Chapter

13.2

Audio Phase and Frequency Measurement

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13.2.1 Introduction

When a signal is applied to the input of a device, the output will appear later. For a sine-wave excitation this delay between input and output may be expressed as a proportion of the sine-wave cycle, usually in degrees. One cycle is 360°, one half-cycle is 180°, etc. This measurement is illustrated in Figure 13.2.1. The phasemeter input signal no. 2 is delayed from, or is said to be *lagging*, input no.1 by 45°. Most audio measuring gear measures phase directly by measuring the proportion of one signal cycle between zero crossings of the signals. This can be done with an edge-triggered set-reset flip-flop as shown in Figure 13.2.1. The output of this flip-flop will be a signal which goes high during the time between zero crossings of the signals. By averaging the amplitude of this pulse over one cycle (i.e., measuring its *duty cycle*) a measurement of phase results.

13.2.2 Phase Measurement

Phase is typically measured and recorded as a function of frequency over the audio range. For most audio devices, phase and amplitude responses are closely coupled. Any change in amplitude that varies with frequency will produce a corresponding phase shift. A device that has no more phase shift than what is required by the amplitude-response variation with frequency is described as having *minimum-phase* characteristics. A typical phase-and-amplitude- versus-frequency plot of a graphic equalizer is shown in Figure 13.2.2.

A fixed time delay will introduce a phase shift that is a linear function of frequency. This time delay can introduce large values of phase shift at high frequencies which are of no significance in practical applications. The time delay will not distort the waveshape of complex signals and will not be audible in any way. There can be problems with time delay when the delayed signal will be used in conjunction with an undelayed signal. This would be the case if one channel of a stereo signal was delayed and the other was not. If we subtract out the absolute time delay from a phase

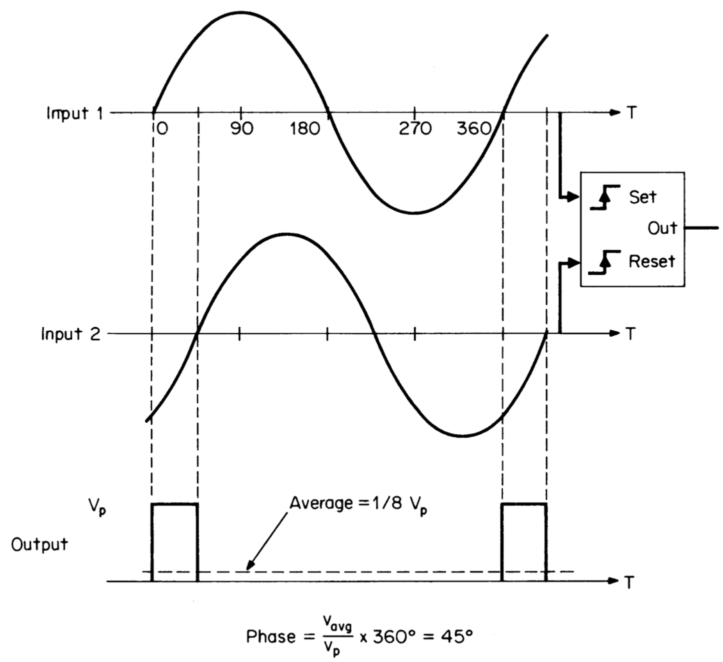


Figure 13.2.1 Basic measurement of a phase shift between two signals.

plot, the remainder will truly represent the audible portions of the phase response. In instances where absolute delay is a concern, the original phase curve is more relevant.

13.2.2a Relation to Frequency

When dealing with complex signals, the meaning of phase becomes unclear. Viewing the signal as the sum of its components according to Fourier theory, we find a different value of phase shift at each frequency. With a different phase value on each component, which one is to be used? If the signal is periodic and the waveshape is unchanged passing through the device under test, a phase value may still be defined. This may be done by using the shift of the zero crossings as a fraction of the waveform period. Indeed, most commercial phasemeters will display this value. However, if there is differential phase shift with frequency, the waveshape will be changed. It is then not possible to define any phase-shift value, and phase must be expressed as a function of frequency.

