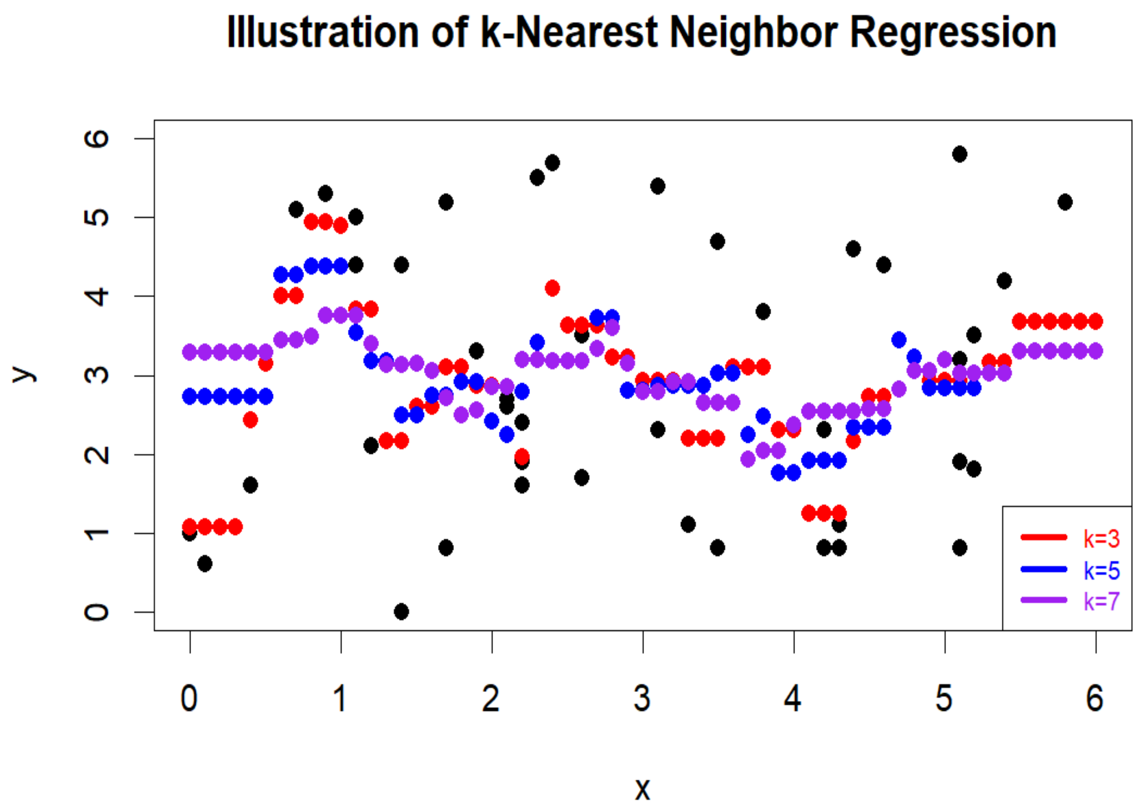
# K-NEAREST NEIGHBOR REGRESSION AND CLASSIFICATION

For regression, the **k Nearest-neighbor (kNN) algorithm** works as follows: for any point (on a grid) in the space of predictor variables, we find $k$ nearest neighbors using the regular Euclidean distance. The **Euclidean distance** between two $d$-dimensional vectors $\mathtt{v} = (v_1, \ldots, v_d)$ and $\mathtt{w} = (w_1, \ldots, w_d)$ is defined as $distance(\mathtt{v}, \mathtt{w}) = \sqrt{(v_1 - w_1)^2 + (v_2 - w_2)^2 + \cdots + (v_d - w_d)^2}$. The predicted value of $y$ for this fixed point on a grid is determined as the average of the target values of the $k$ nearest neighbors.

We illustrate how kNN regression works on a sample set of points with one predictor variable $x$ and a target variable $y$. The black points in figure below represent the observed data on a scatterplot.

We consider each point on a grid between 0 and 6 with a step size of 0.1 and compute the mean values of $y$ for $k = 3, 5,$ and 7 nearest neighbors. The predicted values are shown as colored dots in the figure below.

Note that as $k$ increases, the predicted values approach to a horizontal line. Indeed if $k$ is very large, all points are considered as nearest neighbors for every grid point, resulting in a single predicted value equal to the mean of all observed $y$-values.



Here is the R code that produced this figure.

```
sampledots.reg<- read.csv(file="./SamplePoints_KNN_reg.csv", sep=",", header=TRUE)

plot(sampledots.reg$x, sampledots.reg$y, xlim=c(0,6), ylim=c(0,6), xlab="x",
ylab="y", main="Illustration of k-Nearest Neighbor Regression", pch=16)

for (grid in seq(0, 6, 0.1)) {
distance<- c()

for (z in 1:length(sampledots.reg$x))
 distance[z]<- abs(grid - sampledots.reg$x[z])

min.dist3<- sort(distance)[1:3] # 3 nearest neighbors
y.pred3<- mean(sampledots.reg$y[which(distance %in% min.dist3)])
points(grid,y.pred3,col="red", pch=16)

min.dist5<- sort(distance)[1:5] # 5 nearest neighbors
y.pred5<- mean(sampledots.reg$y[which(distance %in% min.dist5)])
points(grid,y.pred5,col="blue", pch=16)

min.dist7<- sort(distance)[1:7] # 7 nearest neighbors
y.pred7<- mean(sampledots.reg$y[which(distance %in% min.dist7)])
points(grid,y.pred7,col="purple", pch=16)

}
legend("bottomright",c("k=3","k=5","k=7"),lty=1,lwd=3,col=c("red","blue","purple"),
cex = 0.7, text.col=c("red","blue","purple"))
```

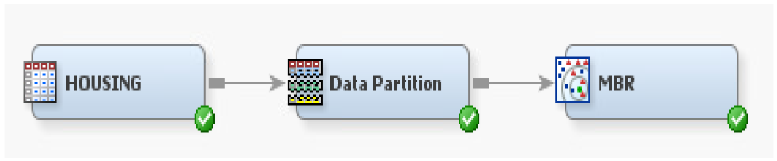
**Historical Note:** The kNN algorithm was first described in "Discriminatory Analysis. Nonparametric Discrimination: Consistency Properties", by Evelyn Fix and Joseph Hodges, report, UC Berkeley, 1951.

