(2) total time units (for/while loops)

$= 1 + (n+1) + n$

$T(n) = 2n + 2$

T(n) is a polynomial of degree 2,

so $2n+2 = O(n)$

(3) total time units (consecutive statements)

$= 1 + 1 + 1$

$T(n) = 3$

T(n) is a polynomial of degree 0,

so $3 = O(n^0)$ or simply $O(1)$

(4) Total time units (if/else statements)

check the longest if/else section plus

on

$(1+1+1) + 1$

$T(n) = 4$

same as question 3, $O(1)$

(5) total time units (nested loop)

$= 1 + \underbrace{(n+1)}_{\text{outside loop}} \underbrace{((n+1)+n)}_{\text{inside loop}}$

$= (n+1)(2n+1)+1$

$T(n) = 2n^2 + 3n + 2$

T(n) is a polynomial of degree 2, so

T(n) is $O(n^2)$

(6) Total time units for nested loop

$$1 + \underbrace{(n+1)}_{\text{outside}}\left(\underbrace{(i+1)}_{\text{inside}} + i\right)$$

where i ranges from values 1 to n.
For purposes of calculation we will replace i with n, giving

$$T(n) = 2n^2 + 3n + 2 \quad \text{and} \quad O(n^2)$$

(7) total time for nested loop

$$= 1 + \underbrace{(\log n + 1)}_{\text{outside}}\underbrace{((n+1) + n)}_{\text{inside}}$$

$$= (\log n + 1)(2n + 1) + 1$$

$T(n) = 2n\log n + \log n + 2n + 2$
using coefficient rule and sum rule
$O(n \log n)$

(8) a) $c_1 n \geq \frac{1}{2}c_1 n$ for all values of n so $c_1 n = \Omega(n)$

$c_1 n \leq 2c_1 n$ for all values of n so $c_1 n = O(n)$

b) $C_2 n^3 + C_3 \geq F n^3$ (where $F = C_2 - C_3$)
for all values of n

so $C_2 n^3 + C_3 = \Omega(n^3)$

$C_2 n^3 + C_3 \leq G n^3$ (where $G = C_2 + C_3$)
for all values of n

so $C_2 n^3 + C_3 = O(n^3)$

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c) $C_4 n \log n + C_5 n \geq F n \log n$ (where $F = C_4 - C_5$)
for all $n > 1$

so $C_4 n \log n + C_5 n = \Omega(n \log n)$

$C_4 n \log n + C_5 n \leq G n \log n$ where $G = (C_4 + C_5)^{\wedge (C_4 + C_5)}$
for all $n > 1$

so $C_4 n \log n + C_5 n = O(n \log n)$

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d) $C_6 2^n + C_7 n^6 \geq F 2^n$ where $F = C_6 - C_7$
for all values of n

so $C_6 2^n + C_7 n^6 = \Omega(2^n)$

$C_6 2^n + C_7 n^6 \leq G 2^n$ where $G = (C_6 + C_7)^{\wedge (C_6 + C_7)}$
for all values of n

So $C_6 2^n + C_7 n^6 = O(2^n)$

9 a) $\lim_{n \to \infty} \frac{f(n)}{g(n)}$

$\lim_{n \to \infty} \frac{\log n^2}{\log n + 5} = 2$

f(n) grows at the same rate as g(n)

f(n) is $\Theta(g(n))$

b) $\lim_{n \to \infty} \frac{f(n)}{g(n)}$

$\lim_{n \to \infty} \frac{\sqrt{n}}{\log n^2} = \infty$

f(n) grows at the same rate or faster than g(n)

f(n) is $\Omega(g(n))$

c) $\lim_{n \to \infty} \frac{f(n)}{g(n)}$

$\lim_{n \to \infty} \frac{\log^2 n}{\log n} = \infty$

f(n) grows at the same rate or faster than g(n)

f(n) is $\Omega(g(n))$.

d) $\lim_{n \to \infty} \frac{f(n)}{g(n)}$

$\lim_{n \to \infty} \frac{n}{\log^2 n} = \infty$

f(n) grows at the same rate or faster than g(n)

f(n) is $\Omega(g(n))$

e) $\lim_{n \to \infty} \frac{f(n)}{g(n)}$

$\lim_{n \to \infty} \frac{n \log n + n}{\log n} = \infty$

f(n) grows at the same rate or faster than g(n)

f(n) is $\Omega(g(n))$

f) $\lim_{n\to\infty} \frac{f(n)}{g(n)}$

$\lim_{n\to\infty} \frac{\log(n^2)}{\log^2(n)} = 0$

f(n) grows at the same rate or slower than g(n)

f(n) is $O(g(n))$

g) $\lim_{n\to\infty} \frac{f(n)}{g(n)}$

$\lim_{n\to\infty} \frac{2^n}{10n^2} = \infty$

f(n) grows at the same rate or faster than g(n)

f(n) is $\Omega(g(n))$

h) $\lim_{n\to\infty} \frac{f(n)}{g(n)}$

$\lim_{n\to\infty} \frac{2^n}{n\log n} = \infty$

f(n) grows at the same rate or faster than g(n)

f(n) is $\Omega(g(n))$

i) $\lim_{n\to\infty} \frac{f(n)}{g(n)}$

$\lim_{n\to\infty} \frac{2^n}{3^n} = 0$

f(n) grows at the same rate or slower than g(n)

f(n) is $O(g(n))$

j) $\lim_{n \to \infty} \frac{f(n)}{g(n)}$

$\lim_{n \to \infty} \frac{2^n}{n^n} = 0$

f(n) grows at the same rate or slower than g(n)

f(n) is O(g(n))