Another useful expression of the phase characteristics of an audio device is *group delay.* Group delay is the slope of the phase response. It expresses the relative delay of the spectral components of a complex waveform. This describes the delay in the harmonics of a musical tone relative to the fundamental. If the group delay is flat, all components will arrive together. A peak or rise in the group delay indicates that those components will arrive later by the amount of the peak

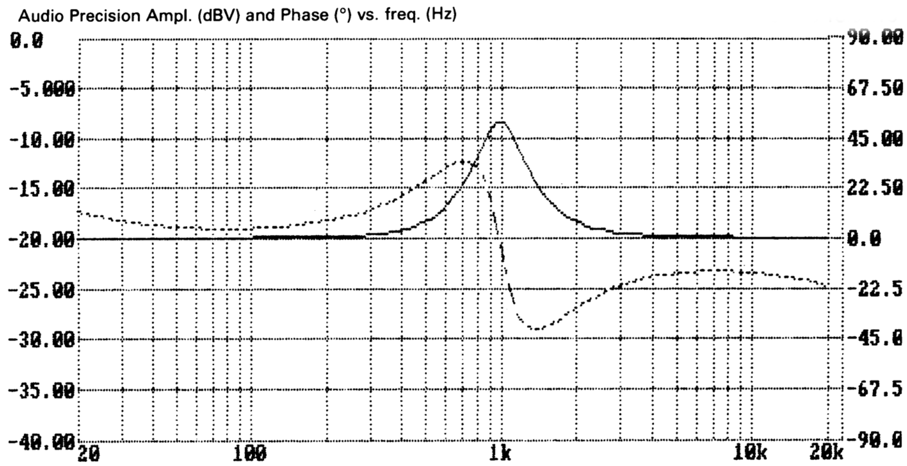


Figure 13.2.2 Typical phase-and-amplitude-versus-frequency plot of a graphic equalizer.

or rise. It is computed by taking the derivative of the phase response versus frequency. Mathematically

$$\text{Group delay} = -(\text{phase at} f_2 - \text{phase at} f_1)/(f_2 - f_1) \tag{13.2.4}$$

This requires that phase be measured over a range of frequencies to give a curve that can be differentiated. It also requires that the phase measurements be performed at frequencies which are close enough together to provide a smooth and accurate derivative.

13.2.3 Frequency Measurement

Frequency is a fundamental characteristic of periodic signals. It is simply the number of times per second that the signal being measured repeats its pattern. An alternative way to specify this parameter is the period of the signal; i.e., the time taken for one cycle of the pattern to occur. Care should be taken not to confuse *pitch* and frequency. Pitch is essentially the perceived frequency. Indeed, for complex waveforms such as narrowband noise of frequency-modulated sine waves frequency is difficult to define. For example, what is the “frequency” of a signal consisting of 2-kHz, 3-kHz, 4-kHz, and 5-kHz sine waves? When this signal is heard, the brain will “insert” the missing l-kHz fundamental and perceive a 1-kHz pitch. Pitch, though not always obvious from electrical measurements, is readily apparent to a listener.