**Example.** We apply the kNN algorithm to build a regression for the data set in the file "housing_data.csv".

In SAS: We save the data file in the sasuser library using the following code.

```
proc import out=sasuser.housing datafile="./housing_data.csv" dbms=csv replace;
run;
```

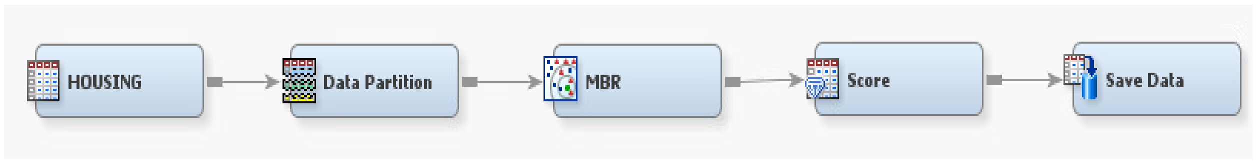
Then we use SAS Enterprise miner to fit a $k$ nearest-neighbor regression (termed **Memory-Based Reasoning (MBR)**, using the path diagram:



For the "Data Partition" node, we specify to split the data into 70% training, 10% validation, and 20% testing sets. The testing set contains 569 observations, the validation test contains 284 observations, and the training set contains 1989 observations. In the MBR node, we specify the number of neighbors: $569/5 = 113.8 \approx 114$ for $k = 5$ classes, $569/6 = 94.8 \approx= 95$ for $k = 6$ classes, etc. We run the paths and note the value for the MSE for the testing set (summarized in the table below).

| **Number of classes** | **Number of neighbors** | **MSE** |
|---|---|---|
| 5 | 114 | 6918425992 |
| 6 | 95 | 6830858760 |
| 7 | 81 | 6719387249 |
| 8 | 71 | 6646008646 |
| **9** | **63** | **6605133365** |
| 10 | 57 | 6586164839 |

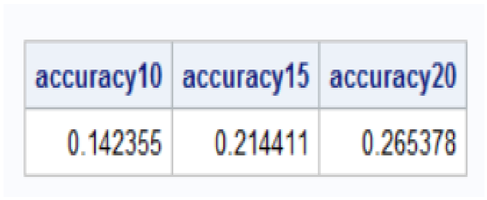
We can see that MSE starts leveling out at $k = 9$, so we pick that number of classes and run the full path that includes scoring of the testing set. The diagram is given in the following figure:



Next, we locate the file with predictions "./Workspaces/EMWS1/EMSave/em_save_test.sas7bdat", open it in SAS (in the library "Tmp1") and run the code below to compute the proportion of predictions with 10%, 15%, and 20% of the actual values, and plot the actual and predicted values.

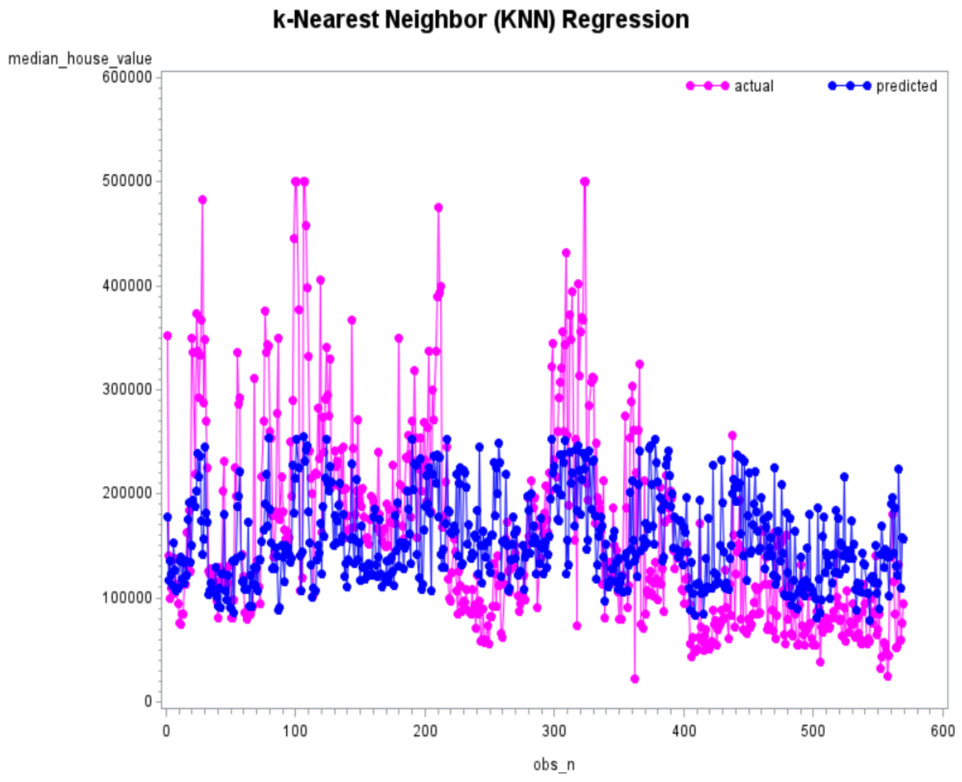
```
/*COMPUTING ACCURACY WITHIN 10%, 15%, AND 20%*/
data accuracy;
```

```
set tmp1.em_save_test;
ind10=(abs(R_median_house_value)<0.10*median_house_value);
ind15=(abs(R_median_house_value)<0.15*median_house_value);
ind20=(abs(R_median_house_value)<0.20*median_house_value);
obs_n=_N_;
run;

proc sql;
select mean(ind10) as accuracy10,
mean(ind15) as accuracy15, mean(ind20) as
accuracy20
from accuracy;
quit;
```



| accuracy10 | accuracy15 | accuracy20 |
|---|---|---|
| 0.142355 | 0.214411 | 0.265378 |

```
/*PLOTTING ACTUAL AND PREDICTED VALUES FOR TESTING DATA*/;
goptions reset=all border;
title1 "k-Nearest Neighbor (KNN) Regression";
symbol1 interpol=join value=dot color=magenta;
symbol2 interpol=join value=dot color=blue;
legend1 value=("actual" "predicted")
position=(top right inside) label=none;
proc gplot data=accuracy;
plot median_house_value*obs_n
EM_PREDICTION*obs_n/ overlay legend=legend1;
run;
```



