RATT

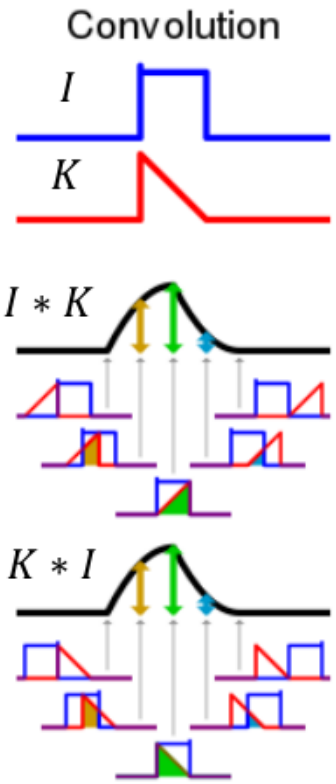
RHODES UNIVERSITY
Where leaders learn

CONVOLUTIONAL NETWORKS

Marcellin Atemkeng
(Rhodes University)

SKA AFRICA
SQUARE KILOMETRE ARRAY

Discrete convolutions



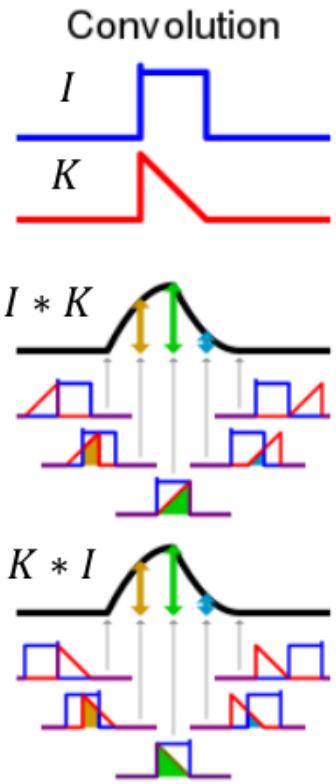
We'll focus on 1 & 2-D signals but signal dimensionality is arbitrary (usually 1,2,3,4-D)

The 2-D discrete convolution of two signals I and K is defined as:

$$(I * K)(i,j) = \sum_m \sum_n I(m,n)K(i-m, j-n)$$
$$= \sum_m \sum_n I(i-m, j-n)K(m,n)$$

Where $-\infty \leq m,n \leq \infty$. Finite signals can be extended by adding zeros (more on this later)

Discrete convolutions



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Convolution vs Cross-correlation

$$(I * K)(i,j) == \sum_m \sum_n I(m,n)K(i-m,j-n)$$

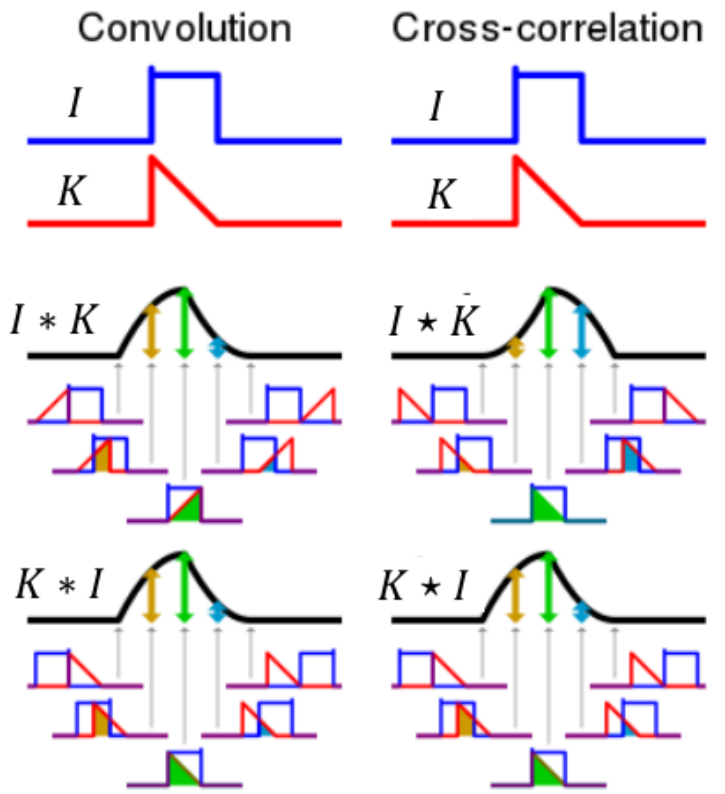
vs

$$(I \star K)(i,j) == \sum_m \sum_n I(m,n)K(i+m,j+n)$$

Cross-correlation quantifies **presence of $I(i,j)$ in (shifted) $K(i,j)$**

Most ML libraries implement convolutional layers as cross-correlation layers.

- Commutativity not important for most ConvNets
- Can avoid flipping one of the signals ⇒ **easier to implement**



Discrete cross-correlation: 2-D example

I

1	0	0	1	2
0	0	0	3	0
0	1	2	1	1
1	1	3	0	0
3	0	0	0	1

$\star$

K

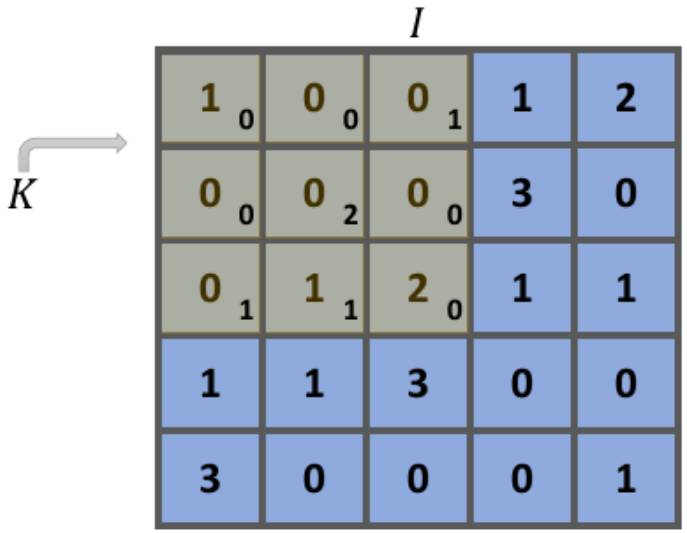
0	0	1
0	2	0
1	1	0

$$
\begin{aligned}
(I \star K)(0,0) = \; & I(0,0)K(0,0) + I(0,1)K(0,1) + I(0,2)K(0,2) + \\
& I(1,0)K(1,0) + I(1,1)K(1,1) + I(1,2)K(1,2) + \\
& I(2,0)K(2,0) + I(2,1)K(2,1) + I(2,2)K(2,2) \\
= \; & 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 + \\
& 0 \cdot 0 + 0 \cdot 2 + 0 \cdot 0 + \\
& 0 \cdot 1 + 1 \cdot 1 + 2 \cdot 0 \\
= \; & 1
\end{aligned}
$$

$$
(I \star K)(i,j) == \sum_m \sum_n I(m,n)K(i+m, j+n)
$$

Discrete cross-correlation: 2-D example

Can be viewed as a “sliding window” operation:



$I \star K$

1	4	11
4	11	5
7	7	7

$$(I \star K)(i,j) == \sum_m \sum_n I(m,n)K(i+m, j+n)$$

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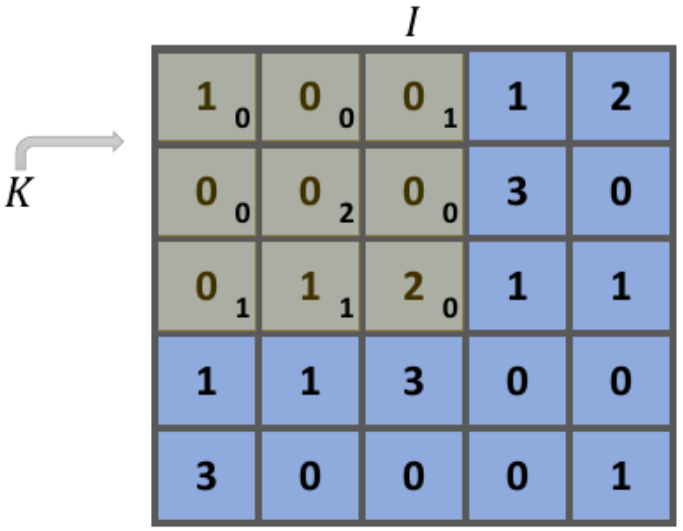

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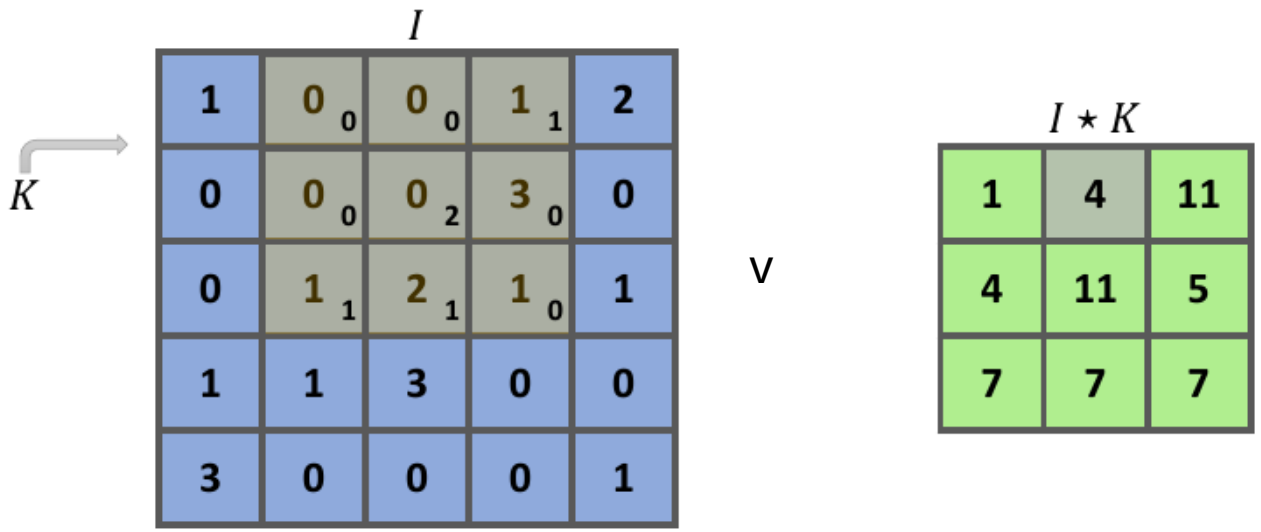

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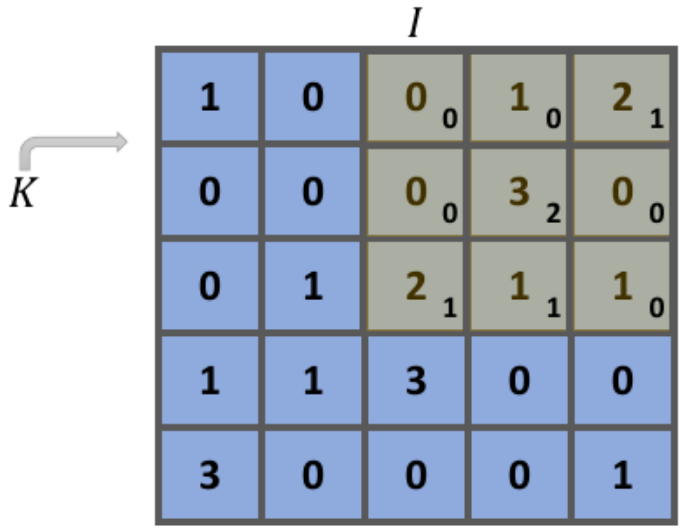

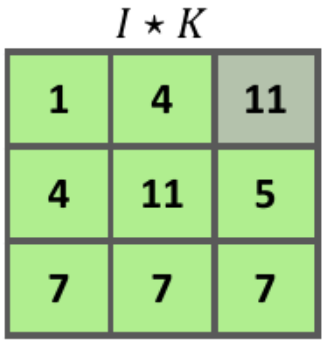

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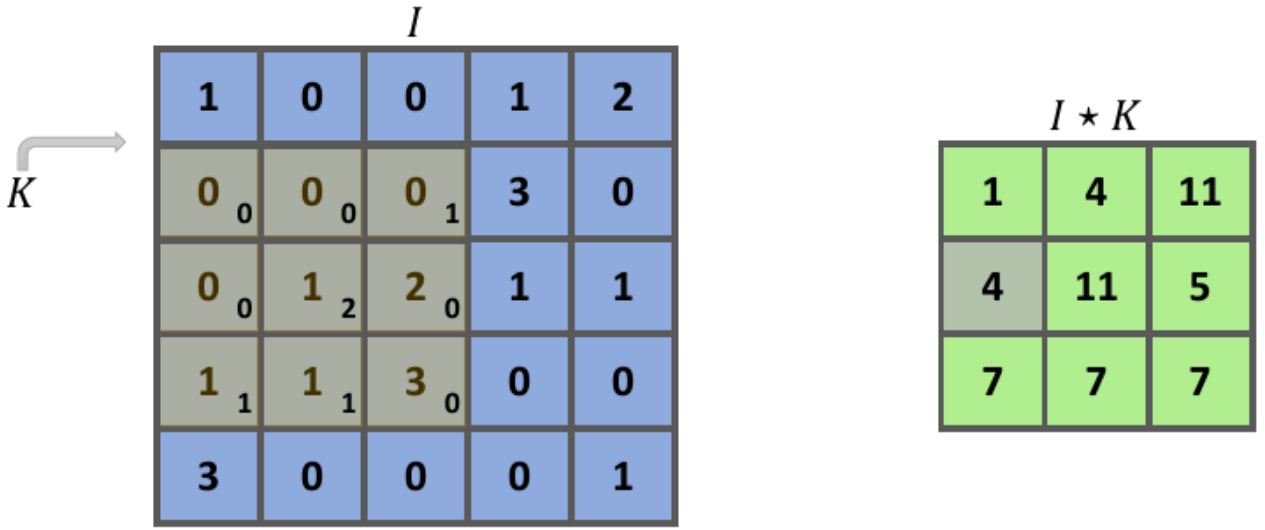

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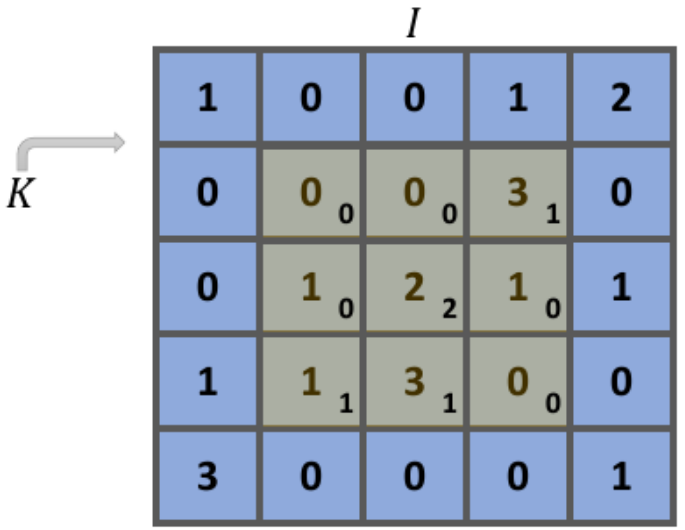

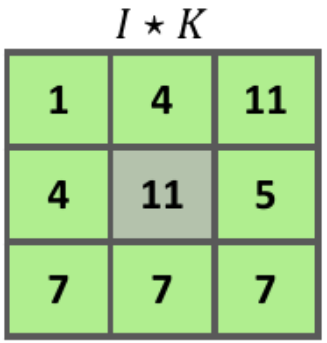

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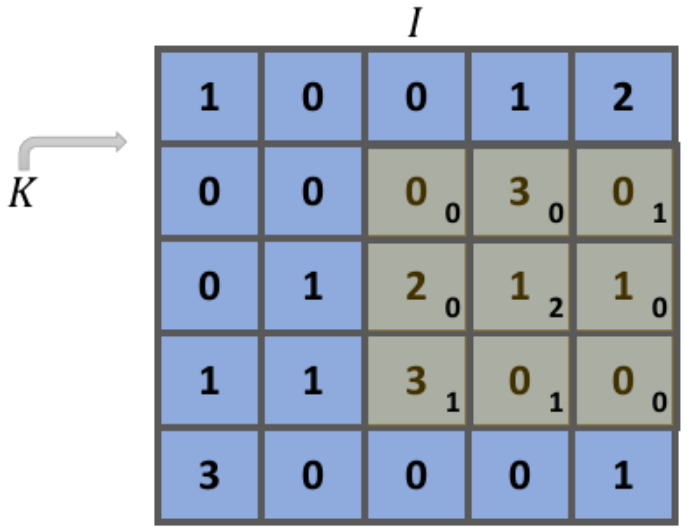

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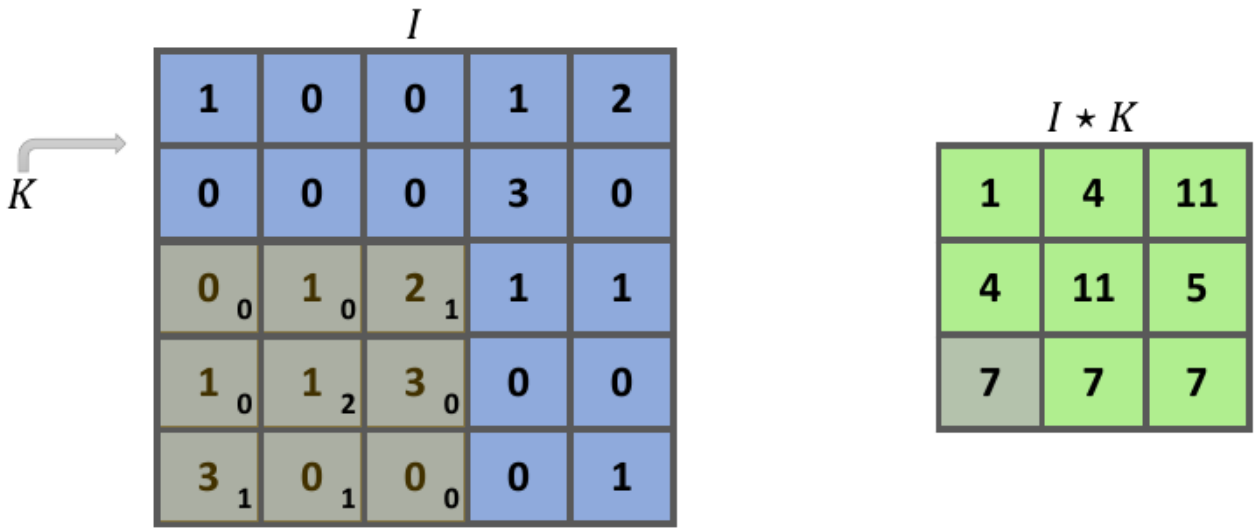

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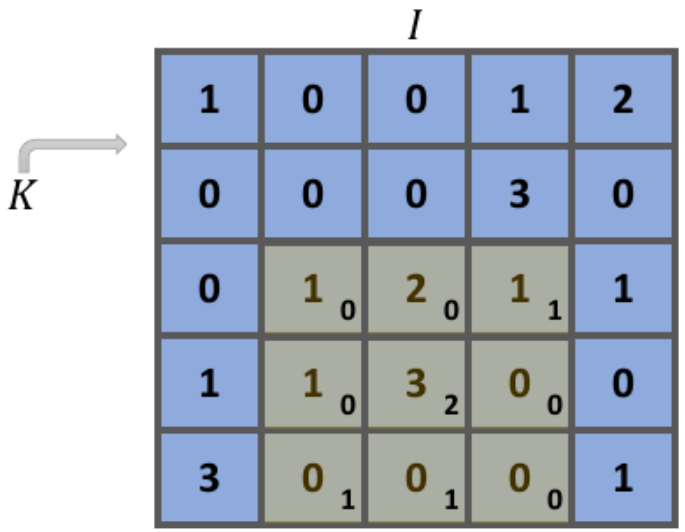

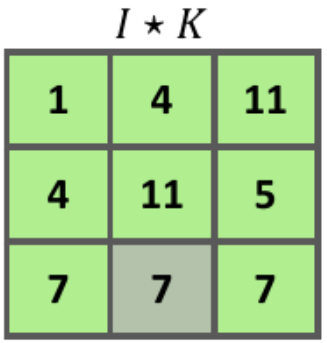

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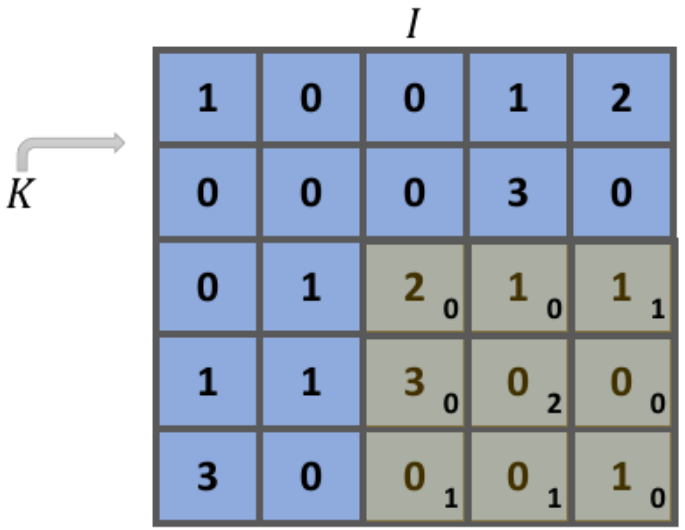