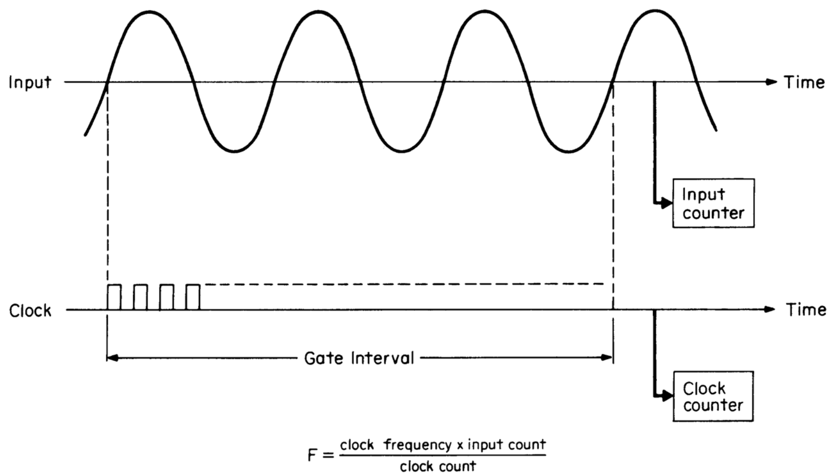


Figure 13.2.3 High-resolution frequency measurement.

13.2.3a Measurement Techniques

Early methods of frequency measurement employed vibrating reeds or frequency-to-voltage conversion circuits. These were hardly precision measurement techniques, but the frequency-to-voltage-converter approach was easy to use and became popular. Frequency measurement has advanced greatly since the development of digital logic. Early designs used digital counters to count the number of zero crossings during a fixed time window. For ease of design, these time windows (*gates*) were decimal fractions or multiples of 1 s.

Modern designs take advantage of the computing power of microprocessors and measure period, reciprocating the result to obtain the frequency. To perform this measurement both a high-frequency reference clock and the input signal are counted during the gate interval, as illustrated in Figure 13.2.3. The frequency of the input signal may then be computed by the formula

$$F = \text{clock frequency} \times \text{no. of signal cycles/count} \tag{13.2.2}$$

Note that the gate interval does not enter into the calculation and may be chosen on the basis of the speed of measurements desired. Longer gate intervals and higher-frequency clocks will result in higher-resolution measurements. However, the gate interval must be an integer multiple of the input-signal period. This is easy to ensure with appropriate logic circuitry. For the fairly typical case of a 10-Hz signal, a 0.1-s gate, and a 10-MHz clock, we would have a 1-cycle gate and a count of

$$10\text{MHz} \times 1 \text{ cycle}/10 \text{ Hz} = 1 \text{ million} \tag{13.2.3}$$

giving a resolution of 6 digits. A 1-s gate would allow 10 cycles of input signal, giving a count of 10 million.

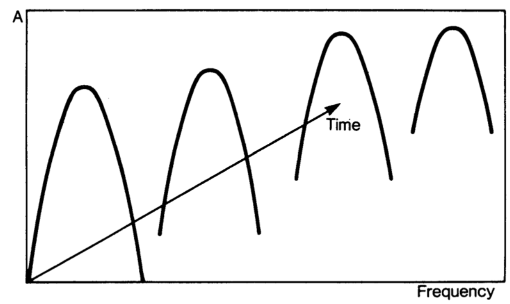


Figure 13.2.4 Sweeping spectrum analyzer response.

Another scheme is sometimes used for measuring low frequencies quickly and to high resolution. This involves locking a voltage-controlled oscillator (VCO) to a multiple of the input frequency, usually 100, with a phase-locked loop (PLL). The counter then counts the VCO output and obtains a factor-of-l00 improvement in resolution for the same gate time. This scheme requires several cycles of input signal for the PLL to acquire and lock to the input. This time must be included in the measurement time, reducing the improvement. Increasing the multiplication factor much above 100 is difficult because of problems of oscillator instability and tuning range. Although the factor-of-two improvement is substantial, period-based measurement schemes achieve even better resolution, yet are more involved.

All these measurement techniques are limited as to achievable accuracy by the accuracy of the reference. Typical quality crystal oscillators provide an accuracy of several parts per million at room temperature at a cost of a few dollars. By temperature-compensating the crystal oscillator the ambient-temperature effects may be removed. Adding an oven around the circuitry to maintain a constant environment further reduces drift. Order-of-magnitude improvements in accuracy tend to require order-of-magnitude increases in cost. Fortunately, for most audio applications the basic crystal accuracy is adequate.