In R:

```
housing.data<- read.csv(file="./housing_data.csv", header=TRUE, sep=",")

#SPLITTING DATA INTO 80% TRAINING AND 20% TESTING SETS
set.seed(880352)
sample <- sample(c(TRUE, FALSE), nrow(housing.data), replace=TRUE, prob=c(0.8,0.2))
train<- housing.data[sample,]
test<- housing.data[!sample,]

train.x<- data.matrix(train[-8])
train.y<- data.matrix(train[8])
test.x<- data.matrix(test[-8])
test.y<- data.matrix(test[8])

#TRAINING K-NEAREST NEIGHBOR REGRESSION
install.packages("caret") #Classification and Regression Training
library(caret)
print(train(median_house_value~ ., data=train, method="knn"))
```

```
  k  RMSE
  5  81040.04
  7  79199.11
  9  78165.81

RMSE was used to select the optimal model using the smallest value.
The final value used for the model was k = 9.
```

```
#FITTING OPTIMAL KNN REGRESSION (K=9)
knn.reg<- knnreg(train.x, train.y, k=9)

#COMPUTING PREDICTION ACCURACY FOR TESTING DATA
pred.y<- predict(knn.reg, test.x)

#accuracy within 10%
accuracy10<- ifelse(abs(test.y-pred.y)<0.10*test.y,1,0)
print(mean(accuracy10))
```

```
0.1372881
```

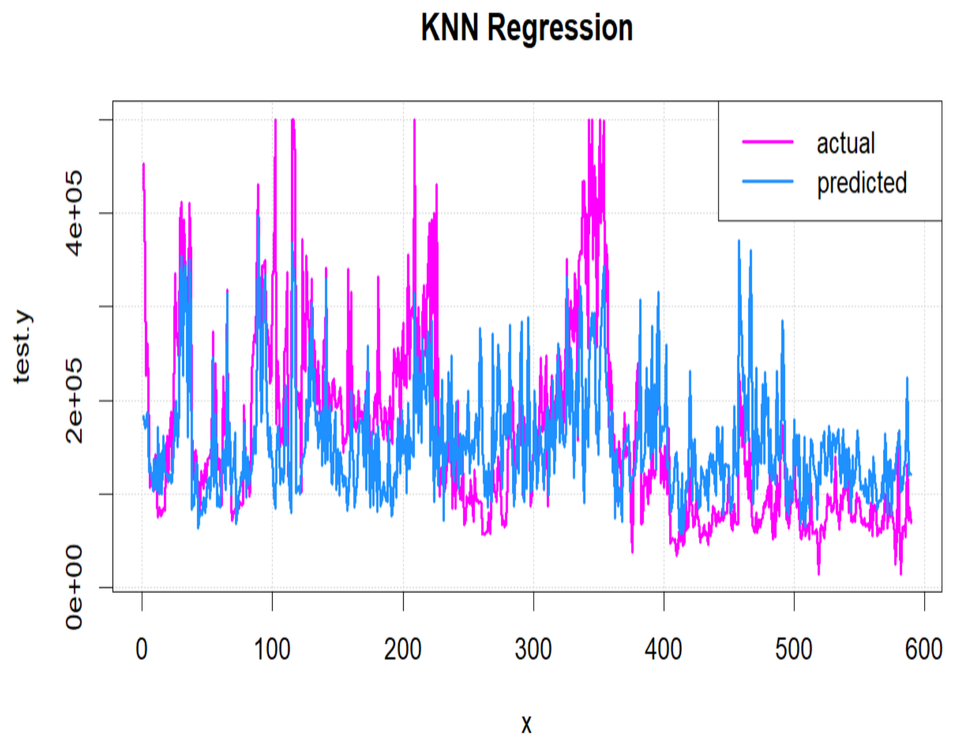
```
#accuracy within 15%
accuracy15<- ifelse(abs(test.y-pred.y)<0.15*test.y,1,0)
print(mean(accuracy15))
```

```
0.1830508
```

