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a) $\lim_{n\to\infty} \frac{f(n)}{g(n)}$

$\lim_{n\to\infty} \frac{(1)n}{\frac{1}{2}n + \frac{1}{2}}$

$\lim_{n\to\infty} \frac{1}{\frac{1}{2}} = 2$

f(n) and g(n) have same rate of growth

b) $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ $\lim_{n\to\infty} \frac{(1)n^2}{(1)n^2+6n}$

$\lim_{n\to\infty} \frac{1}{1} = 1$

f(n) and g(n) have the same rate of growth

c) $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ $\lim_{n\to\infty} \frac{\log(n)}{\log(n^2)}$

$\lim_{n\to\infty} \frac{\ln(x)}{\ln(x^2)}$ $\lim_{n\to\infty} \frac{(1)\ln(x)}{(2)\ln(x)} = \frac{1}{2}$

f(n) and g(n) have the same rate of growth

d) $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ $\lim_{n\to\infty} \frac{2^n}{2^{2n}}$

$\lim_{n\to\infty} 2^{-n}$ $\lim_{n\to\infty} \frac{1}{2^n} = 0$

g(n) grows faster than f(n)

e) $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ $\lim_{n\to\infty} \frac{5n}{4n^{\frac{3}{2}}}$

$\frac{5}{4} \lim_{n\to\infty} n^{-\frac{1}{2}}$ $\frac{5}{4} \lim_{n\to\infty} \frac{1}{\sqrt{n}} = \left(\frac{5}{4}\right)(0) = 0$

g(n) grows faster than f(n)

② a) $f(n) = (n+1)^3$

$g(n) = n^3$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \quad \lim_{n\to\infty} \frac{(n+1)^3}{n^3}$$

$$\lim_{n\to\infty} \frac{(1)n^3 + 3n^2 + 3n + 1}{(1)n^3} = \frac{1}{1} = 1$$

$f(n)$ has the same rate of growth as $g(n)$ thus $(n+1)^3$ is $O(n^3)$

b) $f(n) = 2^{n+1}$ $g(n) = 2^n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \quad \lim_{n\to\infty} \frac{2^{n+1}}{2^n} \quad \lim_{n\to\infty} \frac{(2)2^n}{2^n} = \frac{2}{1} = 2$$

$f(n)$ has same growth rate as $g(n)$, thus 2^{n+1} is $O(2^n)$

c) $f(n) = n$ $g(n) = n \log n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \quad \lim_{n\to\infty} \frac{n}{n \log n} \overset{LH}{=} \lim_{n\to\infty} \frac{1}{\log n} = 0$$

d) $f(n) = n^2 \qquad g(n) = n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \qquad \lim_{n\to\infty} \frac{n^2}{n} \overset{LH}{=} \lim_{n\to\infty} \frac{2n}{1} = \infty$$

n^2 grows faster than n thus

$n^2 = \omega(n)$

e) $f(n) = n^3 \log n \qquad g(n) = n^3$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \qquad \lim_{n\to\infty} \frac{n^3 \log n}{n^3} \overset{LH}{=} \lim_{n\to\infty} \frac{\cancel{6} \log(n)}{\cancel{6}}$$

$$\lim_{n\to\infty} \frac{\log(n)}{1} = \infty$$

$f(n)$ grows faster than $g(n)$

thus $\quad n^3 \log n = \Omega(n^3)$

(3) a) The statement $s = s + i$ will be executed n times, thus $O(n)$.

b) The statement $p = p * i$ will be executed $2n$ times. using rule 3 eliminating the coefficient, it is $O(n)$

c) the statement $p = p * i$ will be executed n^2 times, thus $O(n^2)$

d) the outside loop is run $2n$ times, and the inside loop is run up to $2n$ times. using rule 2 $T_1(n) \times T_2(n) = O(f(n) \times g(n))$ and eliminating the coefficient, it is $O(n^2)$

e) the outside loop is run n^2 times and the inside loop is run up n^2 times, using rule 2 it is $O(n^4)$