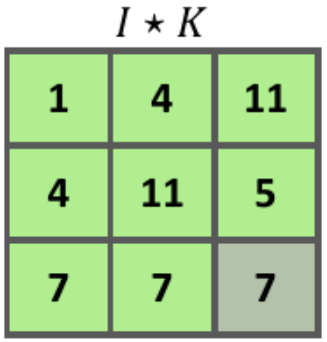

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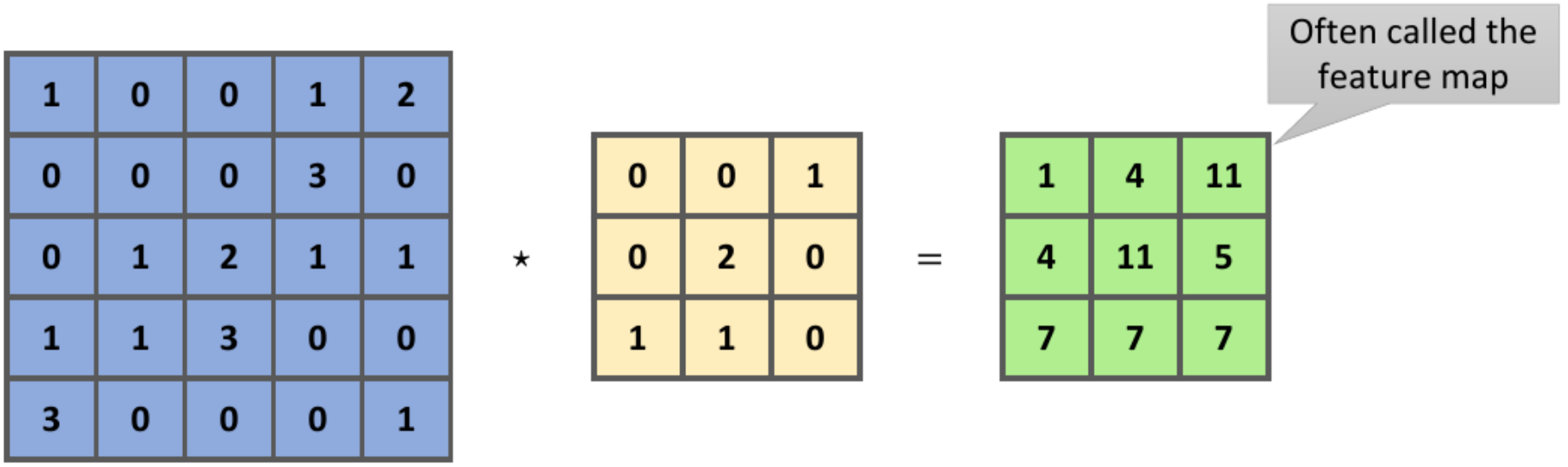

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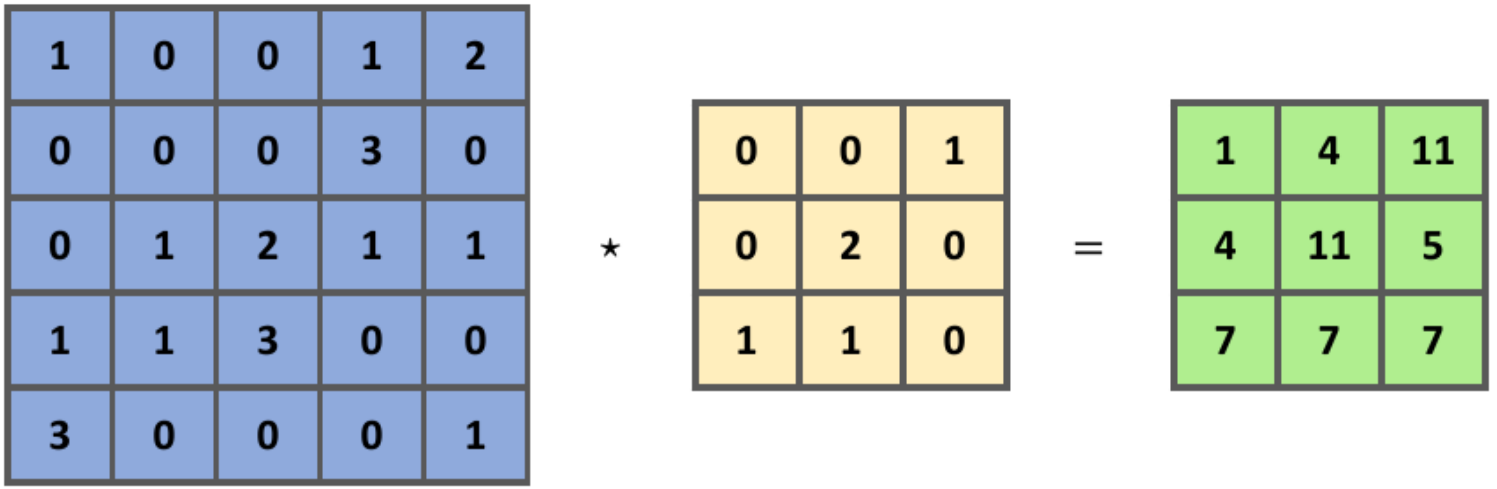
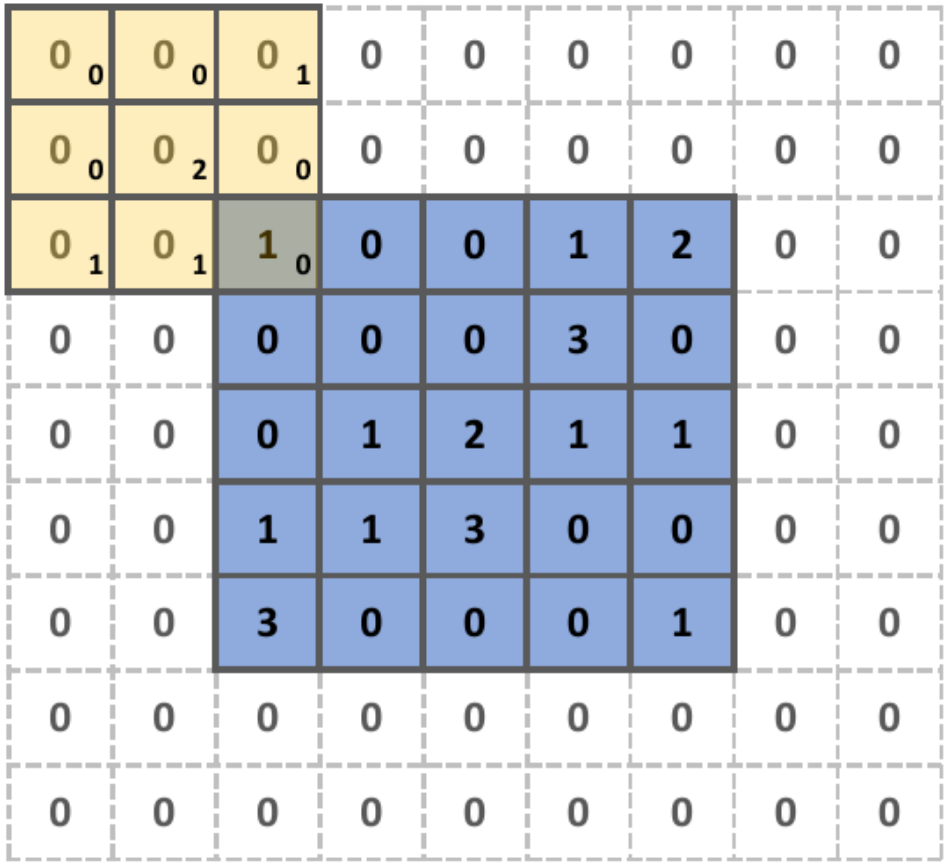


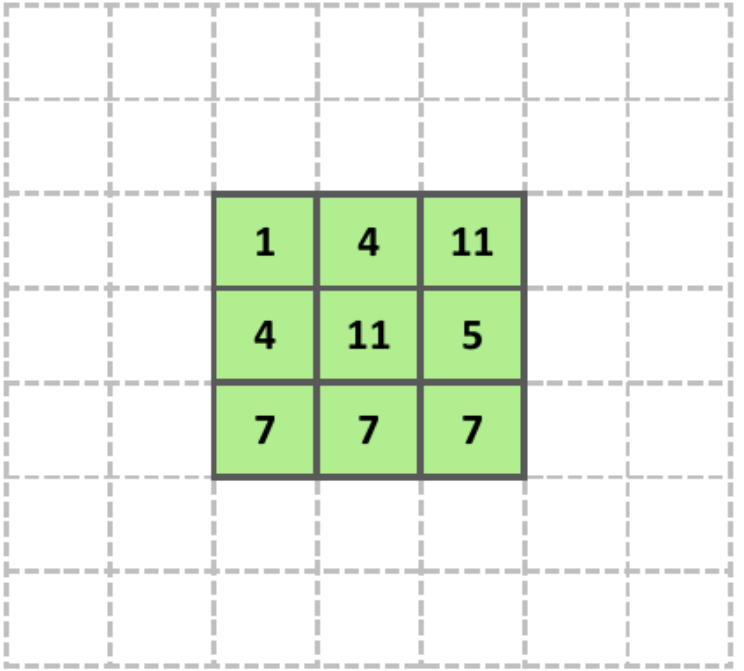
Something's not quite right…

$$(I \star K)(i,j) == \sum_m \sum_n I(m,n)K(i+m, j+n)$$

Discrete cross-correlation: padding



Technically, our signals have infinite extent…
we solve by padding with zeros



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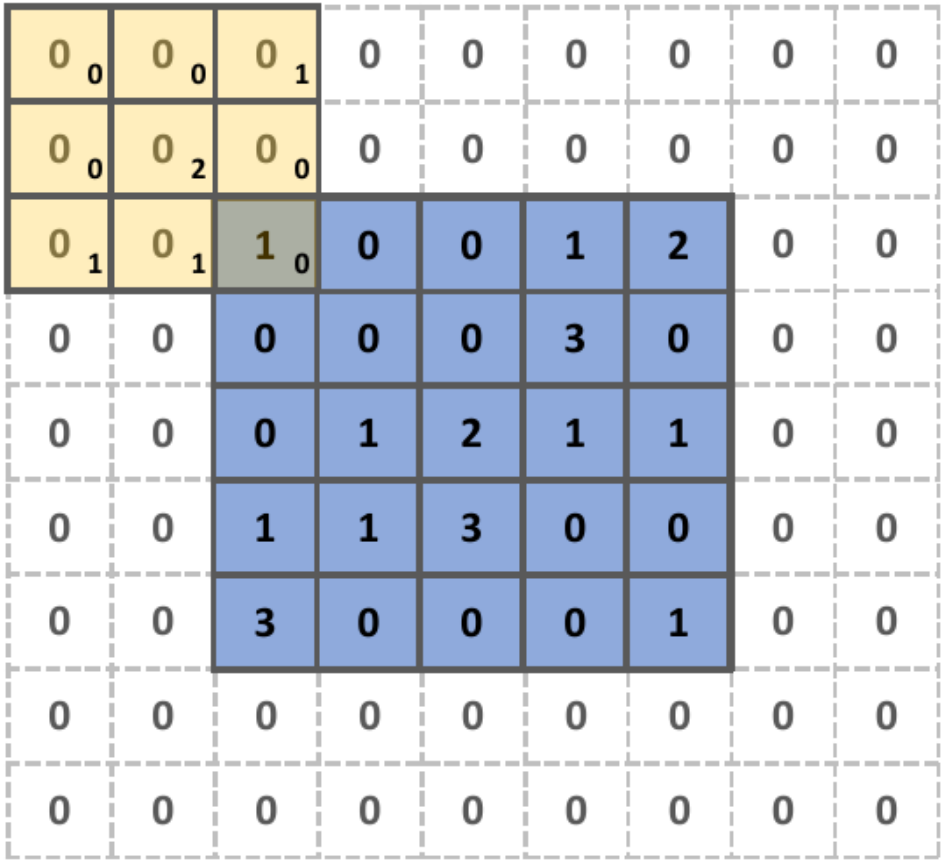

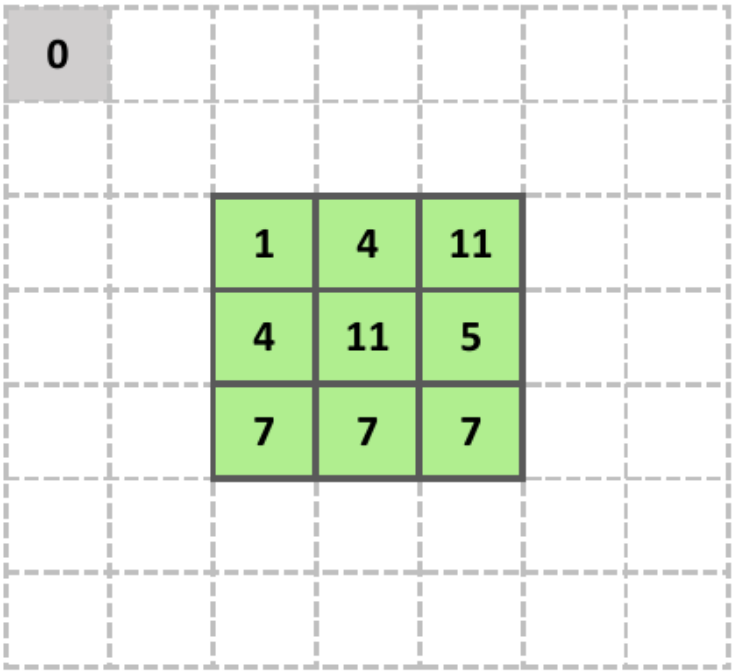

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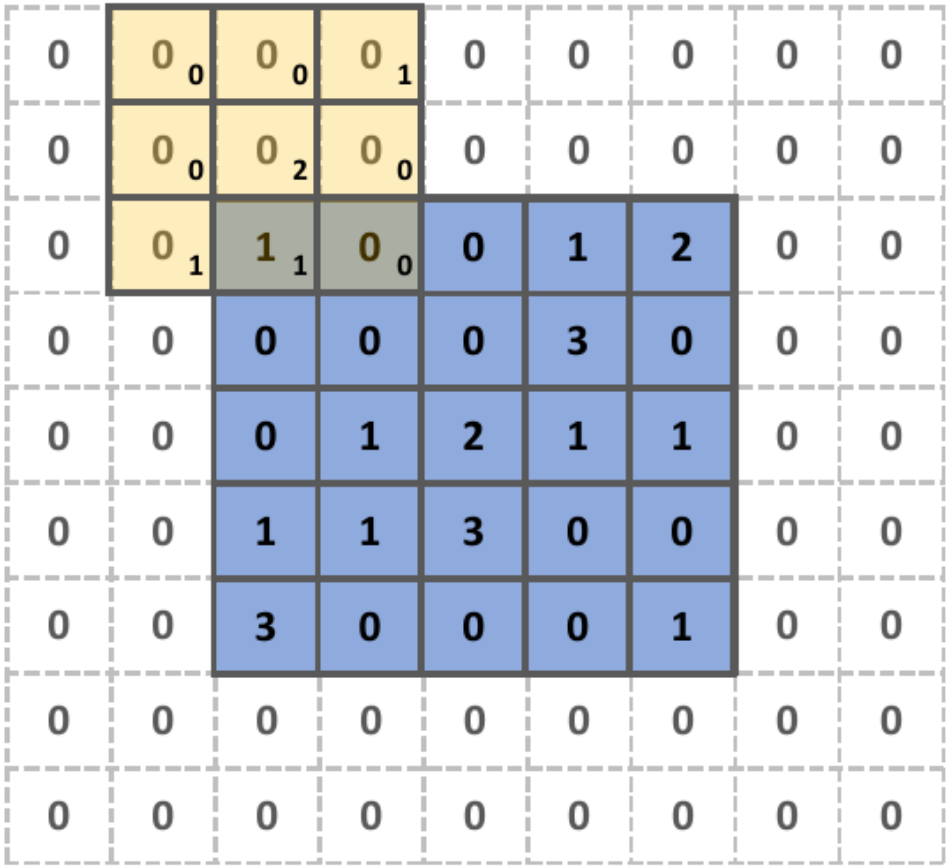

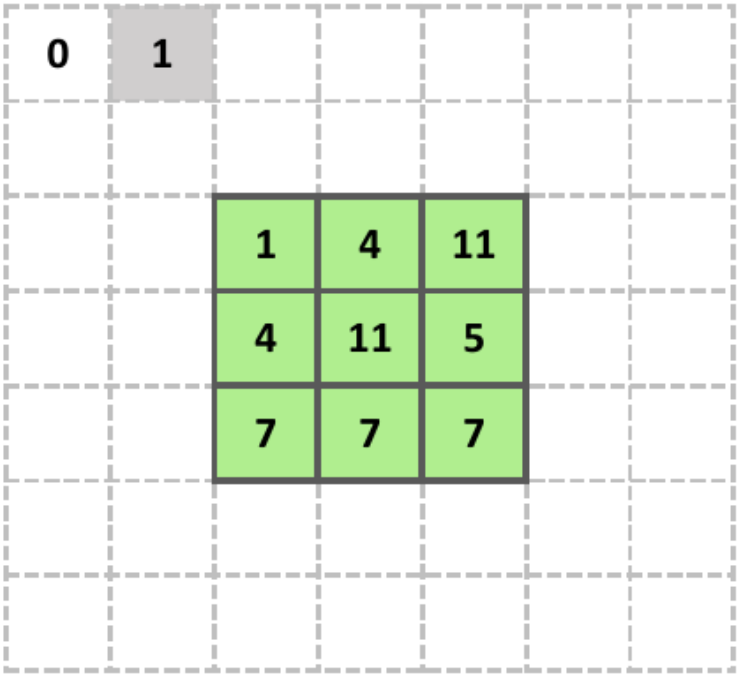

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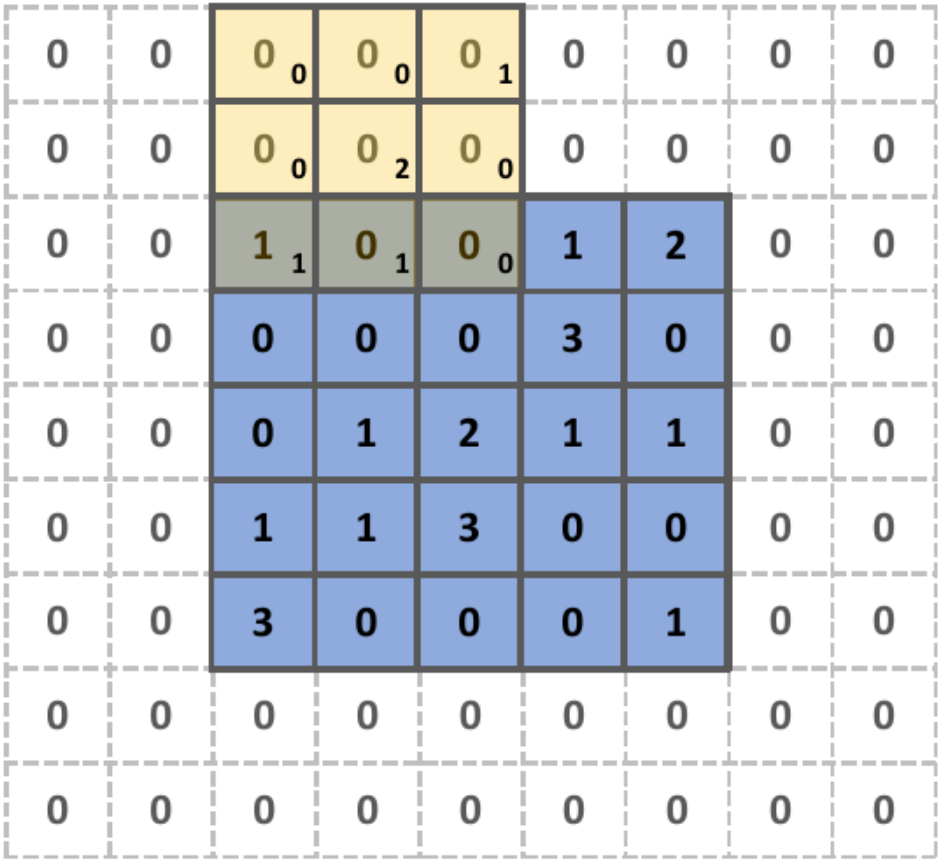