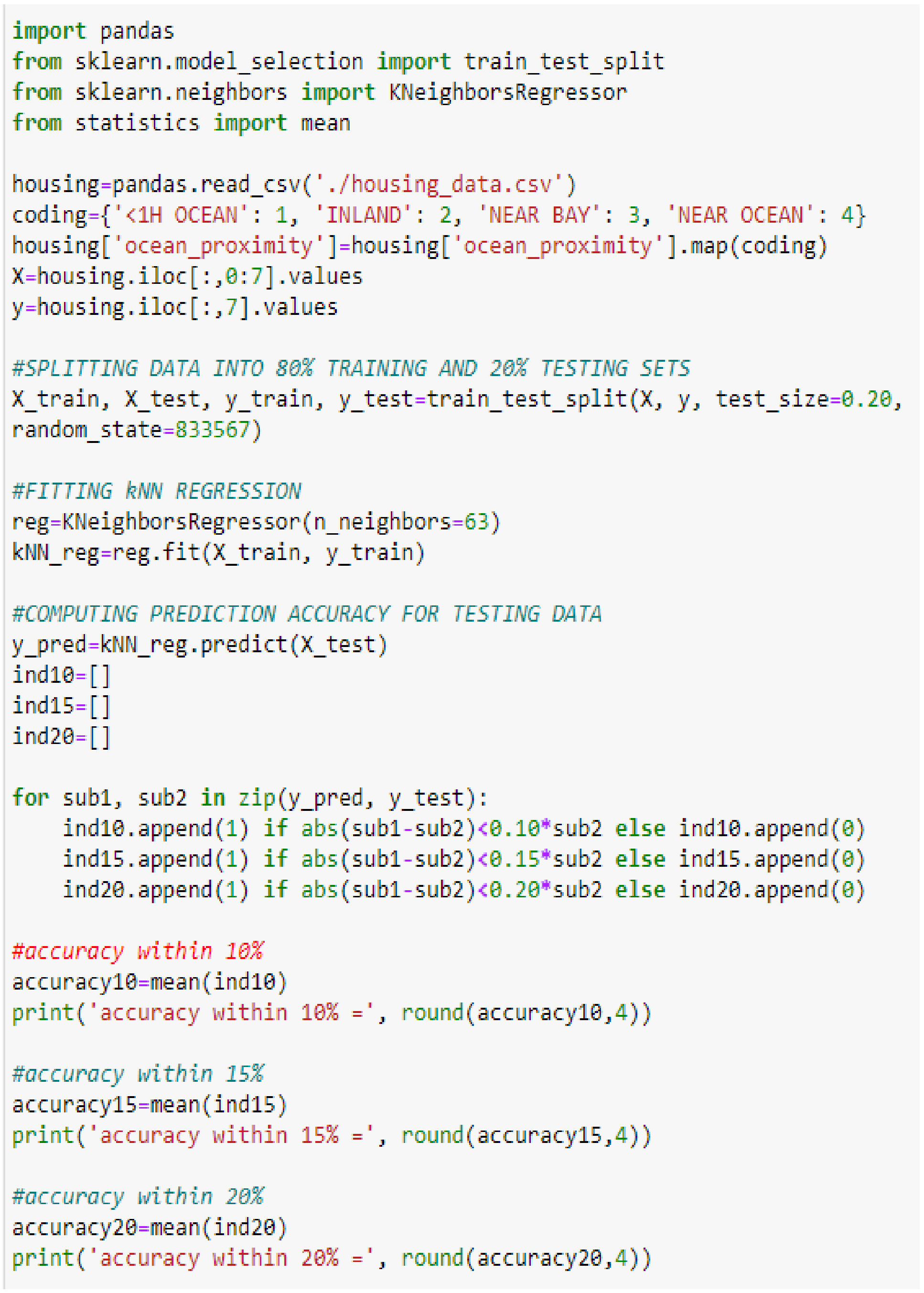
```
#accuracy within 20%
accuracy20<- ifelse(abs(test.y-pred.y)<0.20*test.y,1,0)
print(mean(accuracy20))
```

```
0.2610169
```

```
#PLOTTING ACTUAL AND PREDICTED VALUES FOR TESTING DATA
x<- 1:length(test.y)
plot(x, test.y, type="l", lwd=2, col="magenta", main="KNN Regression", panel.first=grid())
lines(x, pred.y, lwd=2, col="dodgerblue")
legend("topright", c("actual", "predicted"), lty=1, lwd=2,col=c("magenta","dodgerblue"))
```


KNN Regression
actual
predicted
test.y
0e+00
2e+05
4e+05
0
100
200
300
400
500
600
x

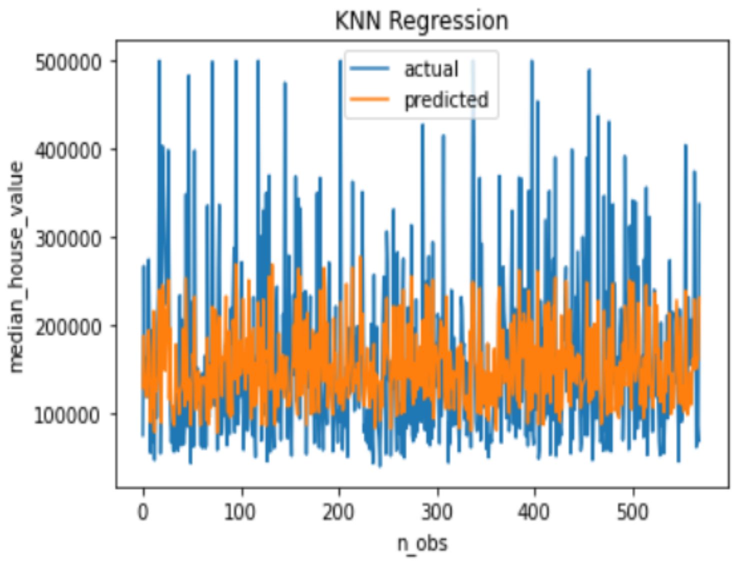
In Python:



```python
import pandas
from sklearn.model_selection import train_test_split
from sklearn.neighbors import KNeighborsRegressor
from statistics import mean

housing=pandas.read_csv('./housing_data.csv')
coding={'<1H OCEAN': 1, 'INLAND': 2, 'NEAR BAY': 3, 'NEAR OCEAN': 4}
housing['ocean_proximity']=housing['ocean_proximity'].map(coding)
X=housing.iloc[:,0:7].values
y=housing.iloc[:,7].values

#SPLITTING DATA INTO 80% TRAINING AND 20% TESTING SETS
X_train, X_test, y_train, y_test=train_test_split(X, y, test_size=0.20,
random_state=833567)

#FITTING kNN REGRESSION
reg=KNeighborsRegressor(n_neighbors=63)
kNN_reg=reg.fit(X_train, y_train)

#COMPUTING PREDICTION ACCURACY FOR TESTING DATA
y_pred=kNN_reg.predict(X_test)
ind10=[]
ind15=[]
ind20=[]

for sub1, sub2 in zip(y_pred, y_test):
    ind10.append(1) if abs(sub1-sub2)<0.10*sub2 else ind10.append(0)
    ind15.append(1) if abs(sub1-sub2)<0.15*sub2 else ind15.append(0)
    ind20.append(1) if abs(sub1-sub2)<0.20*sub2 else ind20.append(0)

#accuracy within 10%
accuracy10=mean(ind10)
print('accuracy within 10% =', round(accuracy10,4))

#accuracy within 15%
accuracy15=mean(ind15)
print('accuracy within 15% =', round(accuracy15,4))

#accuracy within 20%
accuracy20=mean(ind20)
print('accuracy within 20% =', round(accuracy20,4))
```

```
accuracy within 10% = 0.1213
accuracy within 15% = 0.1898
accuracy within 20% = 0.2689
```

```python
#plotting actual and predicted obsevations vs. observation number
import matplotlib.pyplot as plt

n_obs=list(range(0,len(y_test)))
plt.plot(n_obs, y_test, label="actual")
plt.plot(n_obs, y_pred, label="predicted")
plt.xlabel('n_obs')
plt.ylabel('median_house_value')
plt.title('KNN Regression')
plt.legend()
plt.show()
```

