac{f(2+h) - 4}{h} = \lim_{h\to 0^-} \frac{4(2+h) - 4 - 4}{h} = 4,$$

and

$$\lim_{h\to 0^+} \frac{f(2+h) - 4}{h} = \lim_{h\to 0^+} \frac{(2+h)^2 - 4}{h} = \lim_{h\to 0^+} \frac{h^2 + 4h}{h} = 4,$$

it follows that $f'(2) = 4$. Thus

$$f'(x) = \begin{cases} 2x & \text{if } x \geq 2 \\ 4 & \text{if } x < 2 \end{cases}$$