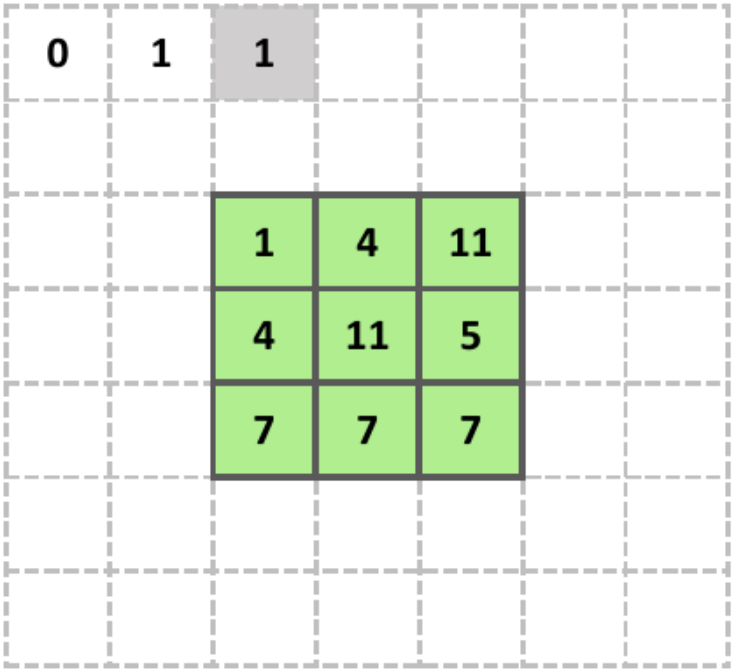

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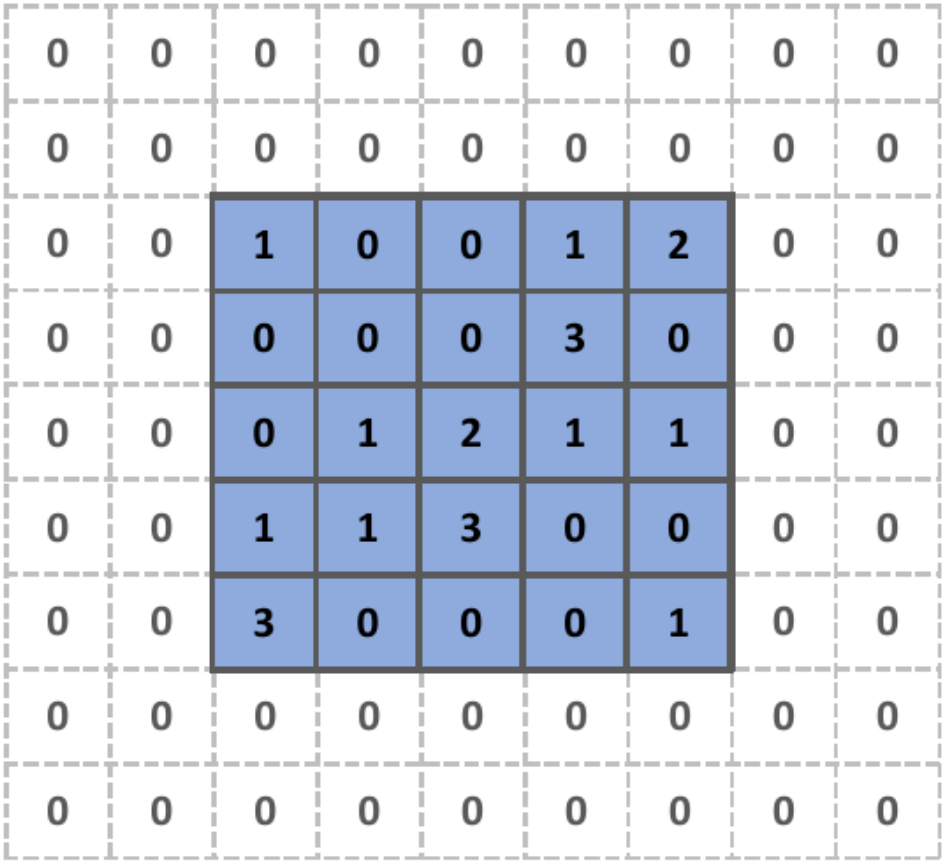
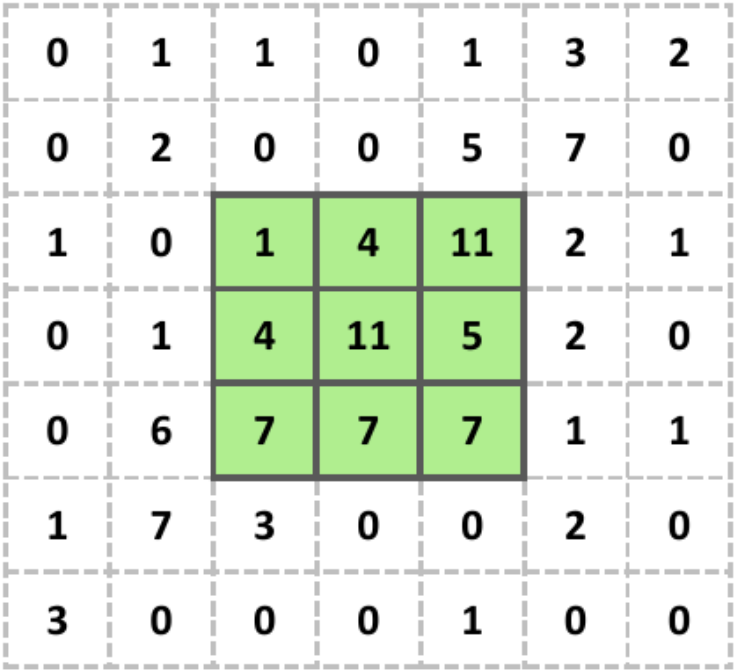


Normally we want the output to maintain the input size



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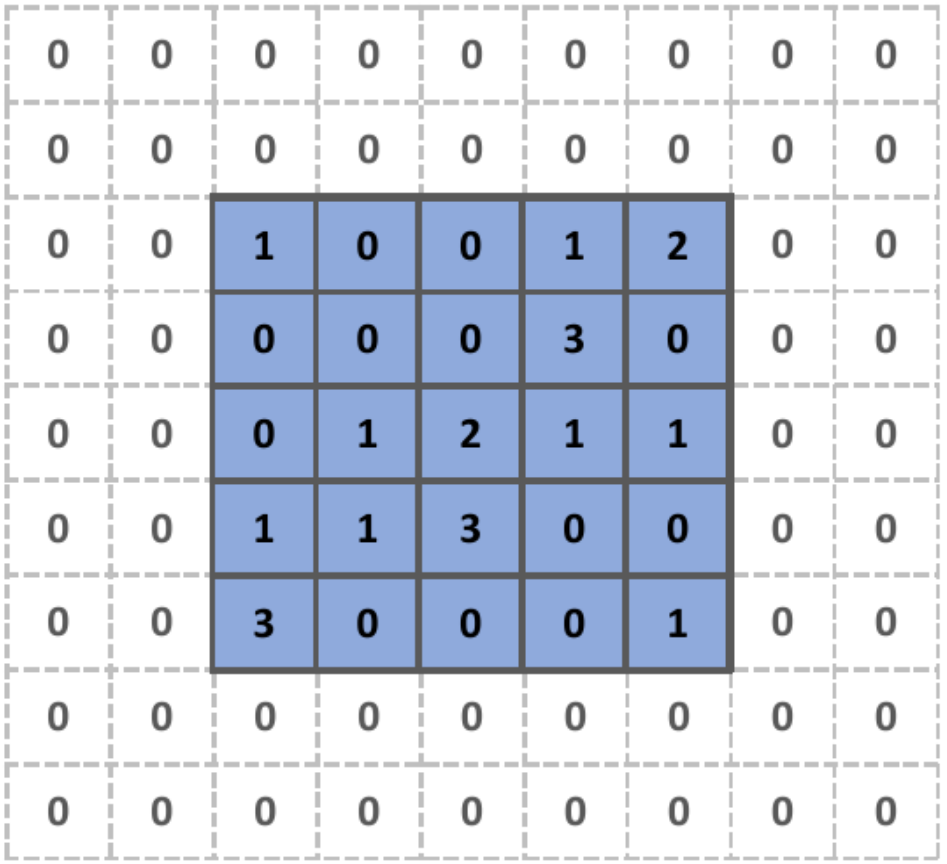

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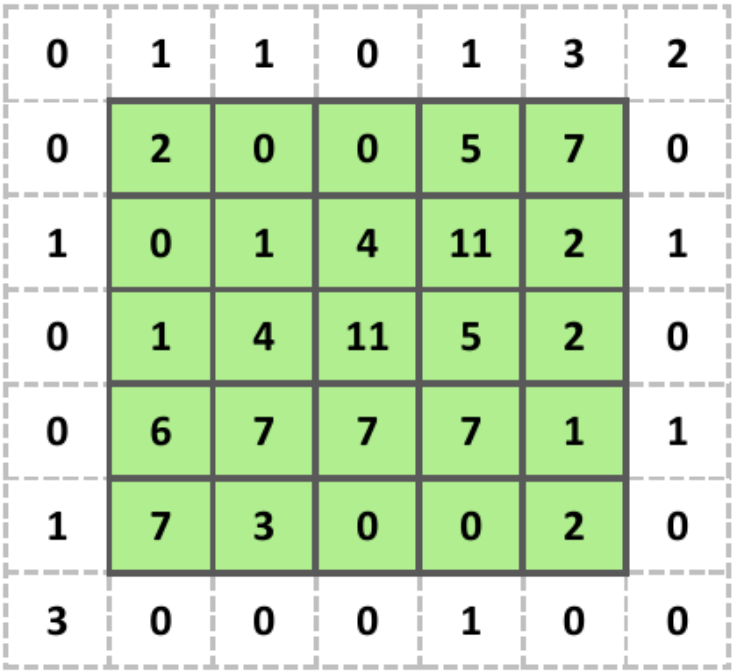
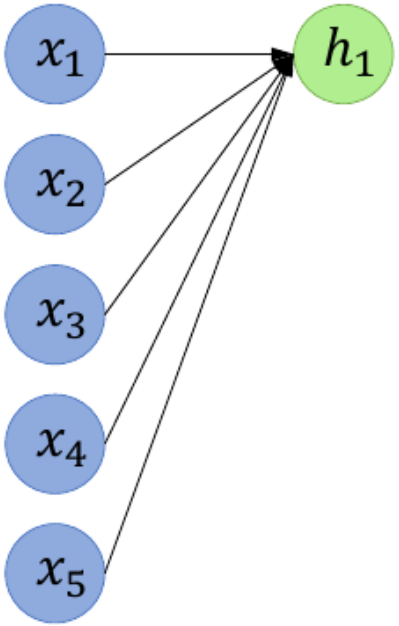


Neural networks ⇒ Convolutional networks



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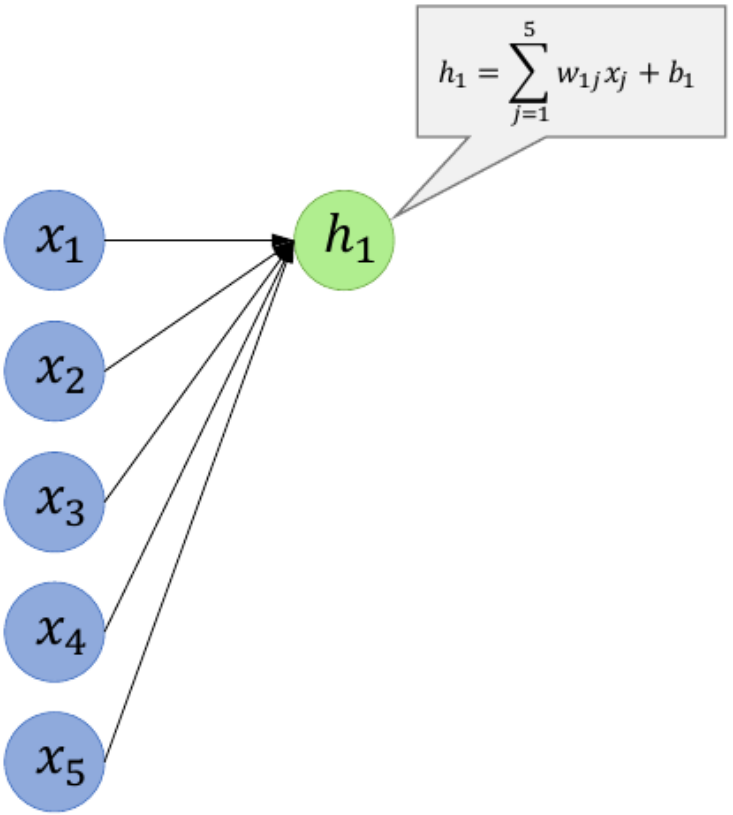

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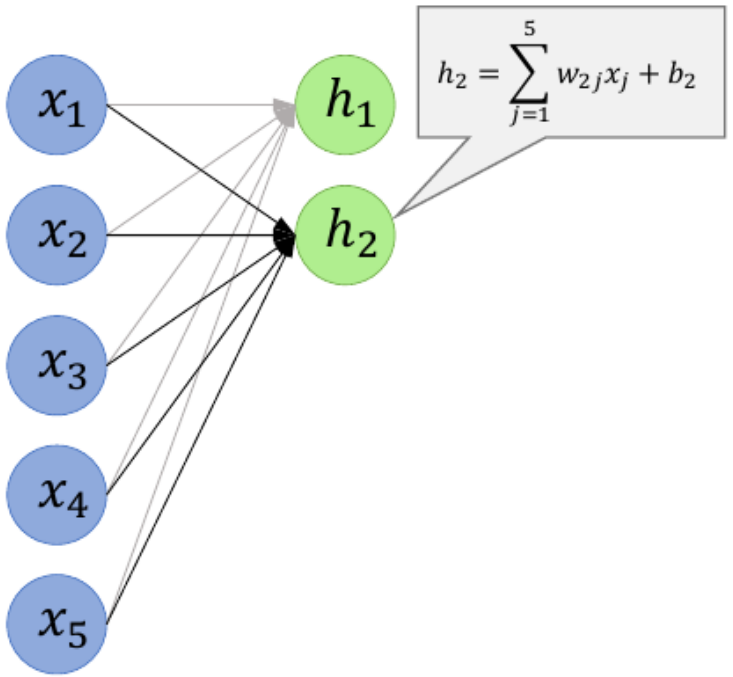

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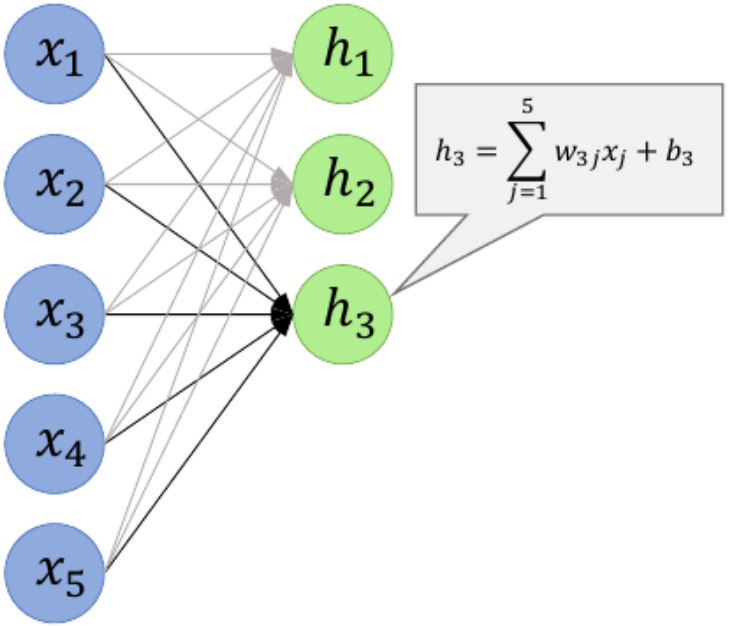

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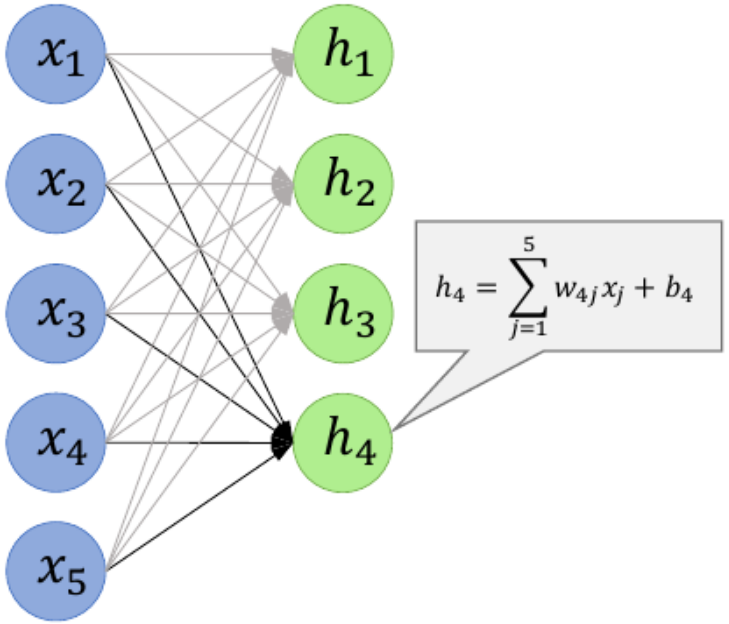

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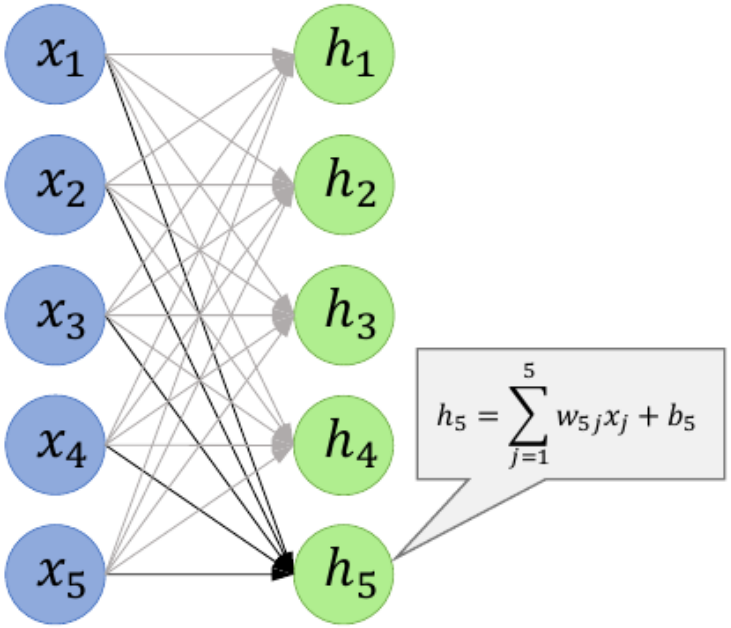

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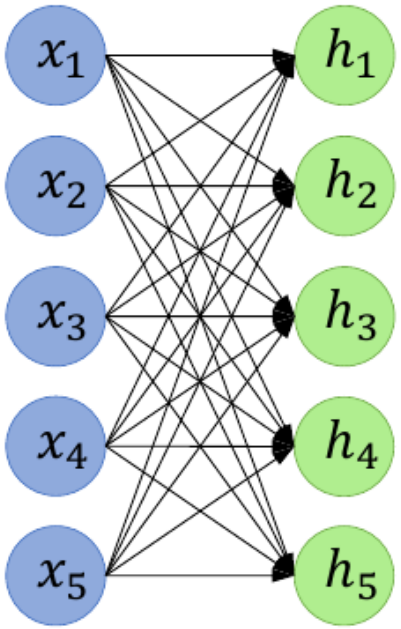

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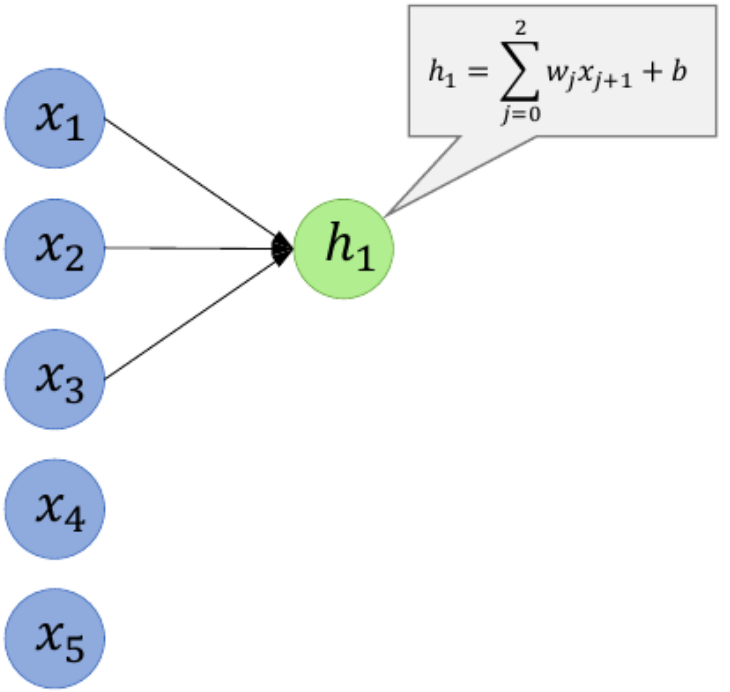
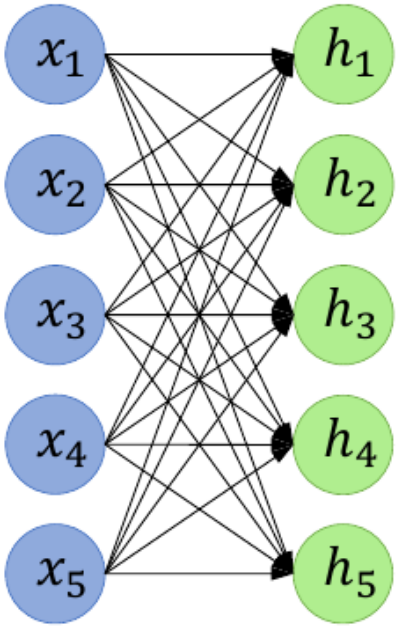
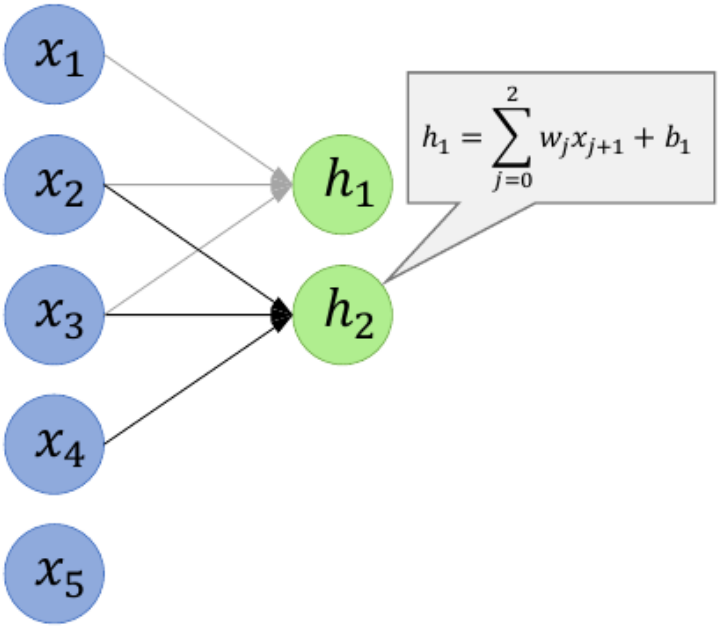