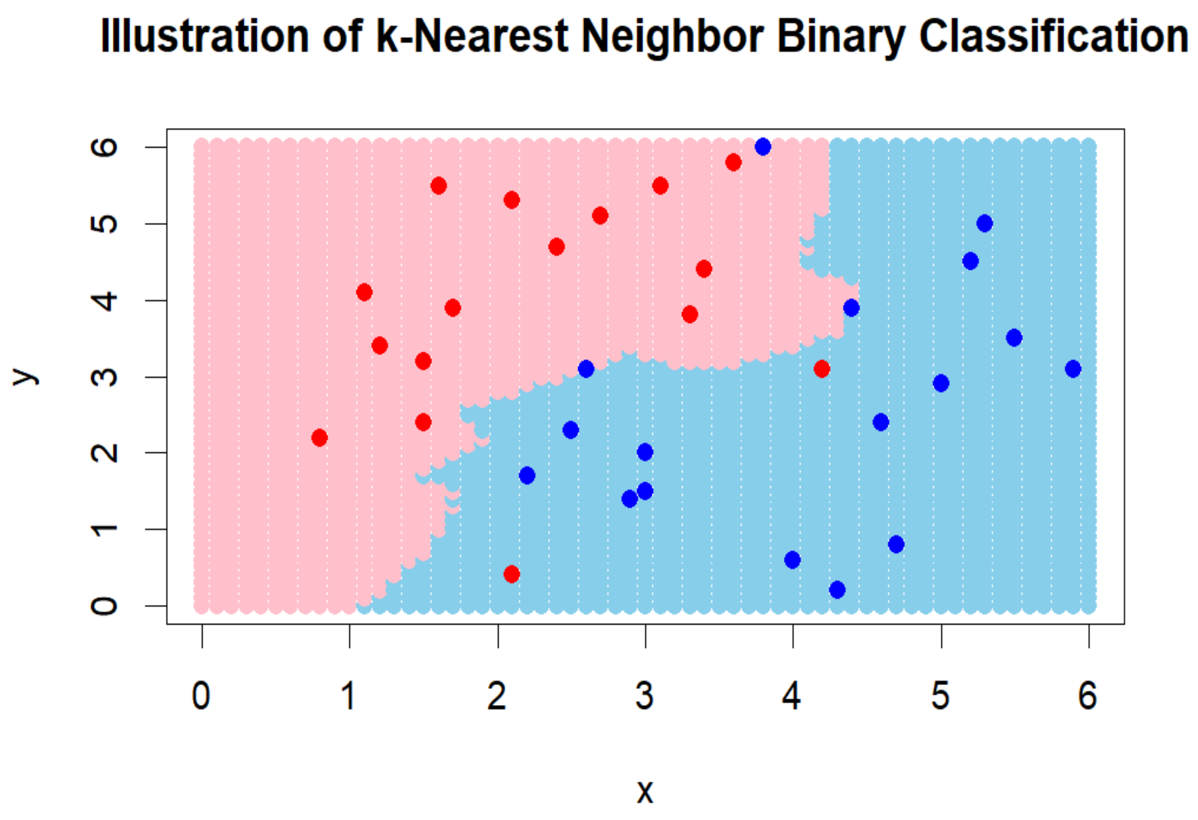


□

## k-Nearest Neighbor Binary Classification

For binary classification, each point on a grid is assigned the class that is most frequent among its $k$ nearest neighbors. Euclidean distance is used to measure the distances between grid points and data points.

We illustrate the algorithm using points of two colors: red and blue. The predictor variables are $x$ and $y$. For each point on a two-dimensional grid with a step size of 0.1, the $k$ nearest neighbors are identified, and the most frequent color among them is assigned to that grid point.



Here is the R code that produced this illustration.

```
sampledots.bc<- read.csv(file="./SamplePoints_KNN_biclass.csv", sep=",", header=TRUE)

plot(sampledots.bc$x, sampledots.bc$y, xlim=c(0,6), ylim=c(0,6), xlab="x",
ylab="y", main="Illustration of k-Nearest Neighbor Binary Classification",
col=sampledots.bc$color, pch=16)

k<- 5 # five nearest neighbors
for (i in seq(0, 6, 0.1)) {
  for (j in seq(0, 6, 0.1)) {
distance.sq<- c()
  for (z in 1:length(sampledots.bc$x))
    distance.sq[z]<- (i-sampledots.bc$x[z])^2+(j-sampledots.bc$y[z])^2
min.values<- sort(distance.sq)[1:k]
    min.colors<- sampledots.bc$color[which(distance.sq %in% min.values)]
      pred.color<- names(sort(table(min.colors), decreasing=TRUE)[1])
if (pred.color=="red") col.dot="pink" else col.dot="skyblue"
points(i,j,col=col.dot, pch=16)
  }
}
points(sampledots.bc$x, sampledots.bc$y, xlim=c(0,6), ylim=c(0,6),
col=sampledots.bc$color, pch=16)
```

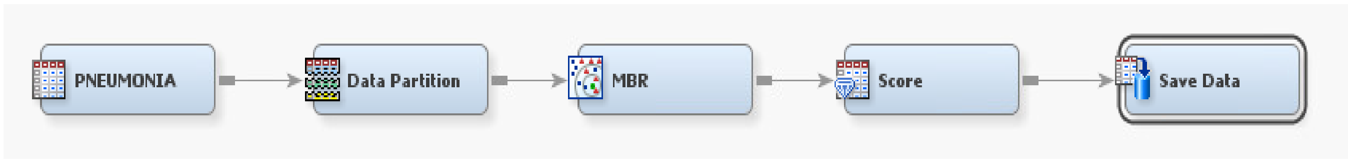
**Example.** For the data set "pneumonia_data.csv", we build the kNN binary classifier.