13.2.3b Spectrum Analysis

A spectrum analyzer, generally speaking, is a device that displays a signal in the frequency domain. The most common approach is the sweeping-filter technique. It may be visualized as a bandpass filter that sweeps in frequency as shown in Figure 13.2.4. The output level from the filter drives the vertical axis of a display, while the signal frequency is the horizontal-axis variable. This gives a graph of the frequency content of the signal. A single sine wave will give a display of one peak at a horizontal point corresponding to the frequency of the input.

Because of the problems in tuning a bandpass filter electronically, instruments have typically been implemented along the lines of the block diagram shown in Figure 13.2.5. A sweeping oscillator (local oscillator, LO) is mixed with the input signal in a multiplier, shifting the frequency of the signal to that of the fixed-frequency bandpass filter. This process is called *heterodyning* and is used in most radio receivers. This has also prompted some manufacturers of these units to call them *heterodyne analyzers.* Two important characteristics identify this type of analyzer. First, the bandwidth of the analysis filter is fixed by the characteristics of the fixed-frequency filter. The bandwidth of the analysis will not depend on the analysis frequency. Second,

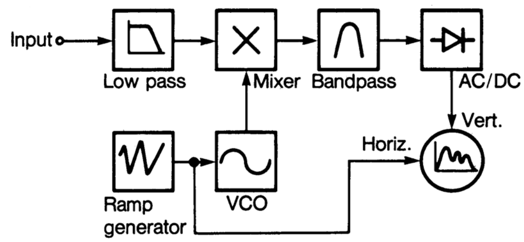


Figure 13.2.5 Block diagram of a heterodyne spectrum analyzer.

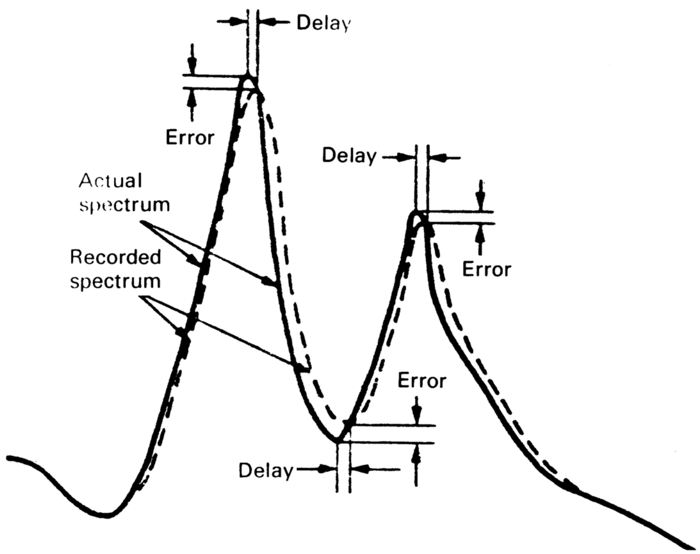


Figure 13.2.6 Error and delay (bias errors) in writing out peaks and valleys in a spectrum. (*After* [1].)

the analyzer must sweep through the frequency range of interest. If something happens at one frequency when the analyzer is tuned to another frequency, it will be missed.

The fixed-frequency-filter stage, called the intermediate frequency (IF) amplifier, is typically a multiple-stage filter with amplifiers between each stage. This allows a sharp rolloff characteristic and very small bandwidths. Means are normally provided for setting the IF bandwidth, allowing the resolution of the spectrum analyzer to be adjusted. The frequency-analysis range, or *span*, of the analyzer is set by the tuning of the LO. A minimum bandwidth is required for any value of span and sweep speed. This requirement allows the IF filter to settle to its steady-state response on the input signal. If the sweep is too fast or the bandwidth too small, the filter output will give an incorrect reading. The shape of the response seen on the analyzer screen for different sweep rates is shown in Figure 13.2.6. As the sweep rate is increased above the optimum value, the peak will start to drop and its frequency will shift in the direction of the sweep. The optimum