$h_i = \sum_j w_j x_{j+i} + b$

Neural networks ⇒ Convolutional networks



$$\boldsymbol{h} = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{b};\ h_i = \sum_j w_{ij} x_j + b_i$$



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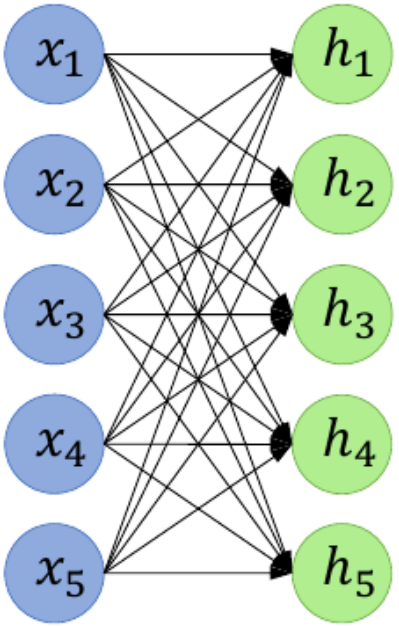

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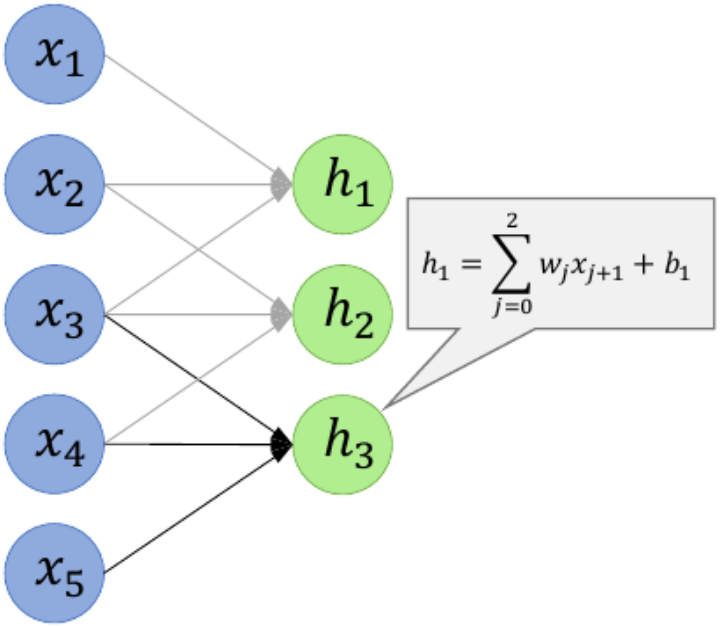

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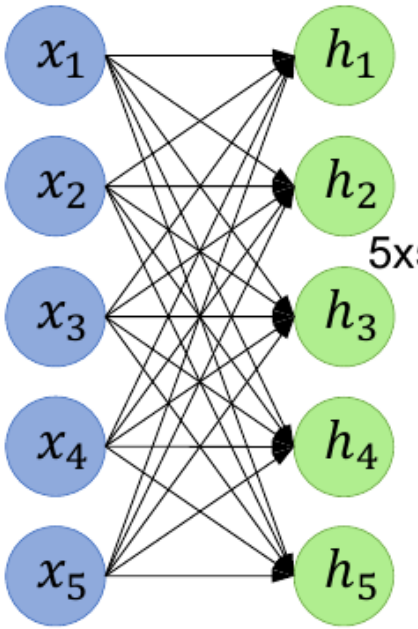
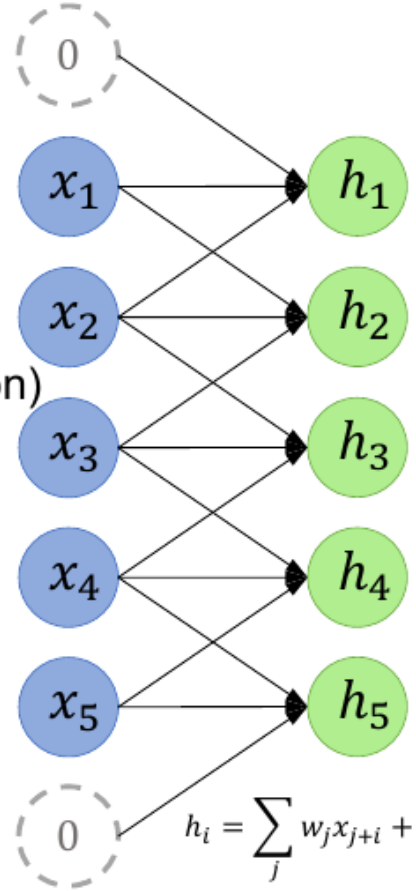


$\boldsymbol{h} = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{b};\ h_i = \sum_j w_{ij} x_j + b_i$

dense connectivity
vs
sparse connectivity

In our example:
5x5 multiplications (naïve implementation)
vs
5x3 multiplications

Sparse connectivity scales better
e.g. 25x25 vs 25x3



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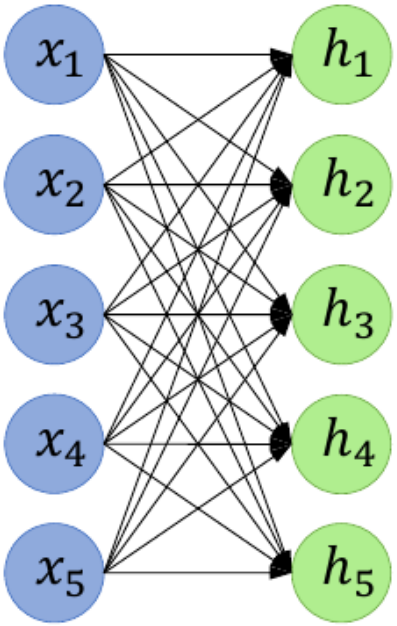
Neural networks ⇒ Convolutional networks

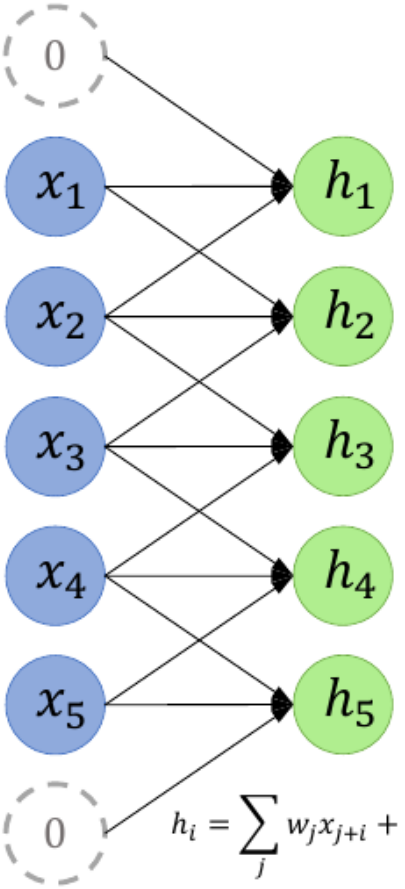


$\boldsymbol{h} = \boldsymbol{W}\boldsymbol{x} + \boldsymbol{b};\ h_i = \sum_j w_{ij} x_j + b_i$

unshared vs shared weights:

In our example:
5x5 vs 3 weights

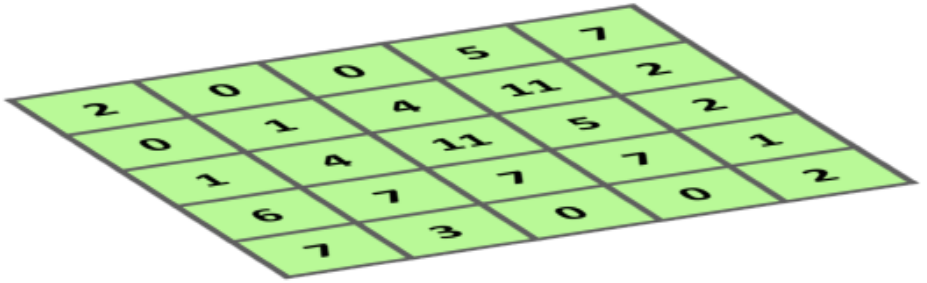
shared weights scale way better:
e.g. 25x25 vs 3

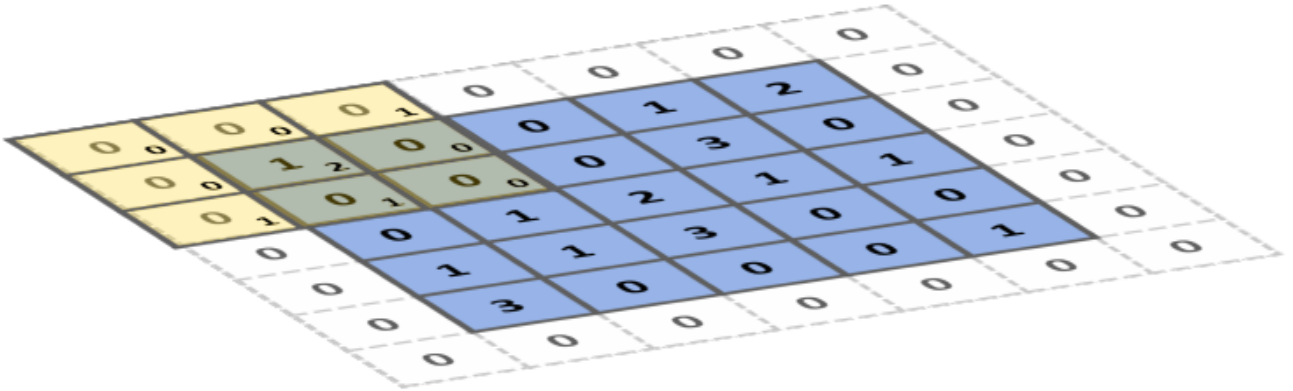


$h_i = \sum_j w_j x_{j+i} + b$

The effect of different strides

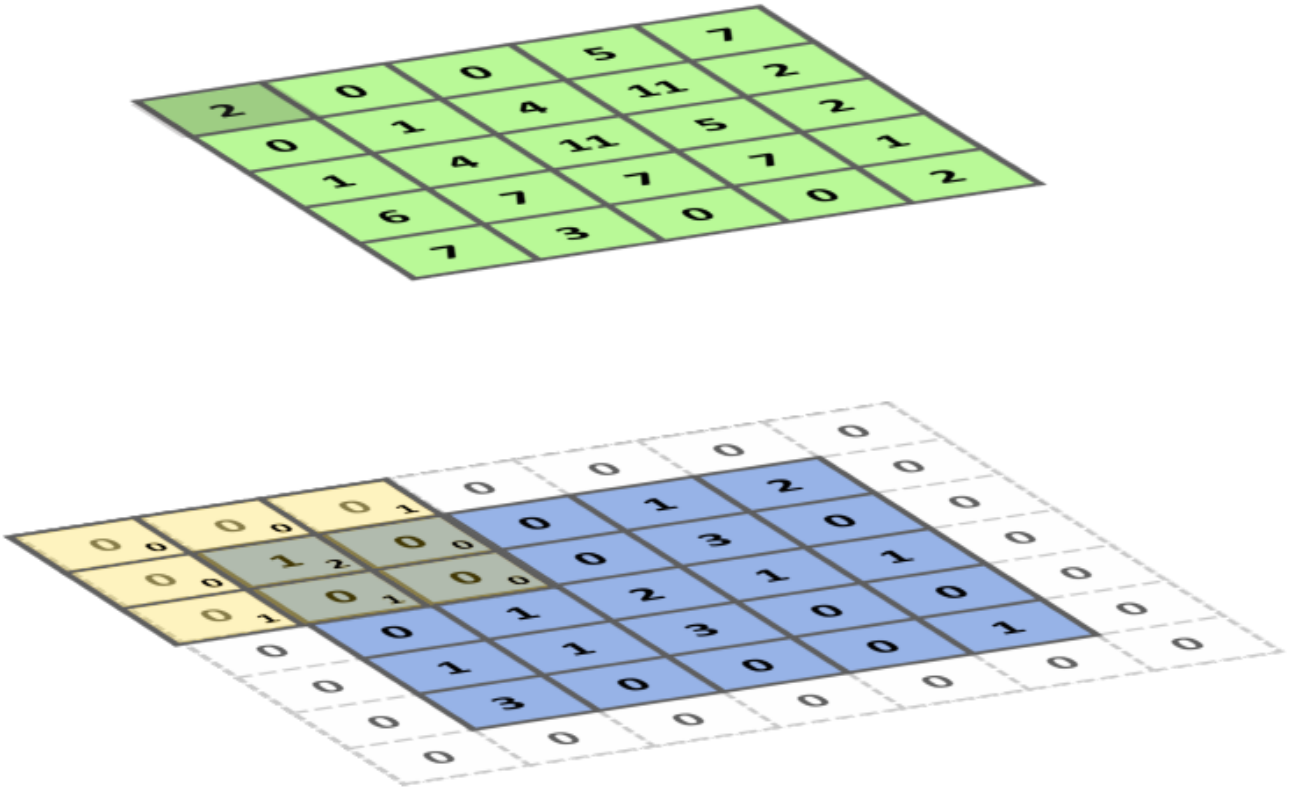
Stride: distance between two consecutive positions of the kernel





The effect of different strides

Stride: 1x1



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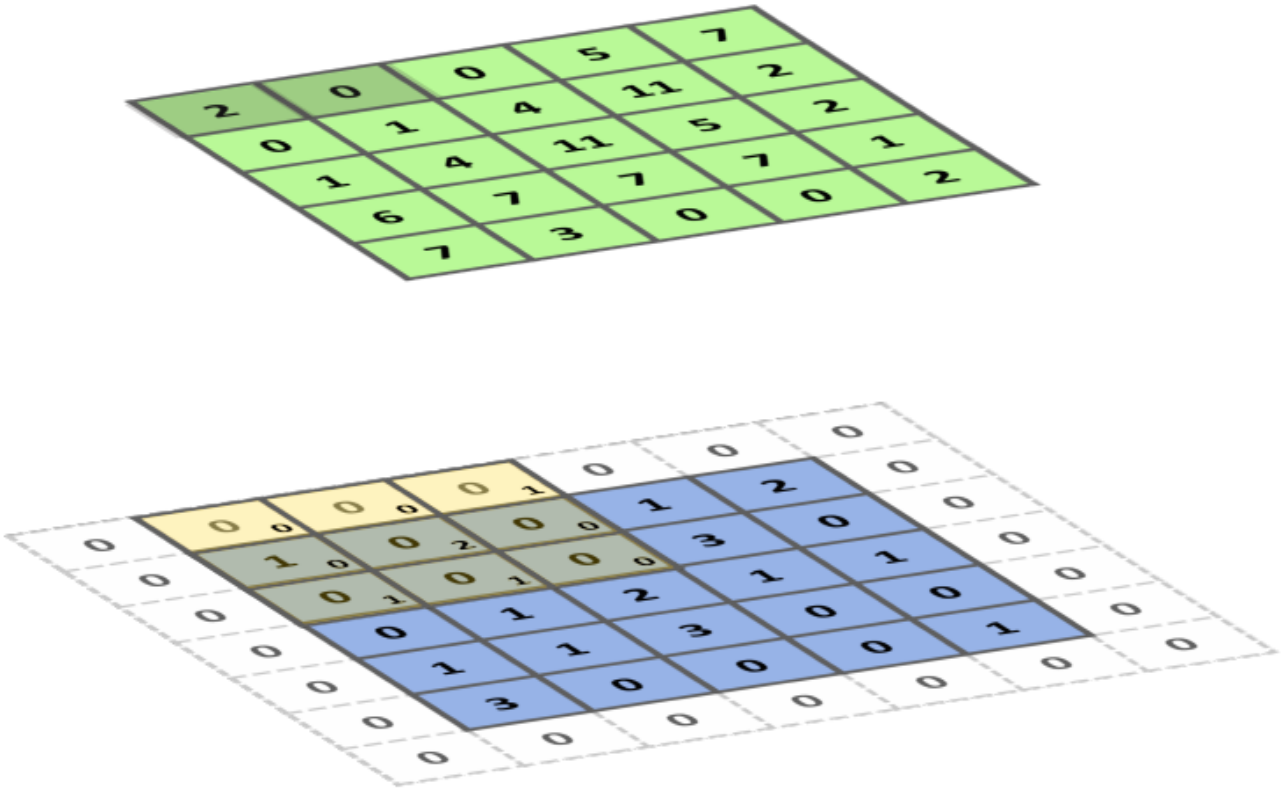

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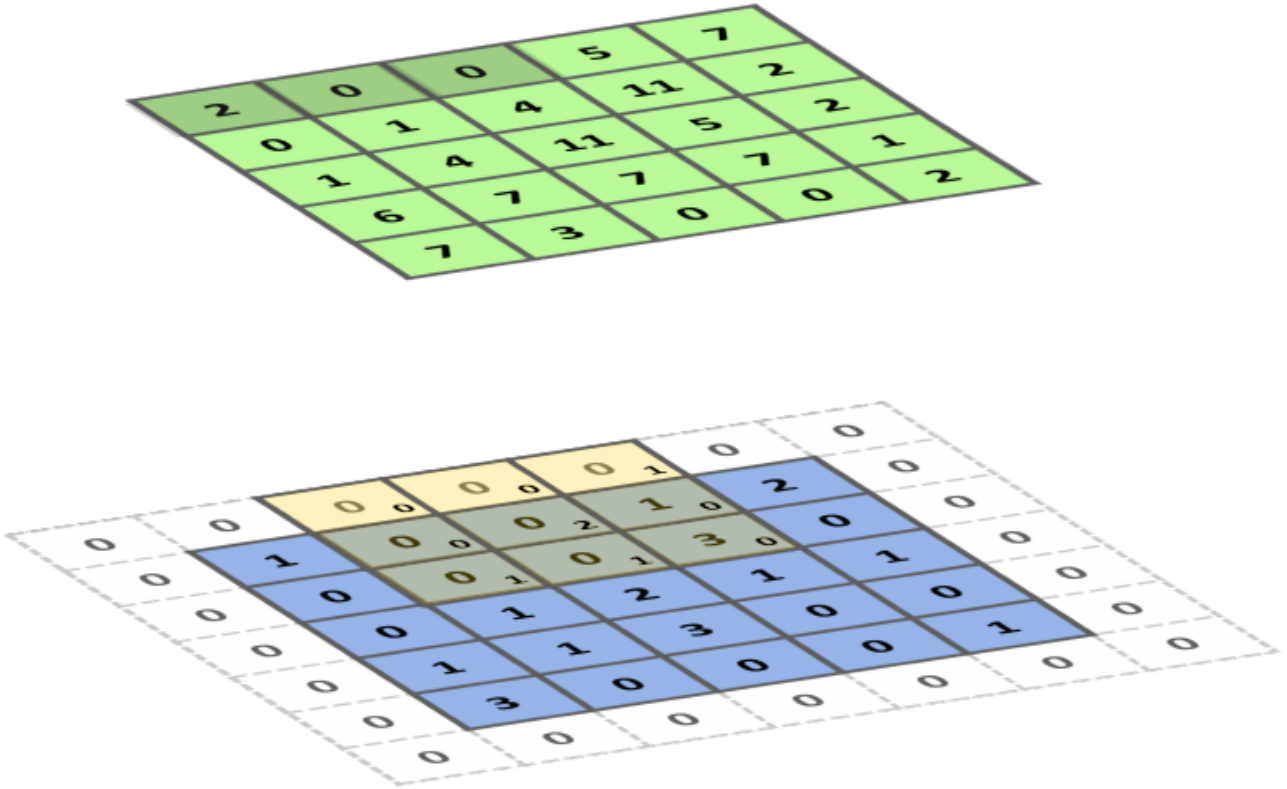
Stride: 1x1