In SAS:

In SAS Enterprise Miner, we partition the data into 70% training, 10% validation, and 20% testing sets (346 rows), and run the MBR node, varying the number of neighbors ($= 346/k$) and record the misclassification rate for the testing set. The results are summarized here:

| **Number of classes** | **Number of neighbors** | **Misclassification Rate** |
|---|---|---|
| 3 | 115 | 0.321 |
| **4** | **87** | **0.301** |
| 5 | 69 | 0.321 |
| 6 | 58 | 0.318 |
| 7 | 49 | 0.321 |
| 8 | 43 | 0.335 |

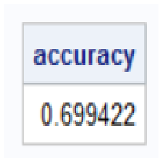
We choose to utilize $k = 4$ classes because it results in the smallest misclassification rate, and run the full path:



Now we open the SAS file with scored data "em_save_test.sas7dat" in the tmp1 folder and compute the accuracy of prediction by running the following SAS code:

```
/*COMPUTING PREDICTION ACCURACY*/
data accuracy;
set tmp1.em_save_test;
match=(em_classification=em_classtarget);
run;
```

```
proc sql;
select mean(match) as accuracy
from accuracy;
quit;
```



| accuracy |
|---|
| 0.699422 |

In R:

```
pneumonia.data<- read.csv(file="./pneumonia_data.csv", header=TRUE, sep=",")

pneumonia.data$pneumonia<- ifelse(pneumonia.data$pneumonia=="yes",1,0)

#SPLITTING DATA INTO 80% TRAINING AND 20% TESTING SETS
set.seed(704467)
sample <- sample(c(TRUE, FALSE), nrow(pneumonia.data), replace=TRUE, prob=c(0.8,0.2))
train<- pneumonia.data[sample,]
test<- pneumonia.data[!sample,]

train.x<- data.matrix(train[-5])
train.y<- data.matrix(train[5])
test.x<- data.matrix(test[-5])
test.y<- data.matrix(test[5])

#TRAINING K-NEAREST NEIGHBOR BINARY CLASSIFIER
library(caret)
print(train(as.factor(pneumonia)~ ., data=train, method="knn"))
```

```
  k  Accuracy
  5  0.6704615
  7  0.6781109
  9  0.6807456

Accuracy was used to select the optimal model using the largest value.
The final value used for the model was k = 9.
```

```
#FITTING OPTIMAL KNN BINARY CLASSIFIER (K=9)
knn.class<- knnreg(train.x, train.y, k=9)
```

```
#COMPUTING PREDICTION ACCURACY FOR TESTING DATA
pred.prob<- predict(knn.class, test.x)

len<- length(pred.prob)
pred.y<- c()
match<- c()
for (i in 1:len){
    pred.y[i]<- ifelse(pred.prob[i]>=0.5, 1,0)
    match[i]<- ifelse(test.y[i]==pred.y[i], 1,0)
}
print(paste("accuracy=",round(mean(match),digits=4)))
```

```
"accuracy= 0.7072"
```

```
#alternative (frugal) way
pred.y1<- floor(0.5+predict(knn.class, test.x))
print(paste("accuracy=", round(1-mean(test.y!=pred.y1),digits=4)))
```

```
"accuracy= 0.7072"
```

In Python:

```
import pandas
from sklearn.model_selection import train_test_split
from sklearn.neighbors import KNeighborsClassifier
from statistics import mean

pneumonia_data=pandas.read_csv('./pneumonia_data.csv')
code_gender={'M':1,'F':0}
code_tobacco_use={'yes':1,'no':0}
code_pneumonia={'yes':1,'no':0}

pneumonia_data['gender']=pneumonia_data['gender'].map(code_gender)
pneumonia_data['tobacco_use']=pneumonia_data['tobacco_use'].map(code_tobacco_use)
pneumonia_data['pneumonia']=pneumonia_data['pneumonia'].map(code_pneumonia)

X=pneumonia_data.iloc[:,0:4].values
y=pneumonia_data.iloc[:,4].values

#SPLITTING DATA INTO 80% TRAINING AND 20% TESTING SETS
X_train, X_test, y_train, y_test=train_test_split(X, y, test_size=0.20,
random_state=459147)

#FITTING kNN BINARY CLASSIFIER WITH k=4
biclass=KNeighborsClassifier(n_neighbors=87)
kNN_biclass=biclass.fit(X_train, y_train)

#COMPUTING PREDICTION ACCURACY FOR TESTING DATA
y_pred=kNN_biclass.predict(X_test)
y_test=pandas.DataFrame(y_test,columns=['pneumonia'])
y_pred=pandas.DataFrame(y_pred,columns=['predicted'])
df=pandas.concat([y_test,y_pred],axis=1)
```