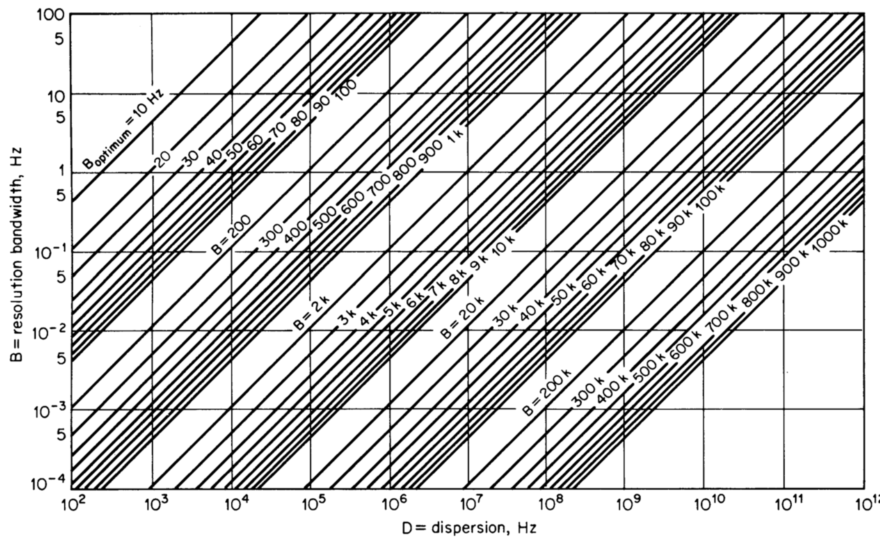


Figure 13.2.7 Optimum resolution setting for spectrum analyzers. Read $B_{optimum}$ for a given dispersion and sweep time. (*After* [2].)

bandwidth B for a particular sweep time T and dispersion or total frequency sweep range D is given in Figure 13.2.7.

Another approach to spectrum analysis is using a real-time analyzer (RTA). A parallel bank of bandpass filters is driven with the signal to be analyzed. The outputs of the filters are rectified and displayed in bar-graph form on a cathode-ray tube (CRT) or other suitable display as shown in Figure 13.2.8. The resulting display is shown in Figure 13.2.9. The filters are at fixed frequencies, usually spaced every one-third octave or full octave from 20 Hz to 20 kHz. This results in 30 filters for the one-third-octave case and 10 filters for octave-band units. These frequencies have been standardized by the IEC and are given in Table 13.2.1. Some units have been built with 12 filters per octave, or a total of 120 filters, for even higher resolution. Because RTAs are normally made with these fractional-octave filters, they are constant-percentage-bandwidth devices. This means that the bandwidth of the filters is always a fixed percentage of the center frequency. The advantage of parallel-filter analyzers is their instantaneous display and their ability to see transient events. Since all filters are constantly monitoring the signal, all transients will be seen. Disadvantages include the low-resolution display and the inability to trade resolution for frequency range after the unit is manufactured.

RTAs are commonly used with a random-noise or multitone test signal to measure a device or system response quickly. A random-noise signal is a signal whose instantaneous amplitude is a random, usually gaussian variable. Random noise has a spectrum (amplitude versus frequency) made up of all frequencies over the bandwidth of the noise. Various terms are used to describe the spectral distribution of noise; the most common are *pink noise* and *white noise*. White noise has an equal energy per hertz of bandwidth. If it is measured by using a filter whose bandwidth is constant with frequency (such as a heterodyne analyzer), the spectrum will be flat. If it is mea-

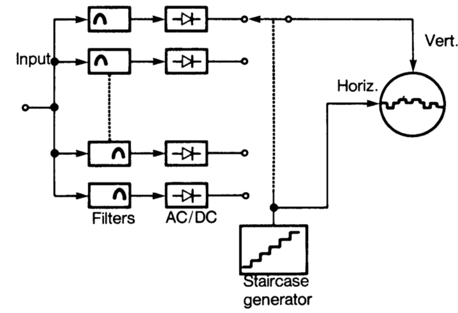


Figure 13.2.8 Real-time-analyzer block diagram.