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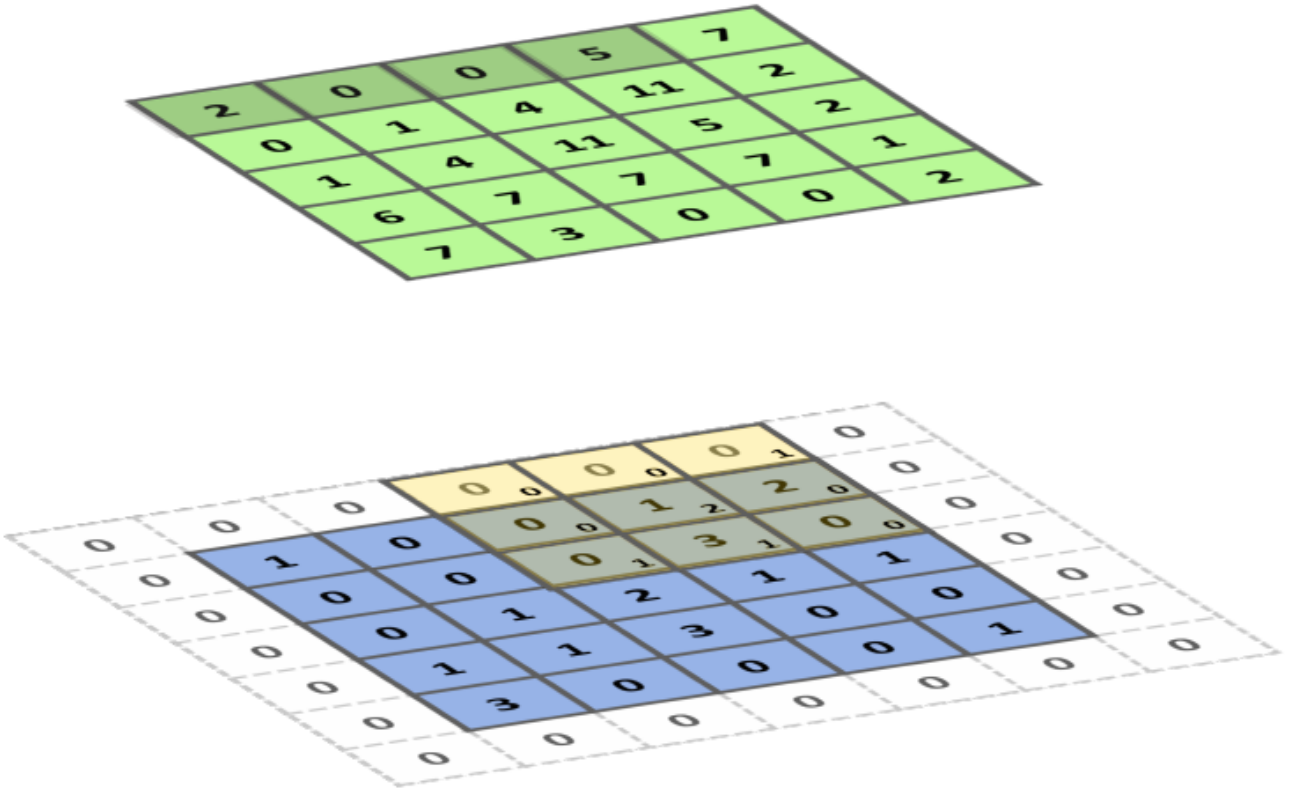
Stride: 1x1



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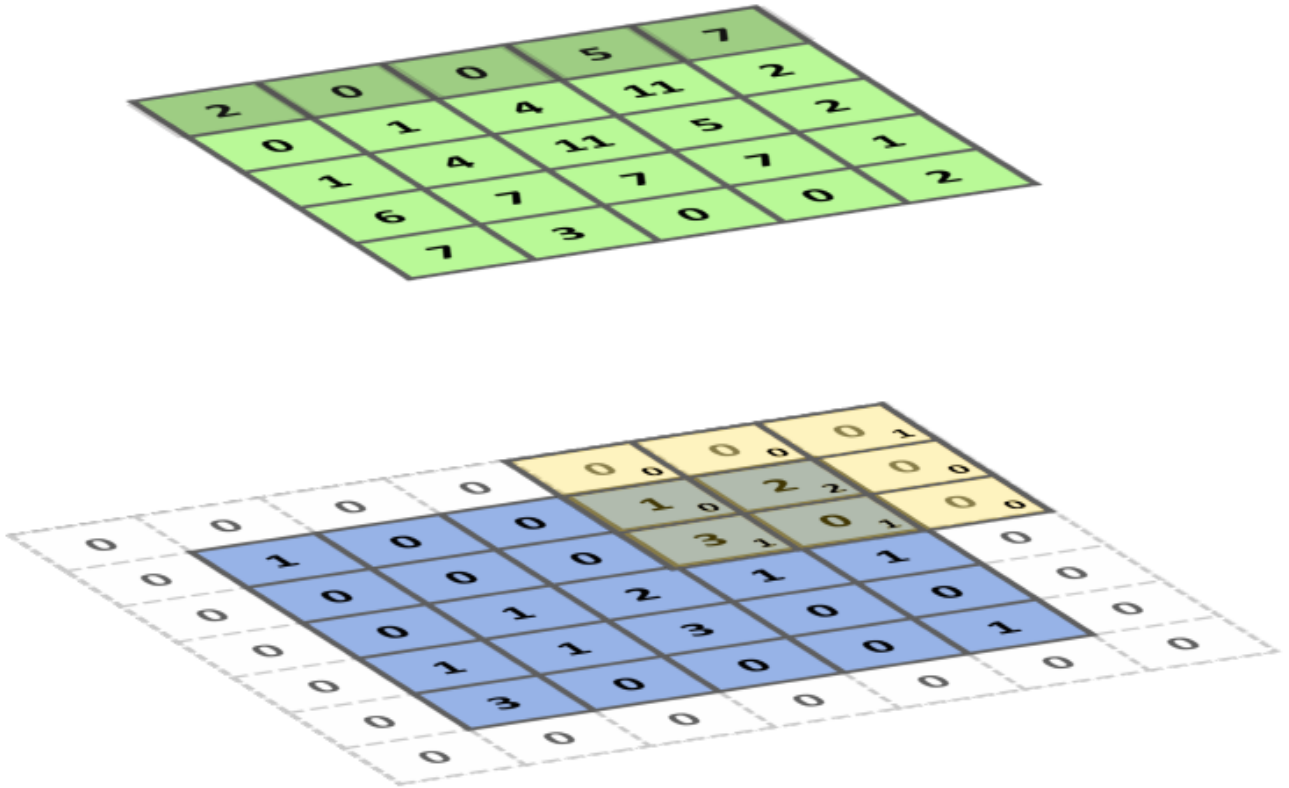
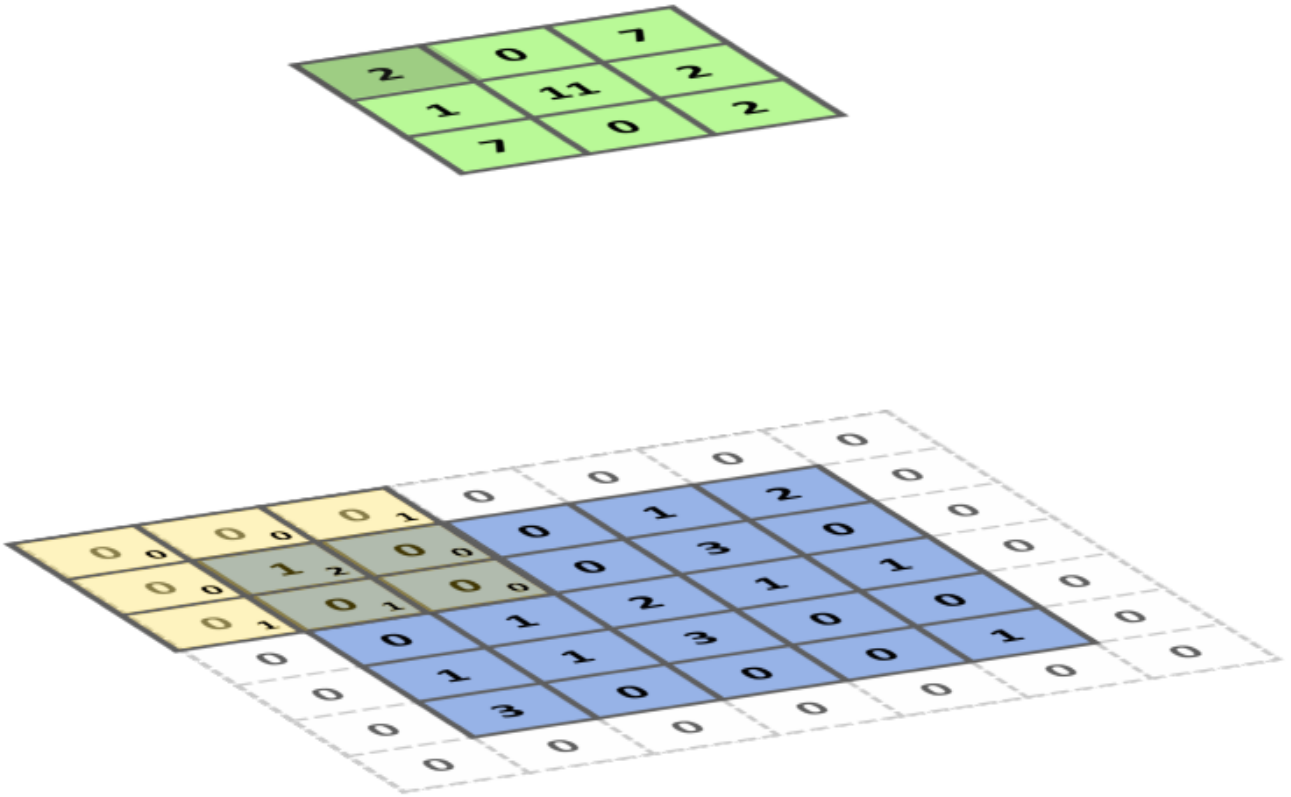


The effect of different strides

Stride: 2x2



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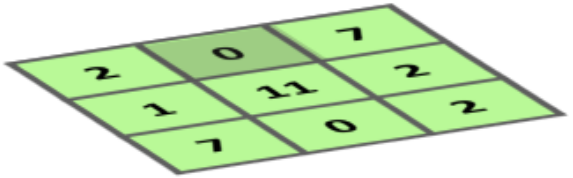
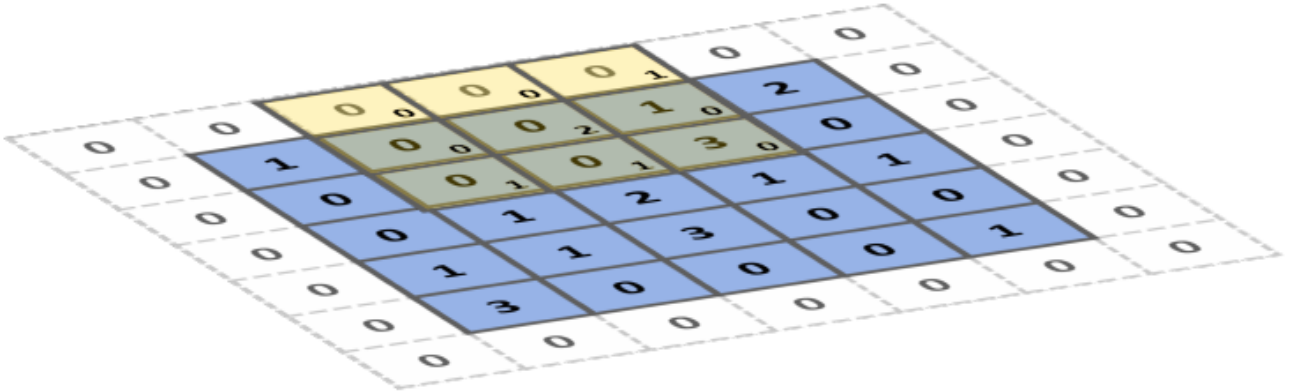
Stride: 2x2





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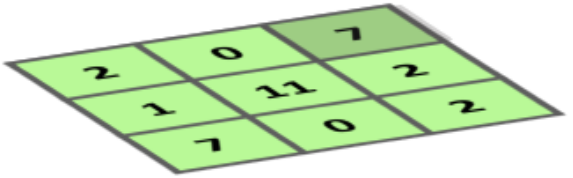

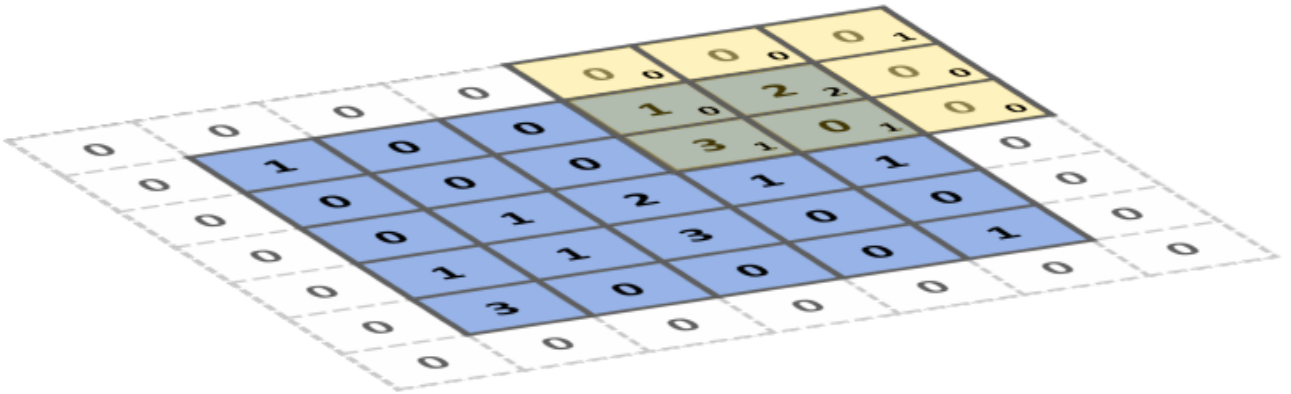

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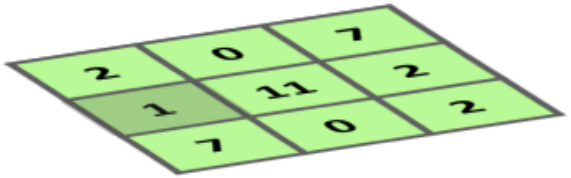

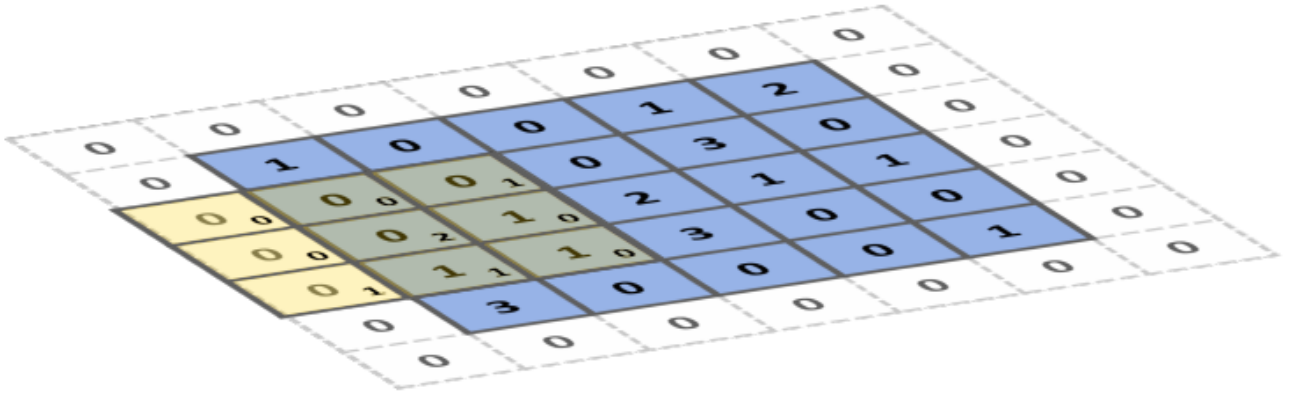

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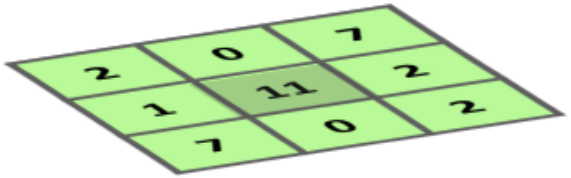

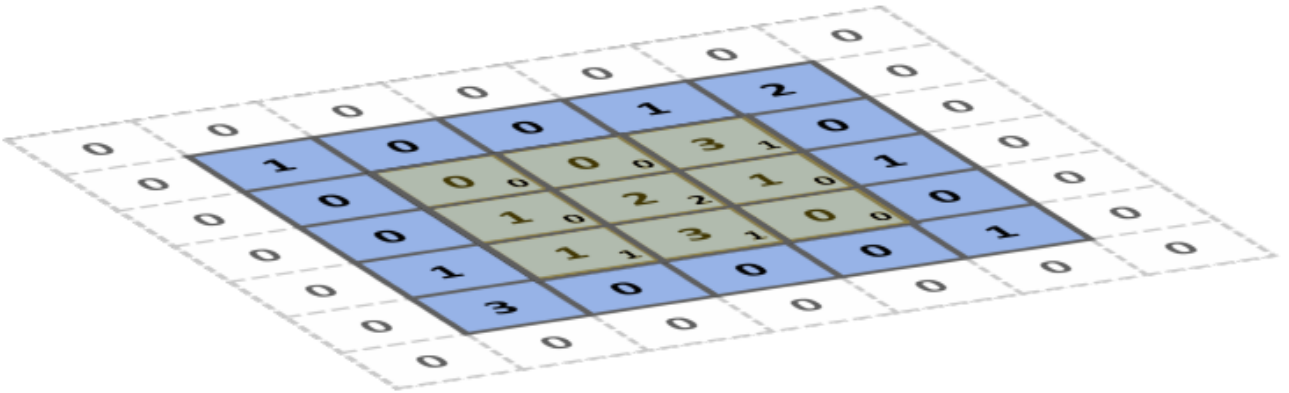

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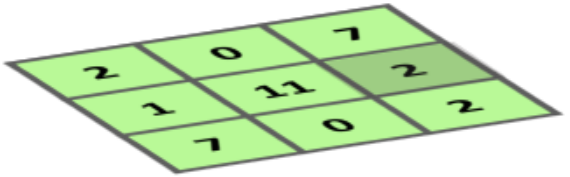

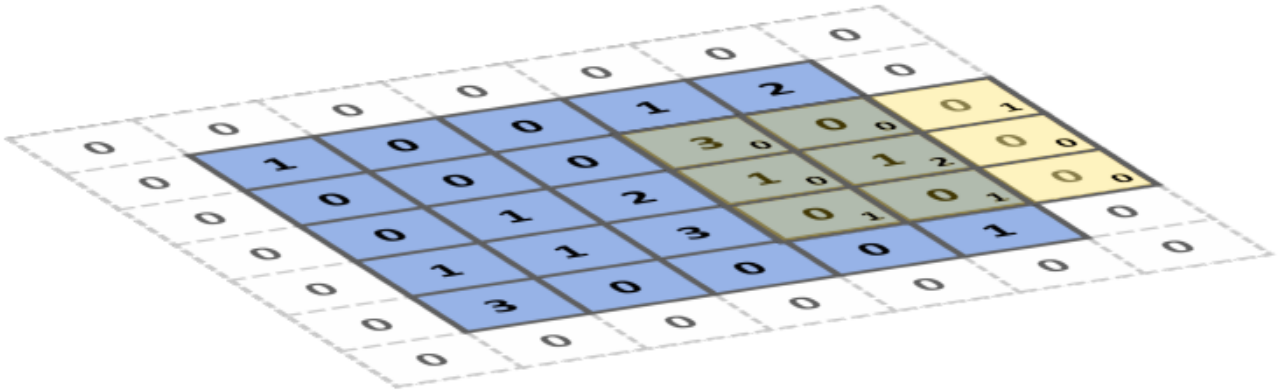

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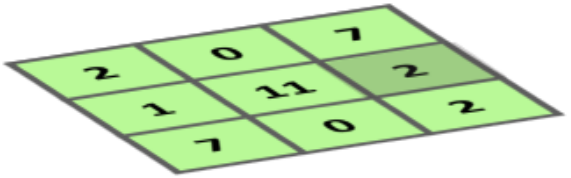
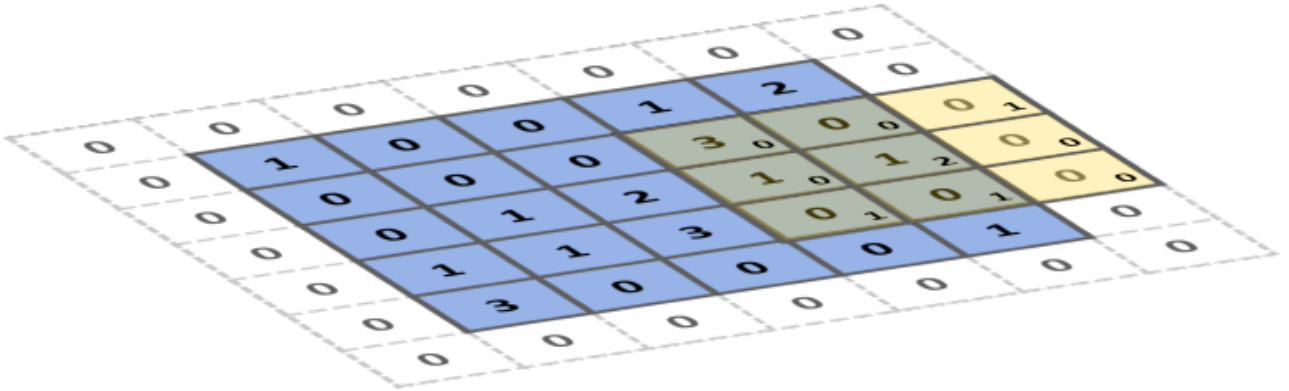


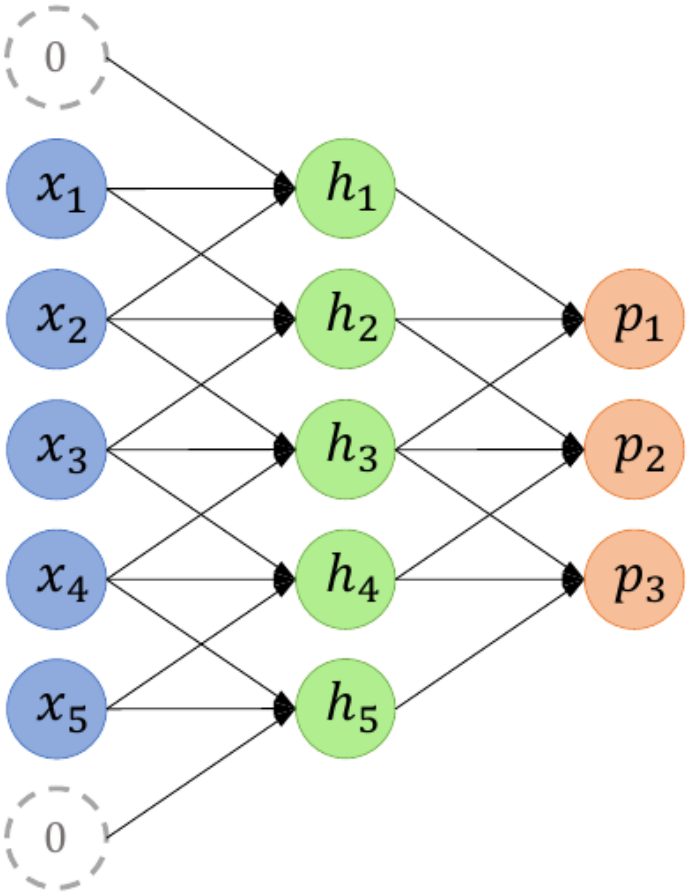
Why use stride > 1?