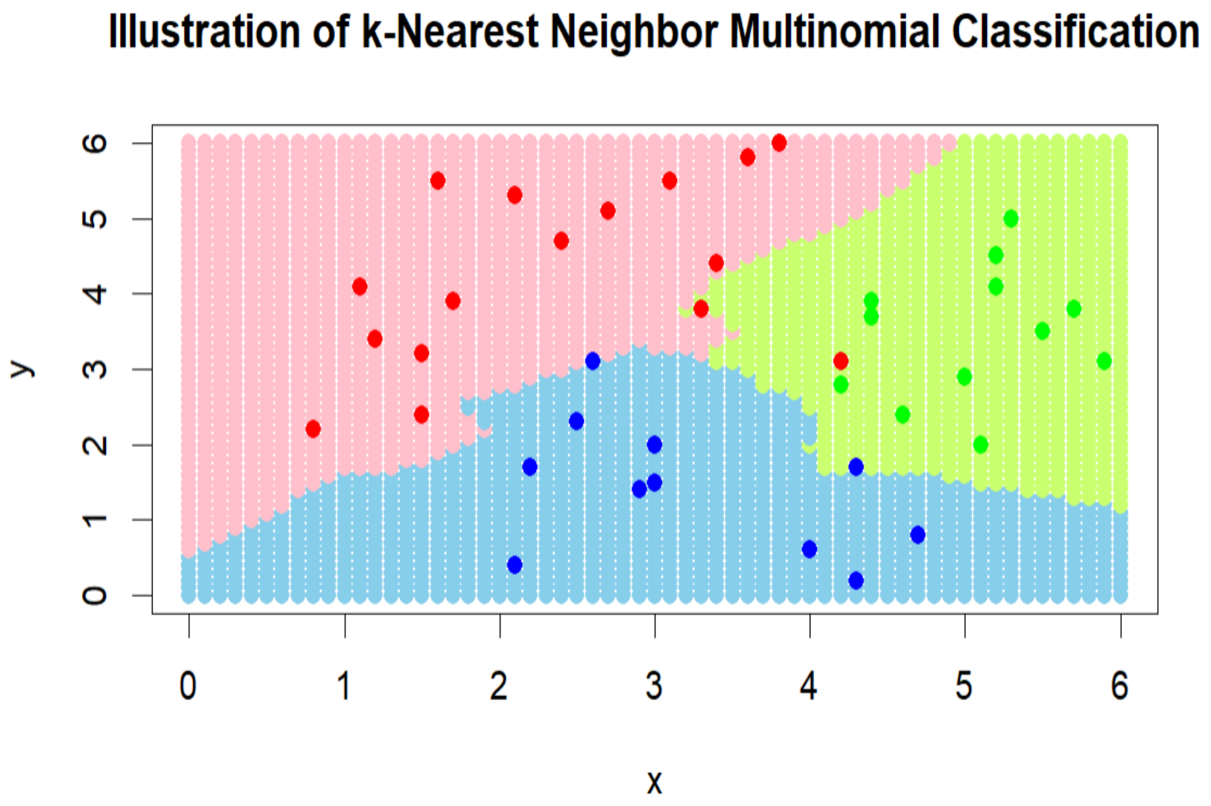
```
match=[]
for i in range(len(df)):
    if df['pneumonia'][i]==df['predicted'][i]:
        match.append(1)
    else:
        match.append(0)

print('accuracy=', mean(match))
```

```
0.6705202312138728
```

□

**k-Nearest Neighbor Multinomial Classification**

The kNN algorithm works with multinomial classification in a manner similar to binary classification. The predicted class for a grid point is determined by a majority vote among the classes of its $k$ nearest neighbors. We illustrate this below using three colors: red, blue, and green.



Here is the R code that produced this illustration.

```
sampledots.bc<- read.csv(file="./SamplePoints_KNN_multiclass.csv", sep=",", header=TRUE)

plot(sampledots.bc$x, sampledots.bc$y, xlim=c(0,6), ylim=c(0,6), xlab="x",
ylab="y", main="Illustration of k-Nearest Neighbor Multinomial Classification",
col=sampledots.bc$color, pch=16)
```

```
k<- 5 # five nearest neighbors
for (i in seq(0, 6, 0.1)) {
  for (j in seq(0, 6, 0.1)) {

    distance.sq<- c()
    for (z in 1:length(sampledots.bc$x))
      distance.sq[z]<- (i-sampledots.bc$x[z])^2+(j-sampledots.bc$y[z])^2

    min.values<- sort(distance.sq)[1:k]
    min.colors<- sampledots.bc$color[which(distance.sq %in% min.values)]
    pred.color<- names(sort(table(min.colors), decreasing=TRUE)[1])

    if (pred.color=="red") col.dot="pink"
    if (pred.color=="blue") col.dot="skyblue"
    if (pred.color=="green") col.dot="darkolivegreen1"
    points(i,j,col=col.dot, pch=16)
  }
}

points(sampledots.bc$x, sampledots.bc$y, xlim=c(0,6), ylim=c(0,6),
col=sampledots.bc$color, pch=16)
```

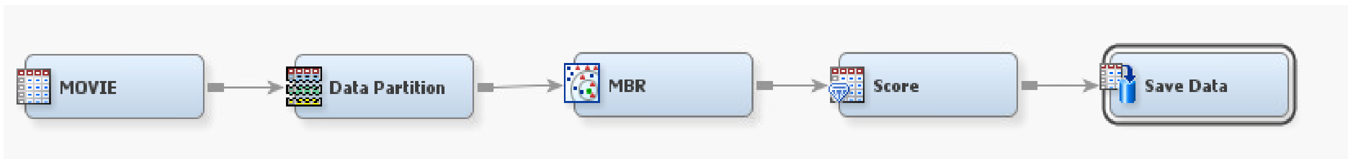
**Example.** Consider the data in the file "movie_data.csv". We fit a multinomial classifier using the kNN algorithm.

In SAS:

In SAS Enterprise Miner, we run the path ending in the MBR node for various numbers of classes (see the table below). For the analysis, we choose $k = 5$ as it corresponds to the minimal misclassification rate for the testing set (157 rows).

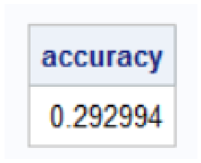
| **Number of classes** | **Number of neighbors** | **Misclassification Rate** |
|---|---|---|
| 3 | 52 | 0.720 |
| 4 | 39 | 0.739 |
| **5** | **31** | **0.707** |
| 6 | 26 | 0.707 |
| 7 | 22 | 0.726 |

We use $k = 5$ and run the full path depicted in the figure below.