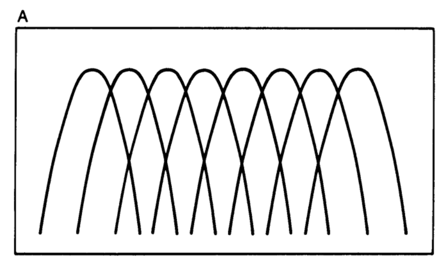


Figure 13.2.9 Real-time (parallel-filter) analyzer response.

sured with a fractional-octave filter, as are RTAs, the spectrum will rise at 3 dB per octave. Pink noise is random noise that has an equal energy per octave. If it is displayed on an RTA, the response will be flat. A heterodyne analyzer will show a rolloff of 3 dB per octave.

For most audio testing work with RTAs, the system or device under test is excited by the pink-noise test signal. The output of the system is measured with the RTA, and the frequency response is displayed without waiting for a sweep. Because of the random nature of the noise signal there is significant fluctuation in the amplitude of each filter output. The average level will be correct, but the instantaneous value will change considerably with time. This puts significant uncertainty in the response being measured. This uncertainty may be reduced, but not eliminated, by increasing the averaging time in the ac-to-dc converters after the filters.

Multitone test signals, which consist of a sine wave at the center frequency of each filter used in the analysis, may be constructed. When this signal is displayed on an RTA, the response will be shown without the random fluctuations common to pink noise. However, because the signal consists of a small number of sine waves, the response shown on the RTA is the steady-state response at each filter's center frequency. Because multitone signals measure at a single frequency in each filter, they will be greatly affected by sharp peaks and dips in the response being measured. When pink noise is used to make this same measurement, the RTA's response will be the average over the frequency range of each filter. This becomes a distinct advantage of random noise as a source for some acoustic measurements. It will average the response over the range of the analyzing filter, resulting in a measurement less affected by room modes and resonances in transducers.

Table 13.2.1 ANSI-ISO Preferred Frequencies.

25 Hz	250 Hz	2.5 kHz
31.5 Hz	315 Hz	3.15 kHz
40 Hz	400 Hz	4.0 kHz
50 Hz	500 Hz	5.0 kHz
63 Hz	630 Hz	6.3 kHz
80 Hz	800 Hz	8 kHz
100 Hz	1 kHz	10 kHz
125 Hz	1.25 kHz	12.5 kHz
160 Hz	1.6 kHz	16 kHz
200 Hz	2	20 kHz

13.2.4 Fast Fourier Transform Measurement

Time and frequency domains are alternate ways of looking at a signal. The French mathematician Fourier proved that any signal may be represented as a series of sine waves summed together. Indeed, this is the justification for analyzing signals with a narrowband-tunable-filter spectrum analyzer to examine these components. Advances in technology have made it possible to implement directly Fourier's theory with digital computing circuits. Instruments that perform Fourier analysis digitize the signal, sampling the waveform at a rate faster than the highest-frequency input signal, and convert these samples into a numerical representation of the signal's instantaneous value. Fourier series provides a way to convert these signal samples into samples of the signal spectrum. This transforms the data from the time domain to the frequency domain. The FFT is merely a technique for efficiently computing the Fourier series by eliminating redundant mathematical operations.

The FFT operates on a piece of the signal that has been acquired and stored in memory for the calculation. Take, for example, the section of a sine wave shown in Figure 13.2.10. This is a piece of a sine wave which continues in time on both sides of the selected segment. The FFT algorithm does not know anything about the waveform outside this piece that it is using for calculations. It therefore assumes that the signal repeats itself outside the "window" it has of the signal. This is important because an incorrectly selected piece of the signal may lead to very strange results.