- Reduce redundancy
- Compress feature map



Pooling

Operation to aggregate or “summarize” sub-region of input.



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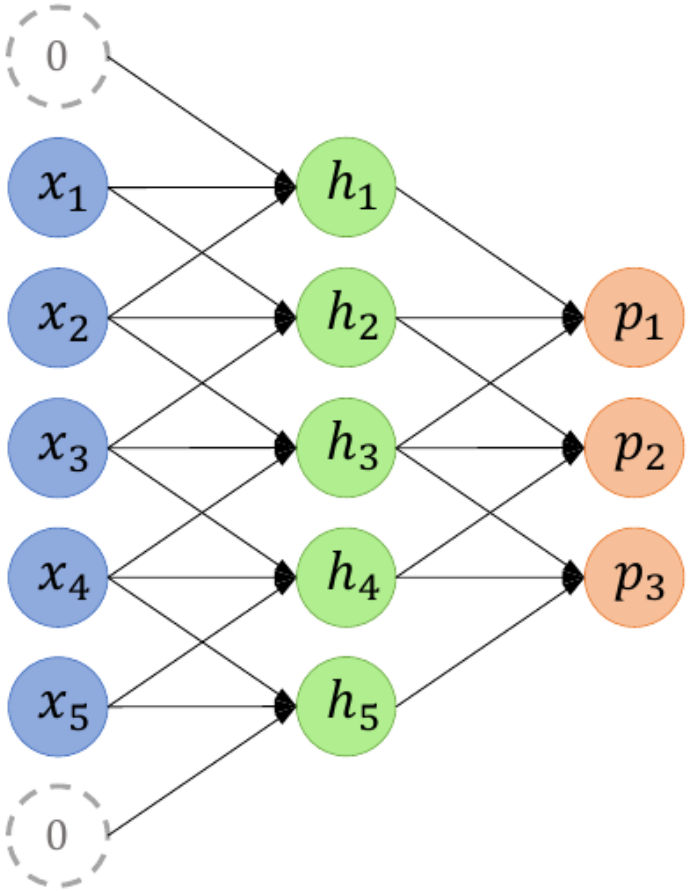
$$p = f(\boldsymbol{a})$$

$$f_{max}(\boldsymbol{a}) = \max_i(a_i)$$

$$f_{avg}(\boldsymbol{a}) = \frac{1}{N}\sum_{i=1}^{N} a_i$$



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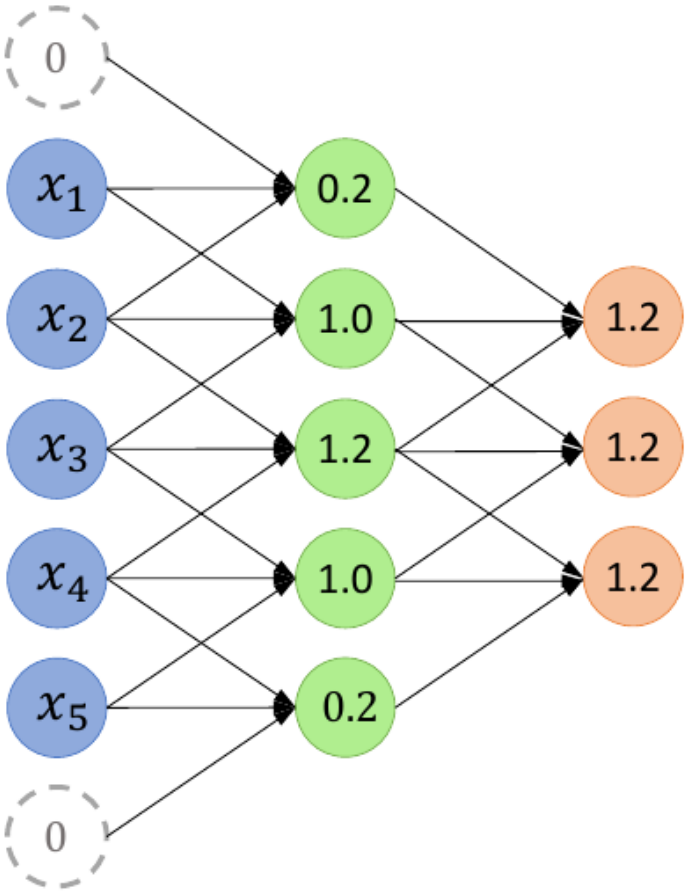

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5/5 inputs change but only 1/3 pooled outputs

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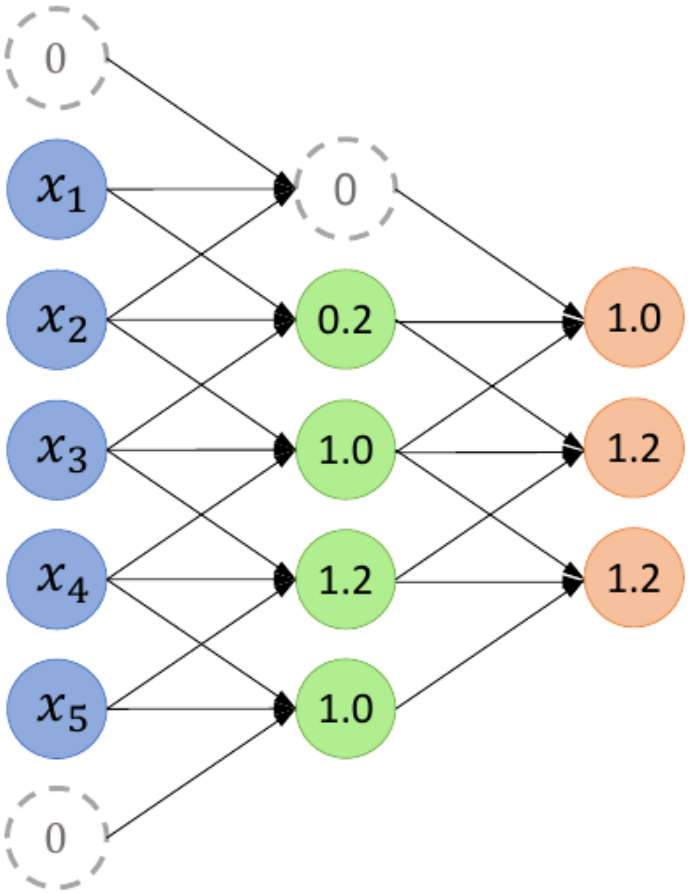

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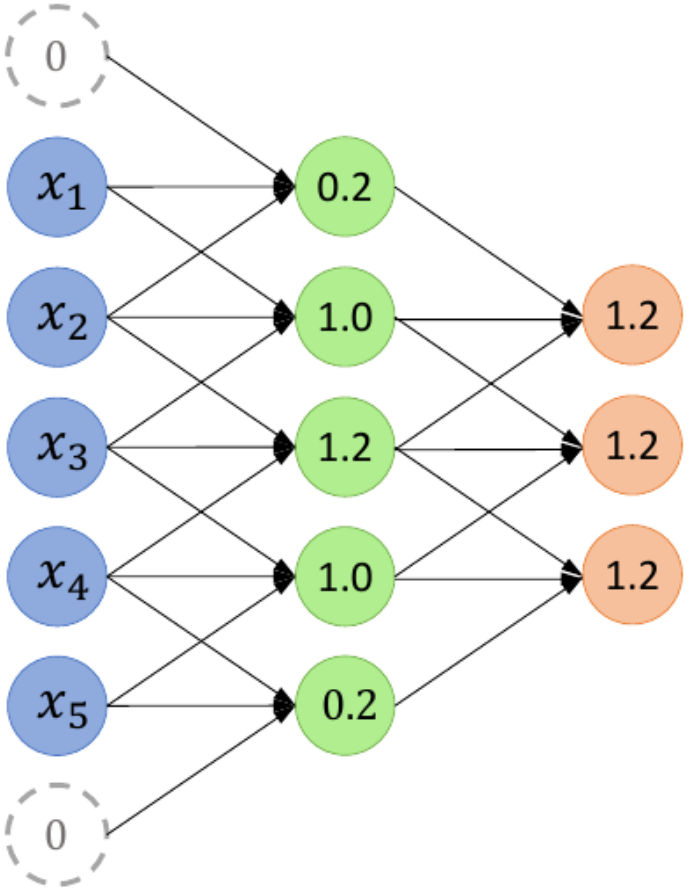

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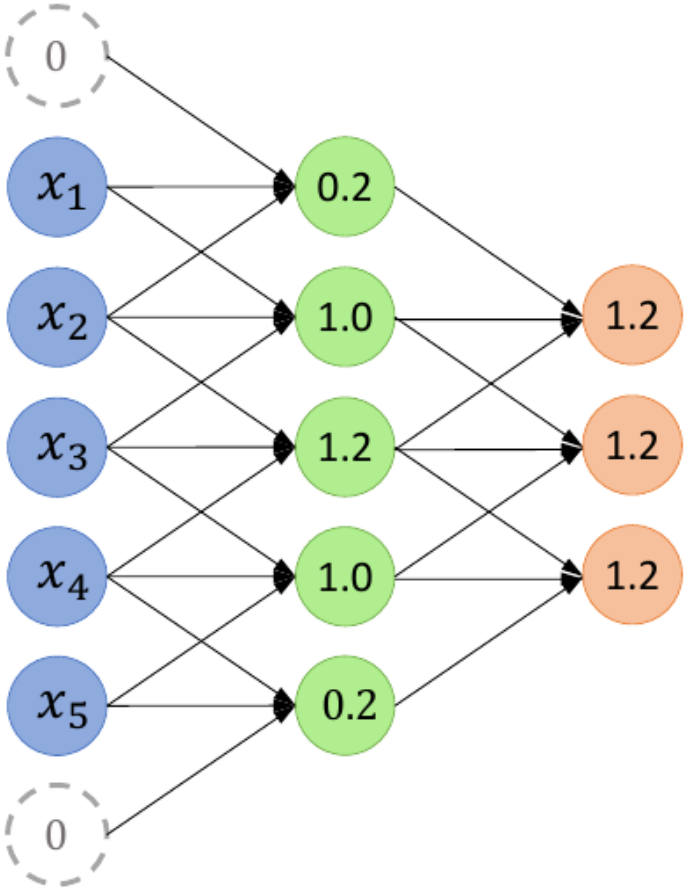

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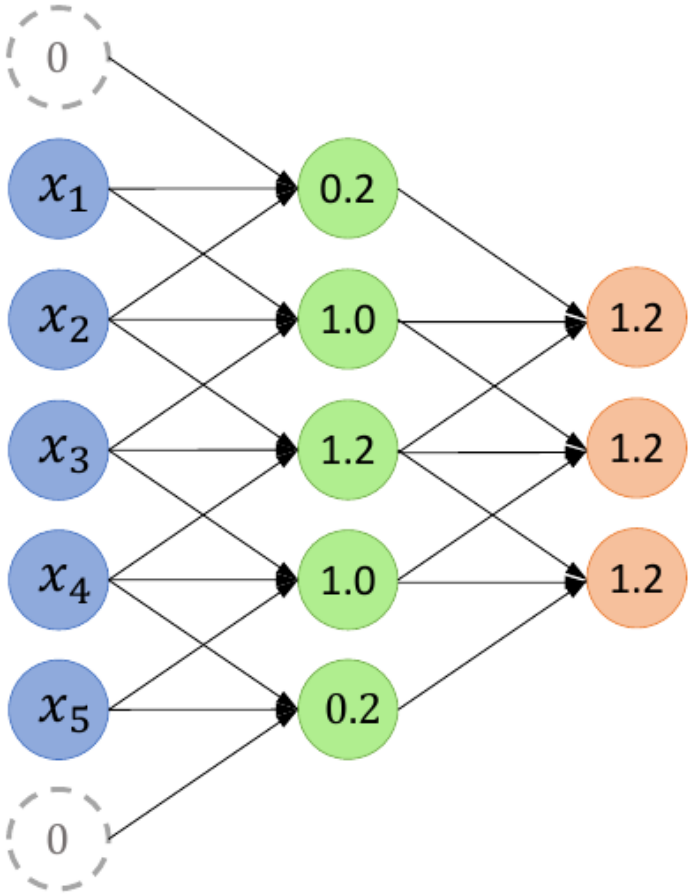
Why is pooling useful?

- Adds **translation invariance** (useful when we care about “what” more than “where”)
- Can be used to **compress signal** (useful to improve computational efficiency)



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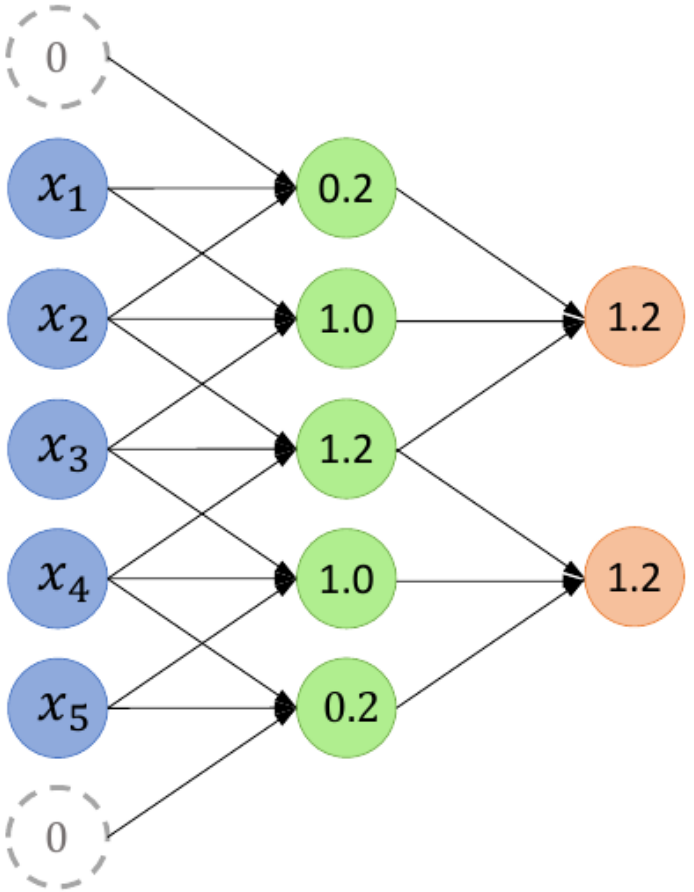
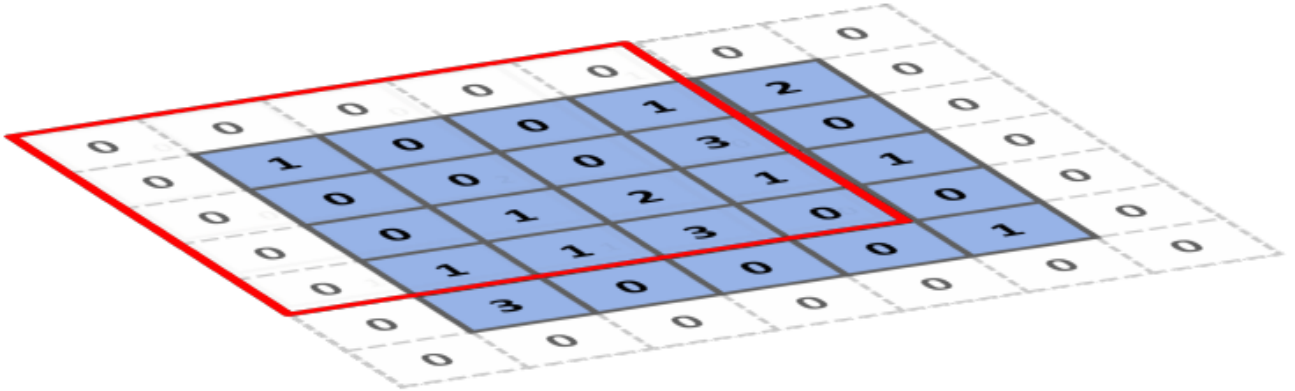


Dilated convolutions

When you want to increase receptive field without increasing computation…



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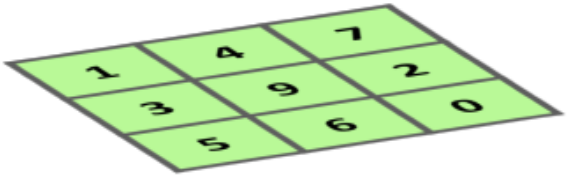

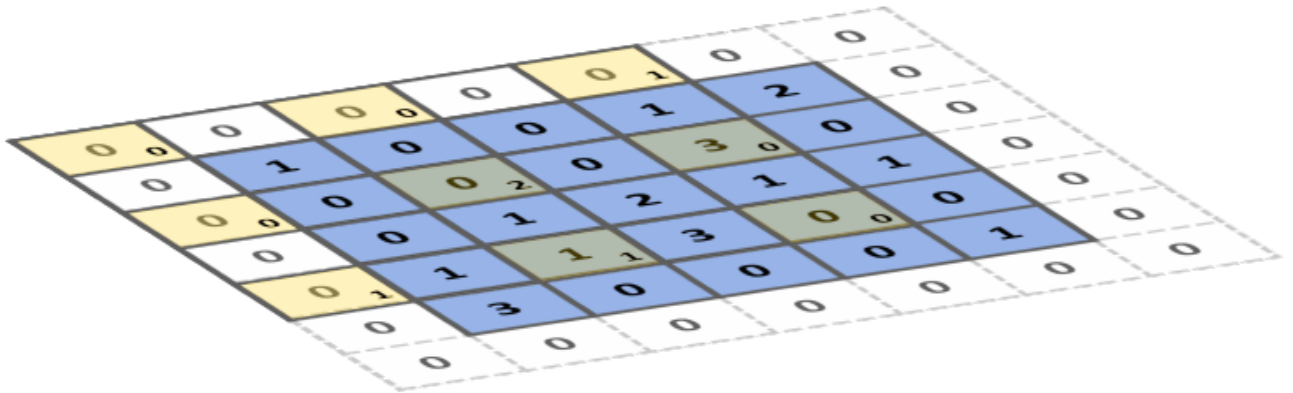

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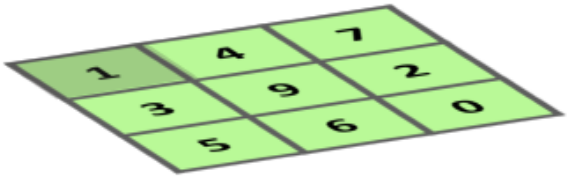