We open the scored testing set in the tmp1 folder in SAS and compute the accuracy of prediction.

```
/*COMPUTING PREDICTION ACCURACY*/
data accuracy;
set tmp1.em_save_test;
match=(em_classification=em_classtarget);
run;

proc sql;
select mean(match) as accuracy
from accuracy;
quit;
```



```
accuracy
0.292994
```

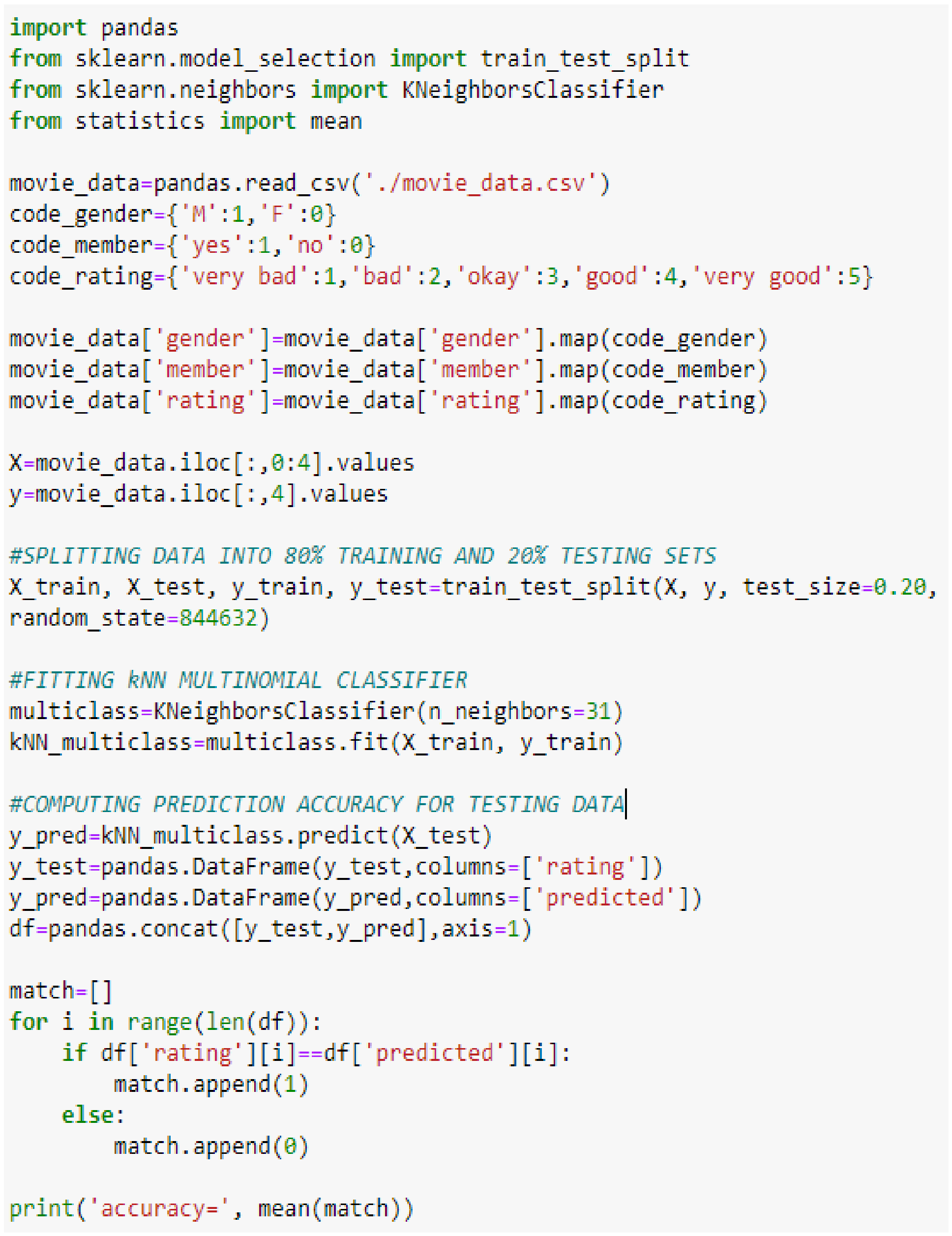
In R:

```
movie.data<- read.csv(file="./movie_data.csv", header=TRUE, sep=",")

#SPLITTING DATA INTO 80% TRAINING AND 20% TESTING SETS
set.seed(123857)
sample <- sample(c(TRUE, FALSE), nrow(movie.data), replace=TRUE, prob=c(0.8,0.2))
train<- movie.data[sample,]
test<- movie.data[!sample,]

train.x<- data.matrix(train[-5])
train.y<- data.matrix(train[5])
test.x<- data.matrix(test[-5])
test.y<- data.matrix(test[5])
```

```
#FITTING K-NEAREST NEIGHBOR MULTINOMIAL CLASSIFIER
#k=3 reasonably maximizes prediction accuracy for testing set
library(caret)
knn.mclass<- knnreg(train.x, train.y, k=3)

#COMPUTING PREDICTION ACCURACY FOR TESTING DATA
pred.y<- round(predict(knn.mclass, test.x), digits=0)
print(paste("accuracy=", round(1-mean(test.y!=pred.y),digits=4)))
```

```
accuracy= 0.2133
```

In Python:



```python
import pandas
from sklearn.model_selection import train_test_split
from sklearn.neighbors import KNeighborsClassifier
from statistics import mean

movie_data=pandas.read_csv('./movie_data.csv')
code_gender={'M':1,'F':0}
code_member={'yes':1,'no':0}
code_rating={'very bad':1,'bad':2,'okay':3,'good':4,'very good':5}

movie_data['gender']=movie_data['gender'].map(code_gender)
movie_data['member']=movie_data['member'].map(code_member)
movie_data['rating']=movie_data['rating'].map(code_rating)

X=movie_data.iloc[:,0:4].values
y=movie_data.iloc[:,4].values

#SPLITTING DATA INTO 80% TRAINING AND 20% TESTING SETS
X_train, X_test, y_train, y_test=train_test_split(X, y, test_size=0.20,
random_state=844632)

#FITTING kNN MULTINOMIAL CLASSIFIER
multiclass=KNeighborsClassifier(n_neighbors=31)
kNN_multiclass=multiclass.fit(X_train, y_train)

#COMPUTING PREDICTION ACCURACY FOR TESTING DATA
y_pred=kNN_multiclass.predict(X_test)
y_test=pandas.DataFrame(y_test,columns=['rating'])
y_pred=pandas.DataFrame(y_pred,columns=['predicted'])
df=pandas.concat([y_test,y_pred],axis=1)

match=[]
for i in range(len(df)):
    if df['rating'][i]==df['predicted'][i]:
        match.append(1)
    else:
        match.append(0)

print('accuracy=', mean(match))
```

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accuracy= 0.3157894736842105
```

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