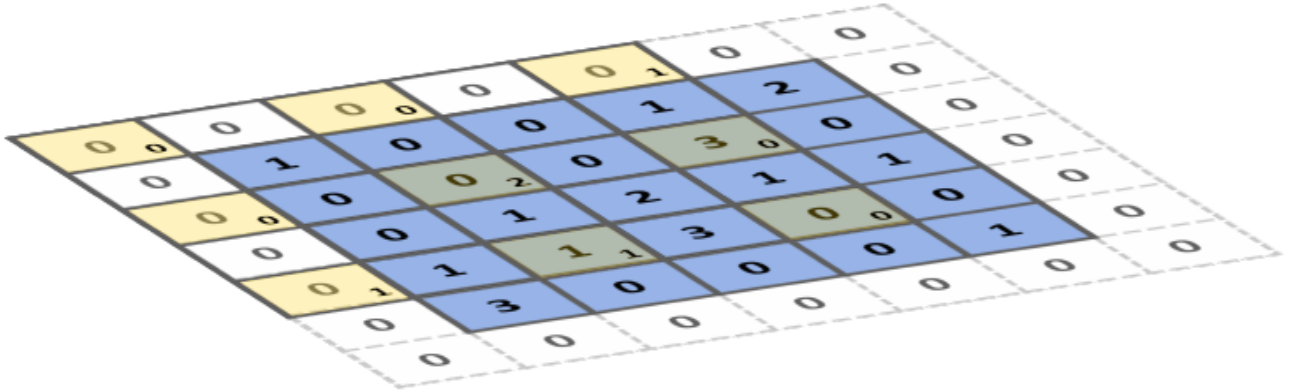

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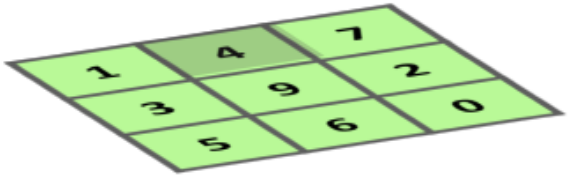

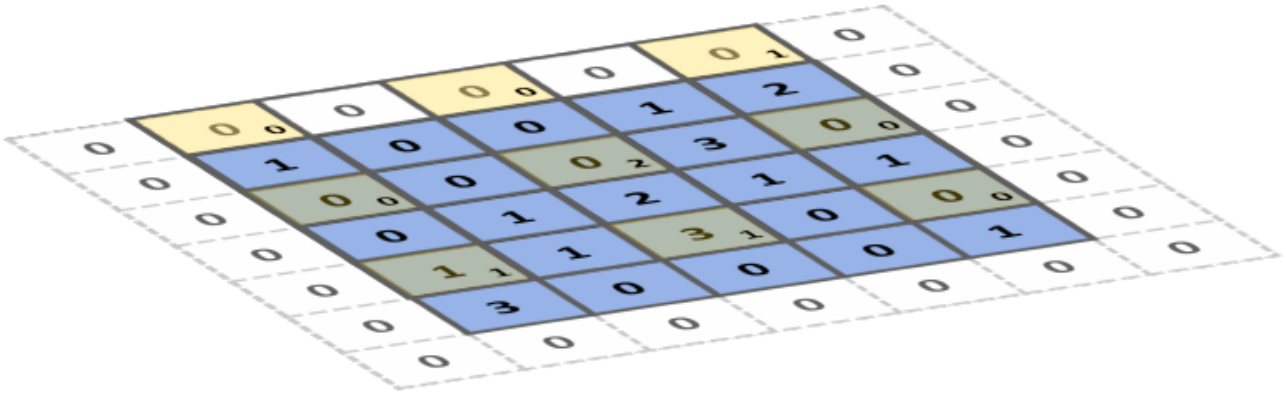

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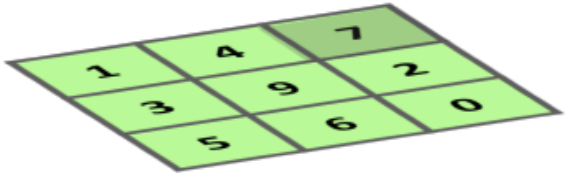
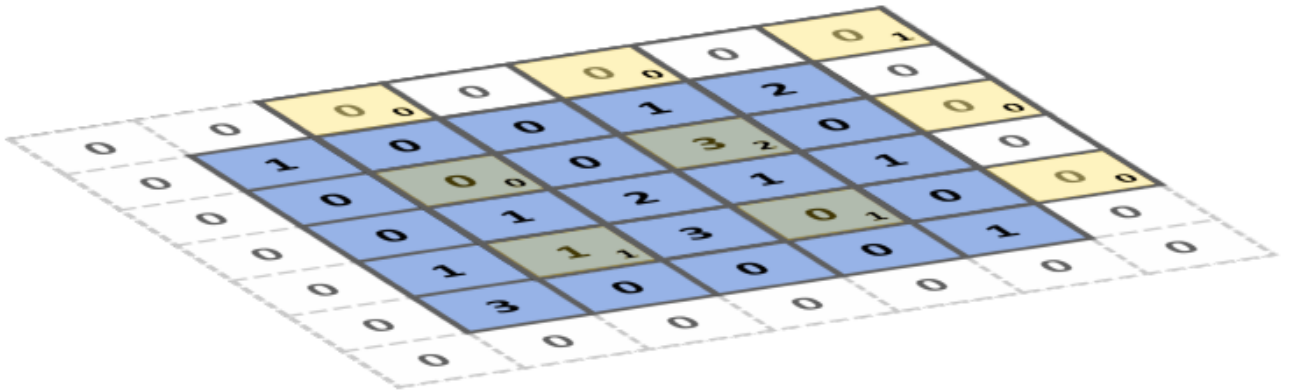
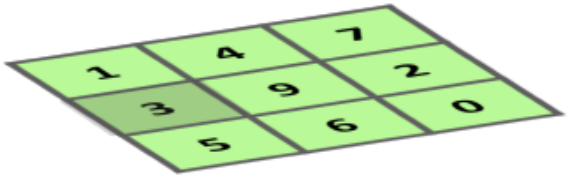
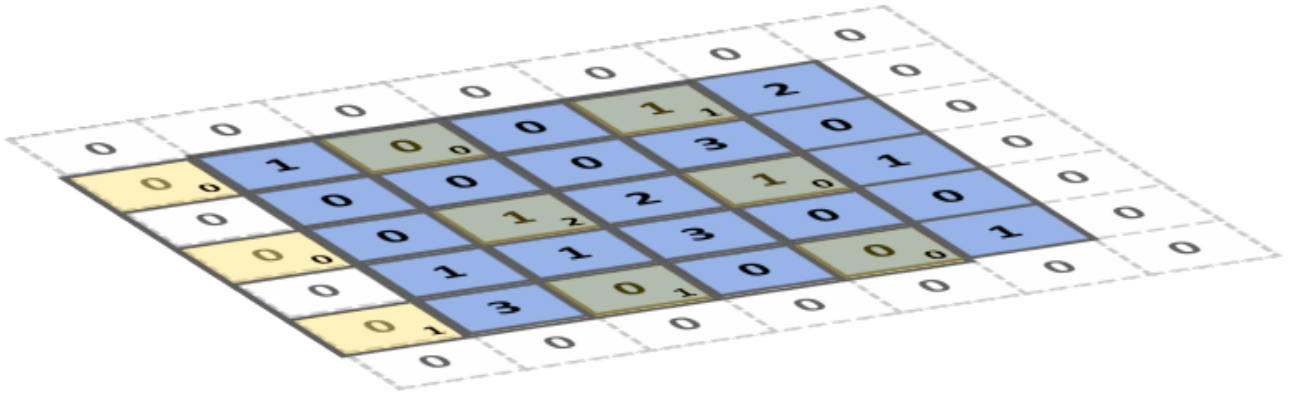




Dilated convolutions

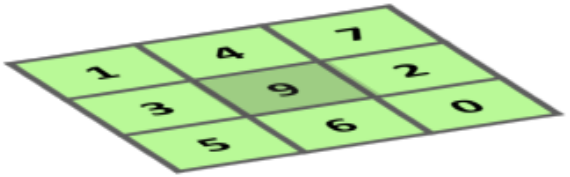
When you want to increase receptive field without increasing computation…

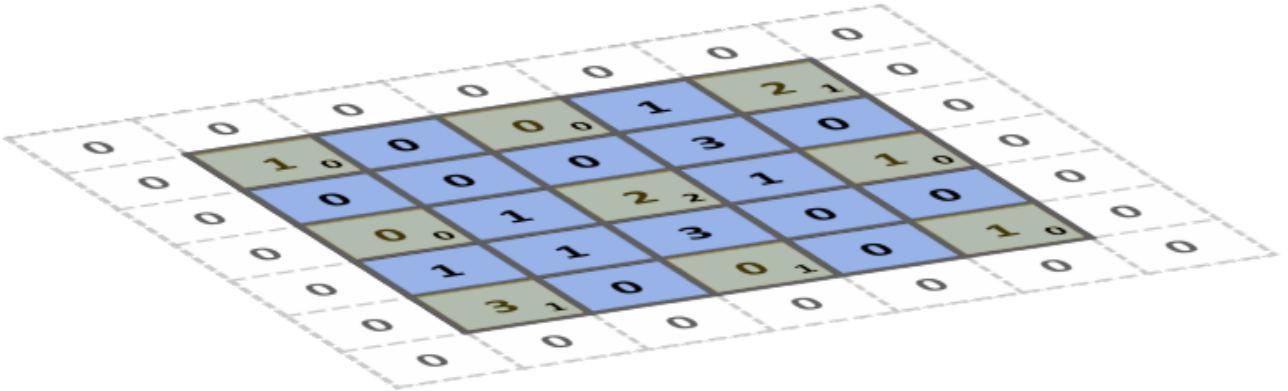




Dilated convolutions

When you want to increase receptive field without increasing computation…





Dilated convolutions

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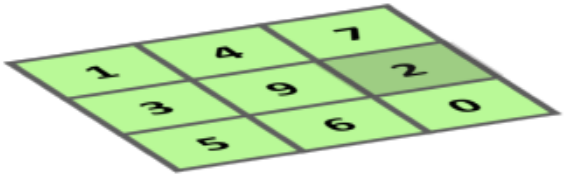

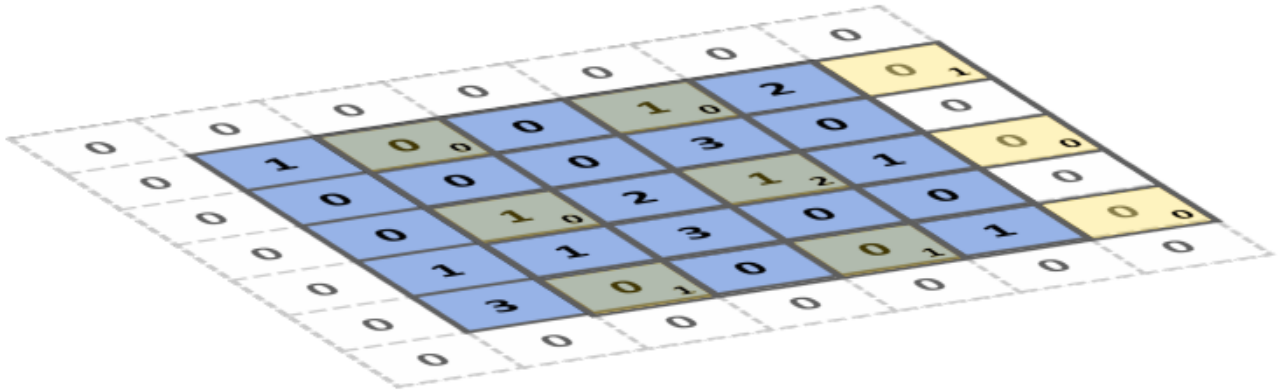

Dilated convolutions

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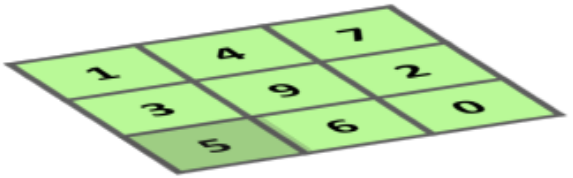
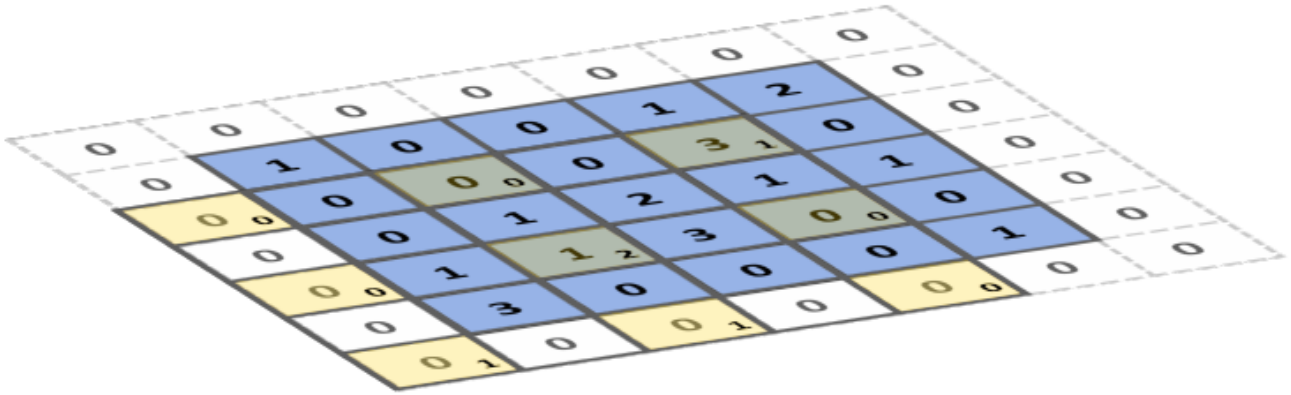
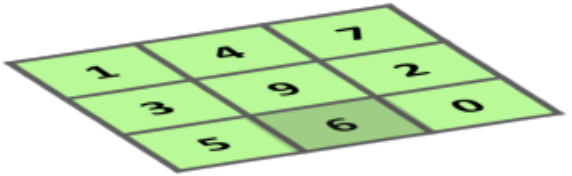
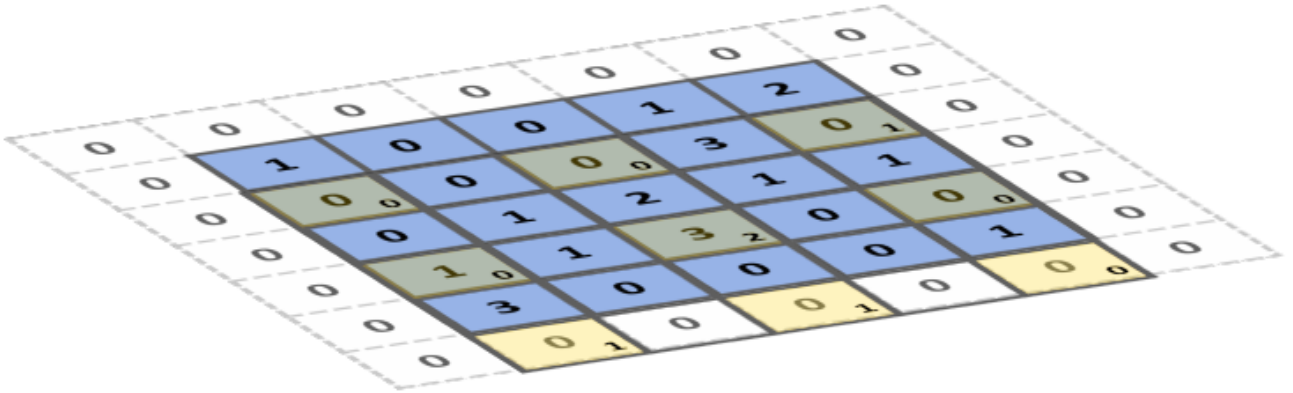




Dilated convolutions

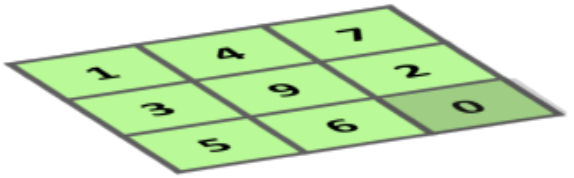
When you want to increase receptive field without increasing computation…



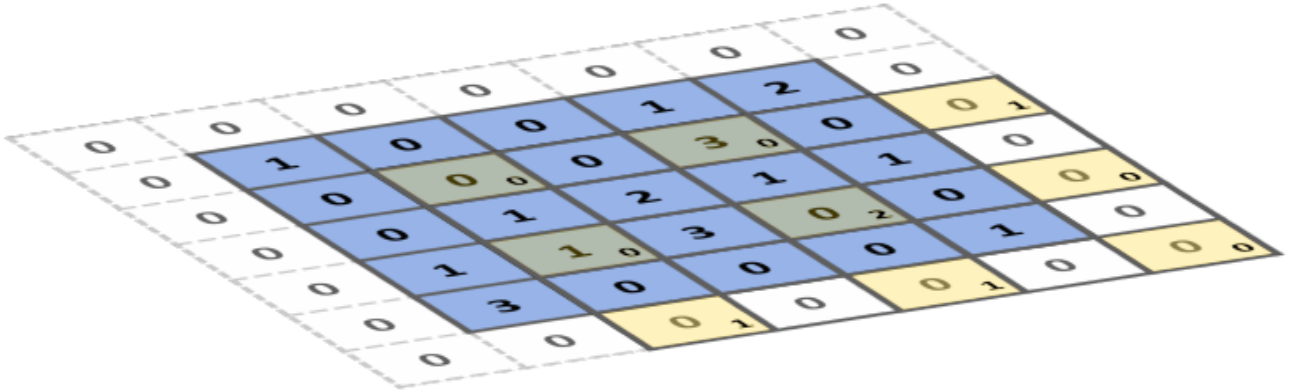


Dilated convolutions

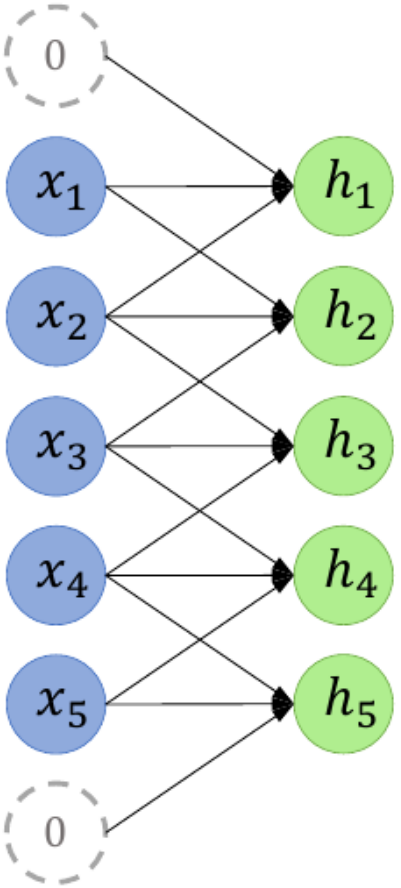
When you want to increase receptive field without increasing computation…



Also known as *“à trous”* (with holes) convolutions

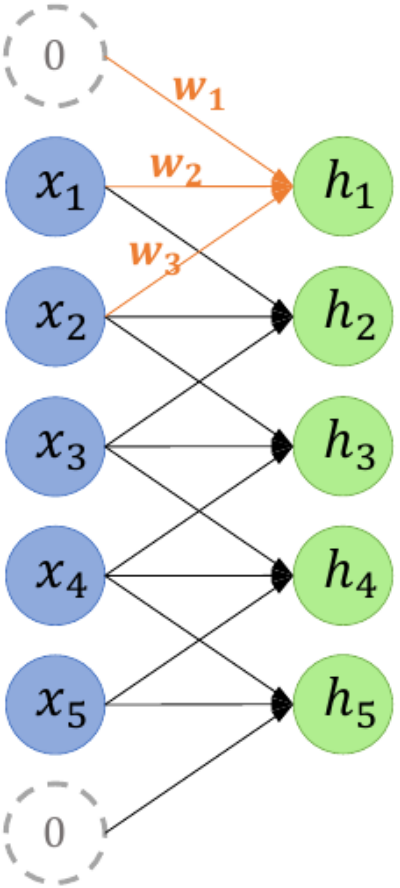


Tiled convolutions



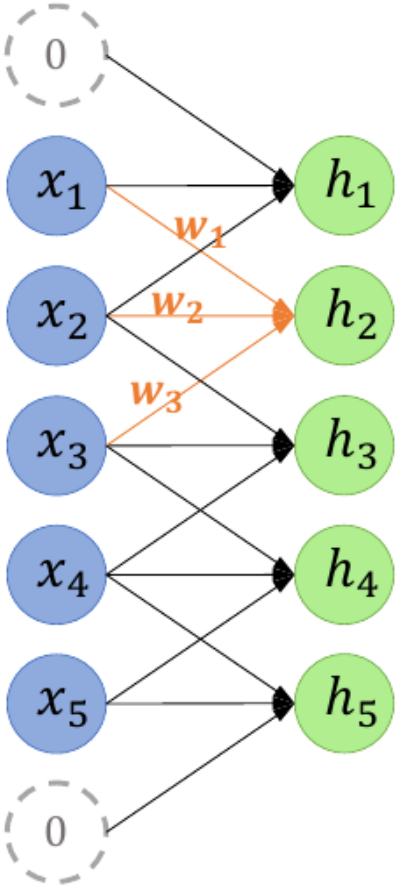
Reminder: regular convolutions slide same filter across signal

Tiled convolutions



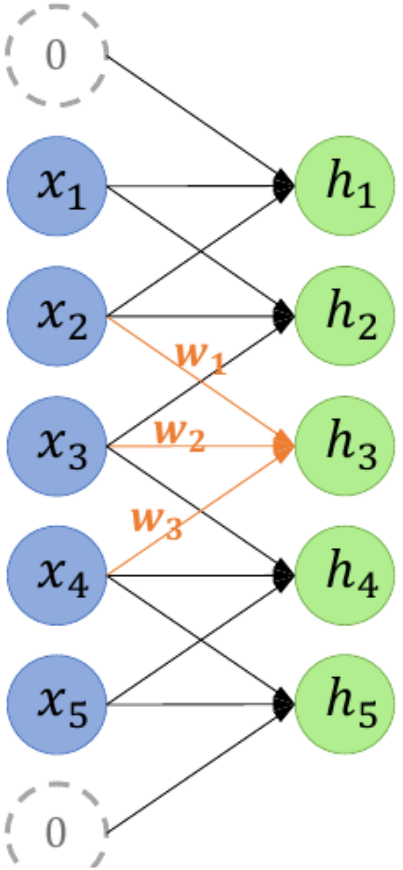
Reminder: regular convolutions slide same filter across signal

Tiled convolutions



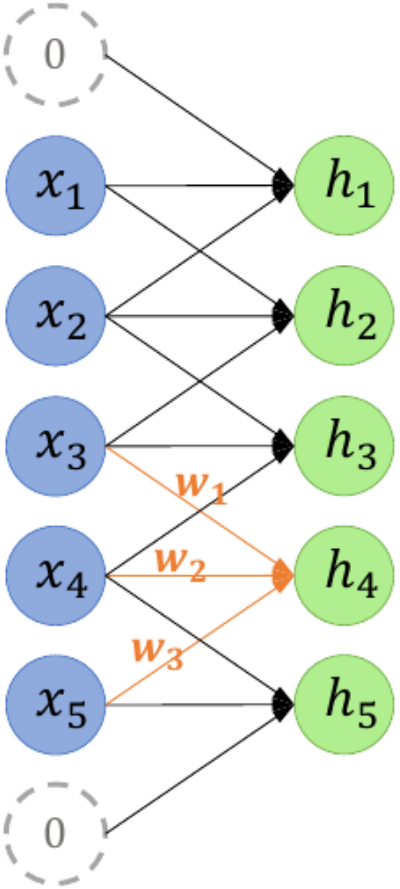
Reminder: regular convolutions slide same filter across signal

Tiled convolutions



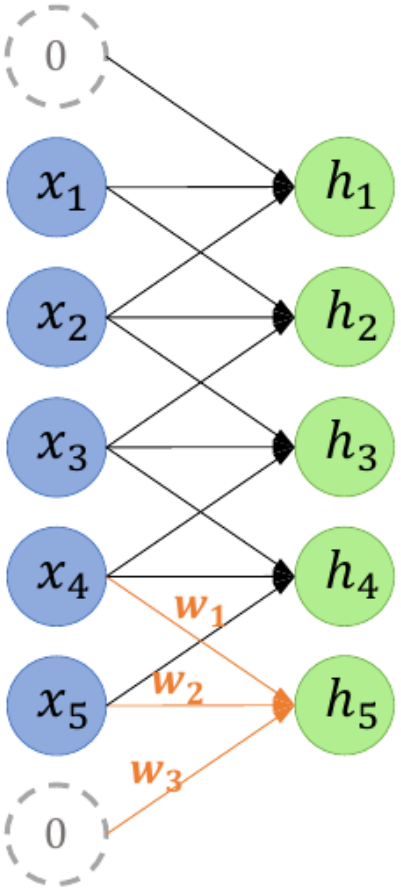
Reminder: regular convolutions slide same filter across signal

Tiled convolutions



Reminder: regular convolutions slide same filter across signal

Tiled convolutions



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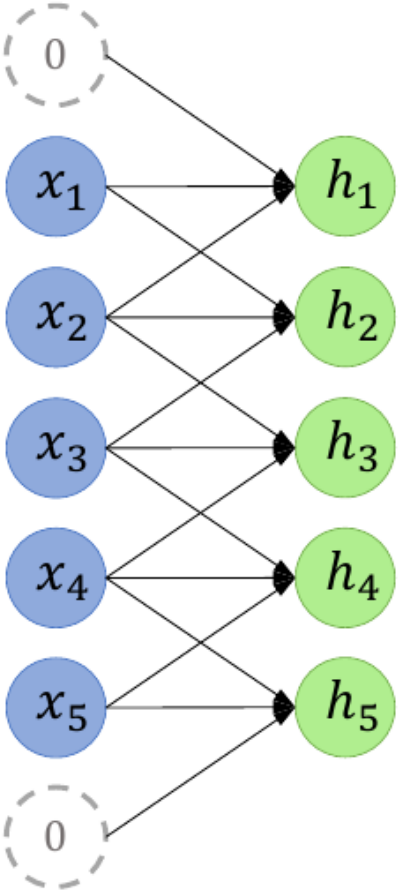
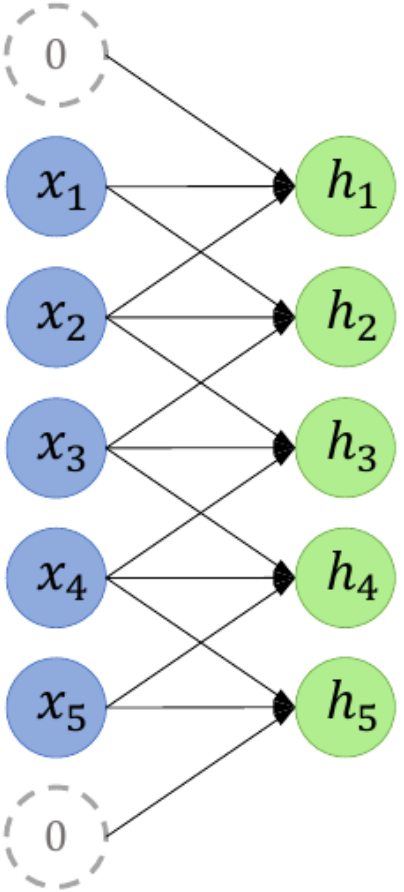
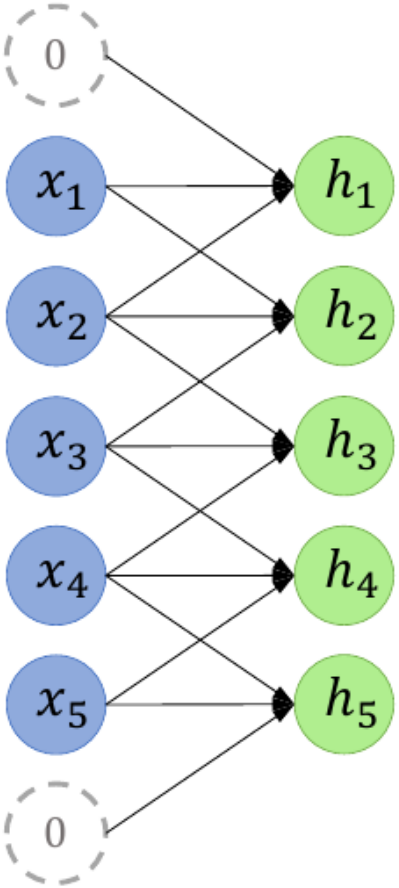
Tiled convolutions



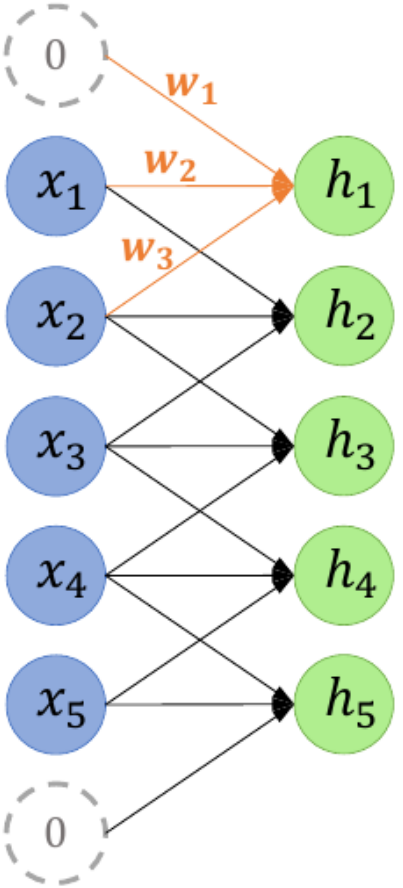
Cycle through different filters



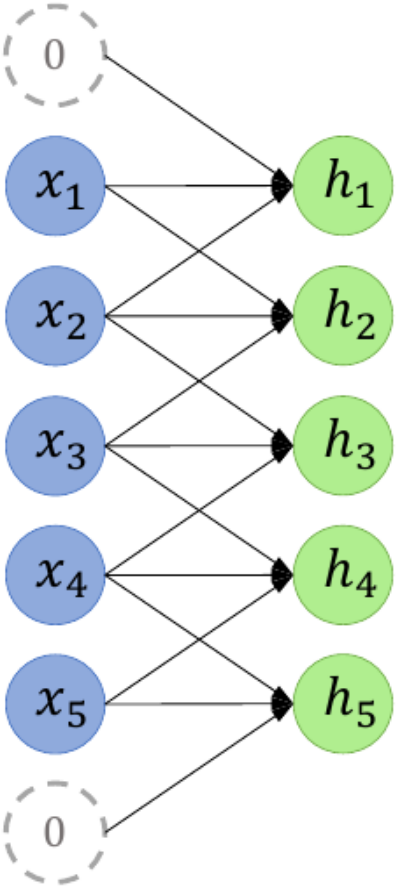
Tiled convolutions



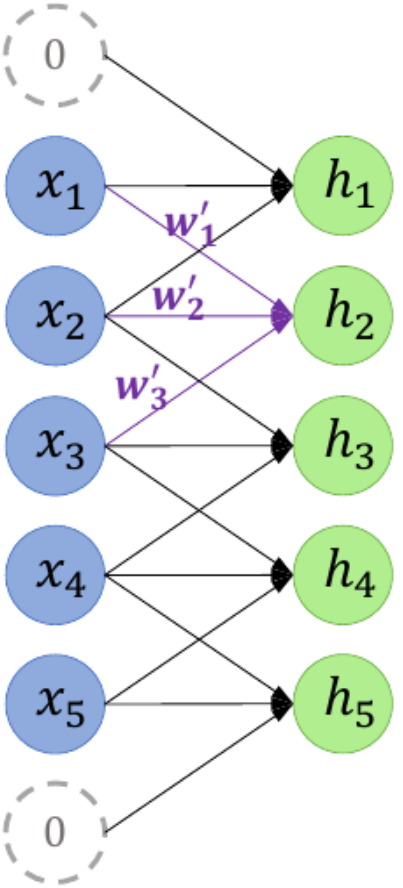
Cycle through different filters



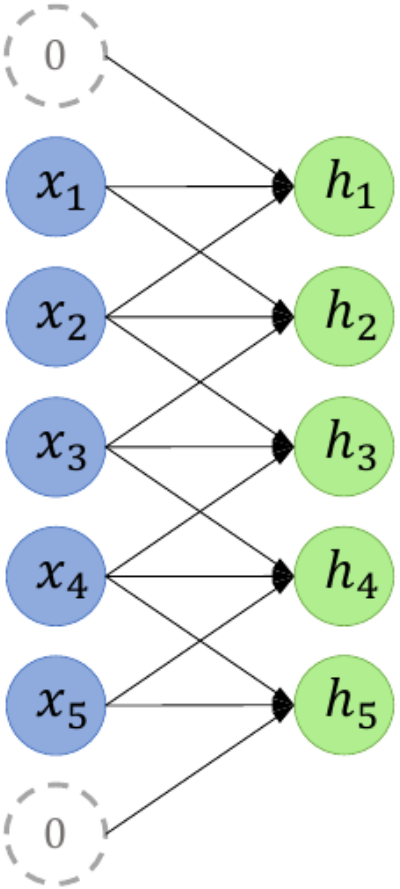
Tiled convolutions



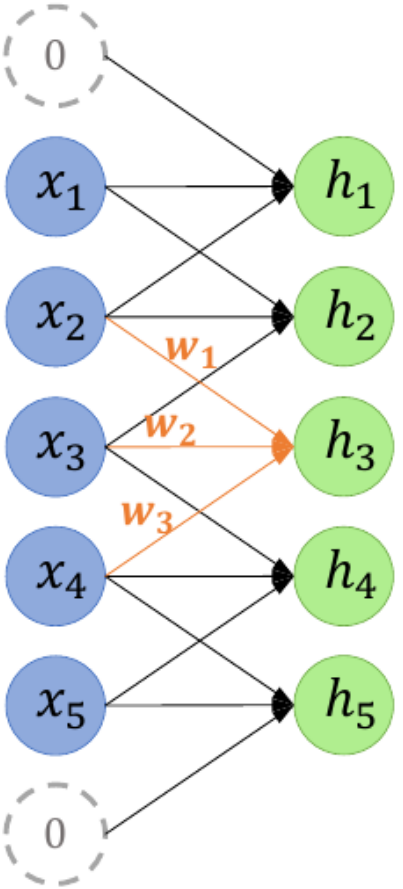
Cycle through different filters



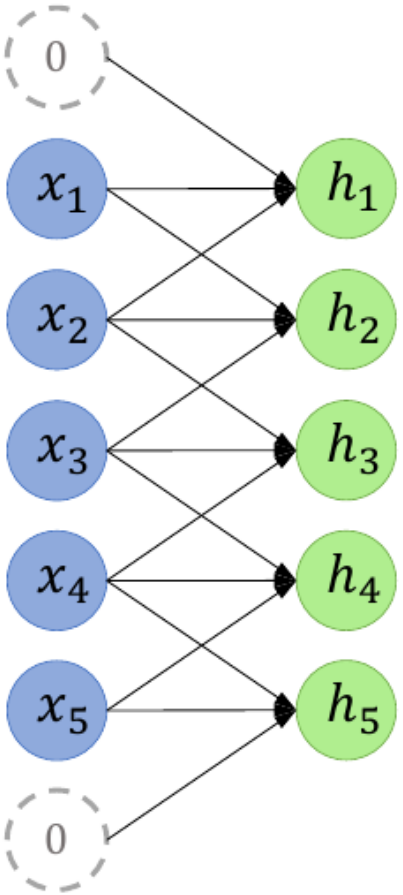
Tiled convolutions



Cycle through different filters



Tiled convolutions



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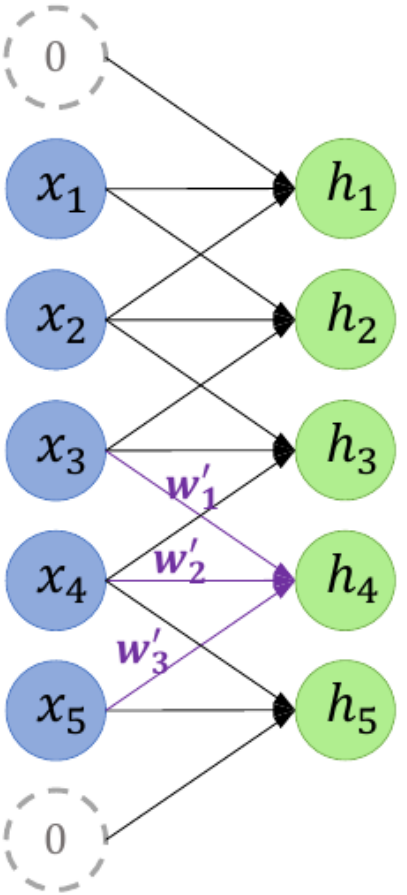
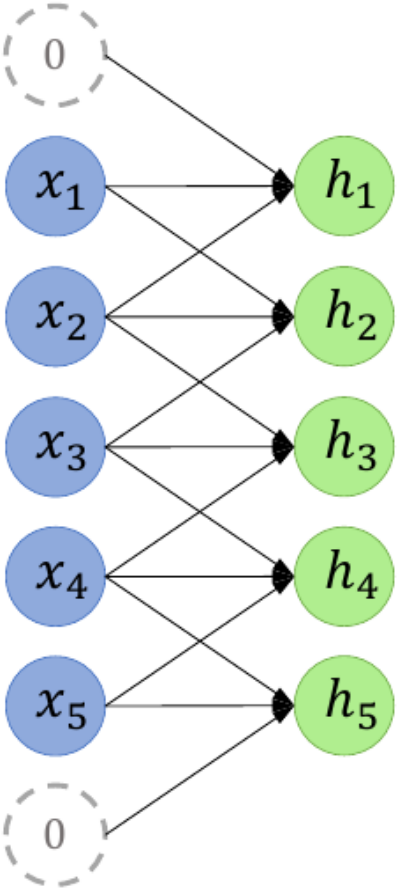
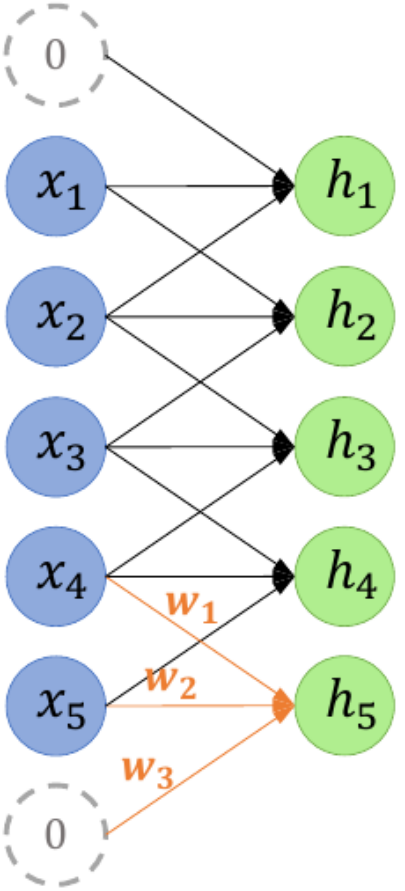


Tiled convolutions



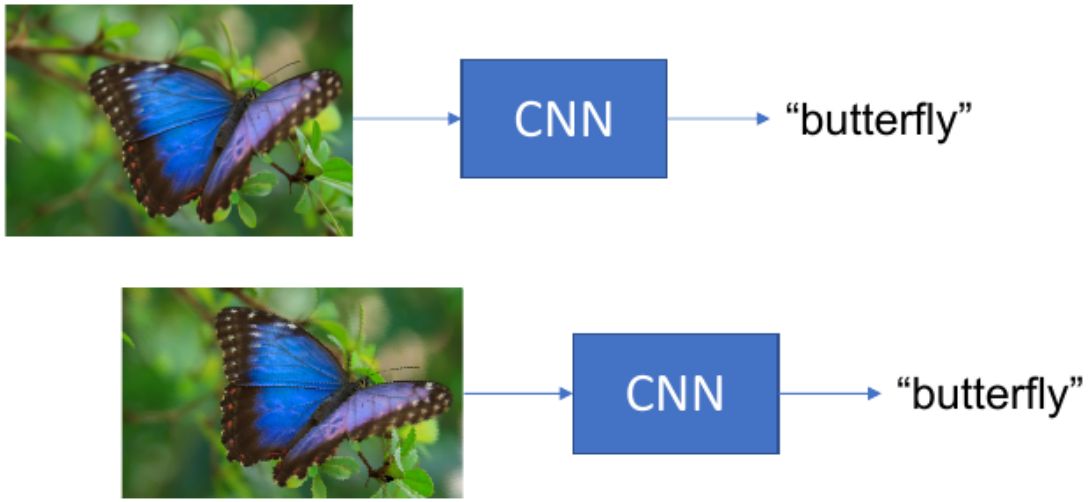
Cycle through different filters

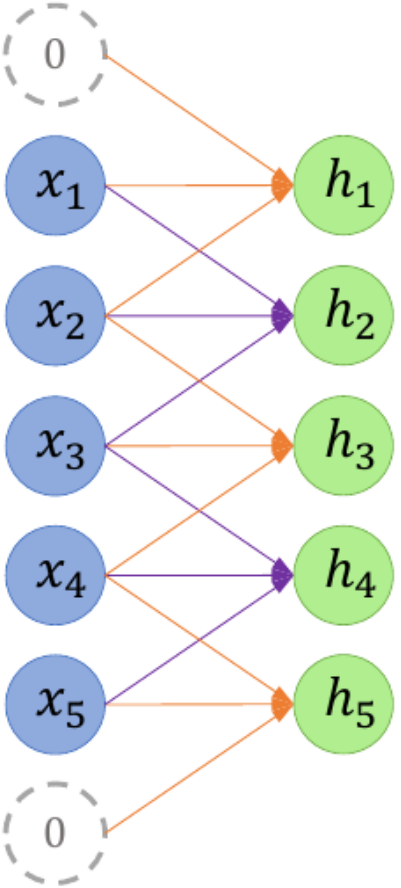


Tiled convolutions

Why is this useful?

Can capture other types of invariance (e.g. rotation)





ACKNOWLEDGEMENTS

Adapted version of the DeepLearning Indaba 2018