Consider the sine wave of Figure 13.2.10 and the possible pieces of it that have been selected for analysis. In the first example, the beginning and end of the window have been chosen to coincide with zero crossings of the signal. If the selected segment is repeated, an accurate representation of the signal is obtained. The FFT of this will give the correct spectrum, a single-frequency component. If the window is chosen incorrectly, as in the second example, there is a noninteger number of cycles in the waveform. When this segment is repeated, the resulting waveform will not look like the original sine wave. The computed spectrum will also be in error; it will be the spectrum of the discontinuous waveform. Transients that start at zero and decay to zero before the end of the sample segment will not suffer from any discontinuities. Continuous random or periodic signals will, however, be affected in a manner analogous to the effects on sine waves.

Clearly, then, the choice of windows for a signal that is to be transformed is critical to obtaining correct results. It is often difficult to select the correct end-points of the window. and even more difficult without operator involvement. The window function may be thought of as a rectangular-shaped function which multiplies the signal. Intuitively it seems that the sharp disconti-

(*a*)

(*b*)

(*c*)

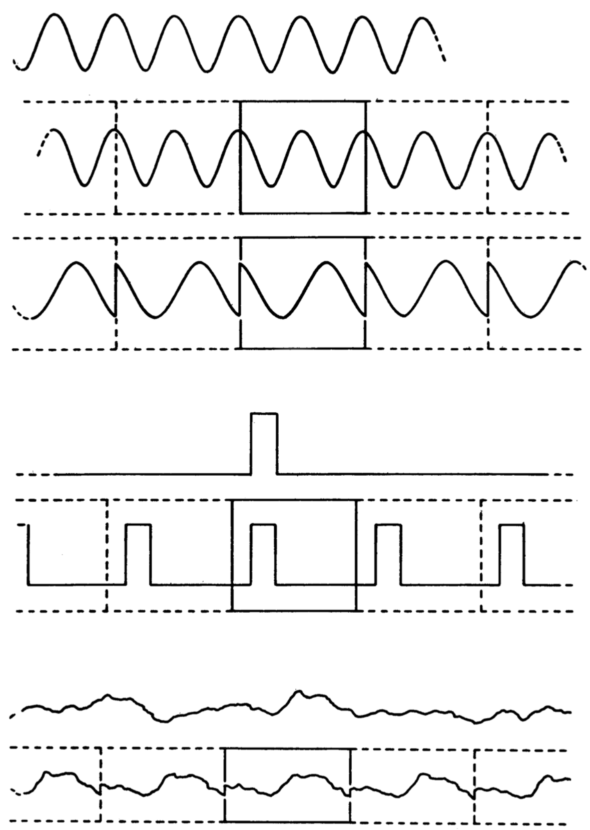


Figure 13.2.10 The FFT assumes that all signals are periodic and that they duplicate what is captured inside the FFT window: (*a*) periodic signal, integral number of cycles in the measurement window; (*b*) nonperiodic signal transient, (*c*) nonperiodic signal, random. (*After* [3].)

nuities introduced by the endpoints of the window are at fault for the spurious components in the FFT. This may be proved theoretically but is beyond the scope of this discussion. A simple solution to the windowing problem is to use nonrectangular windows. Multiplying the signal by a window that decreases to zero gradually at its end-points eliminates the discontinuities. Using these windows modifies the data and results in a widening of the spectral peaks in the frequency domain. This is illustrated in Figure 13.2.11. This tradeoff is unavoidable and has results in the development of many different windowing functions that emphasize the spectral widening or the rejection of spurious components. Perhaps the most common window function the *Hamming window*, illustrated in Figure 13.2.12, which is a cosine function raised above zero. It rejects spurious signals by at least 42 dB but spreads the spectral peaks by only 40 percent.

The FFT algorithm always assumes that the signal being analyzed is continuous. If transient signals are being analyzed, the algorithm assumes that they repeat at the end of each window. If a transient may be guaranteed to have decayed to zero before the end of the window time, a rectan-

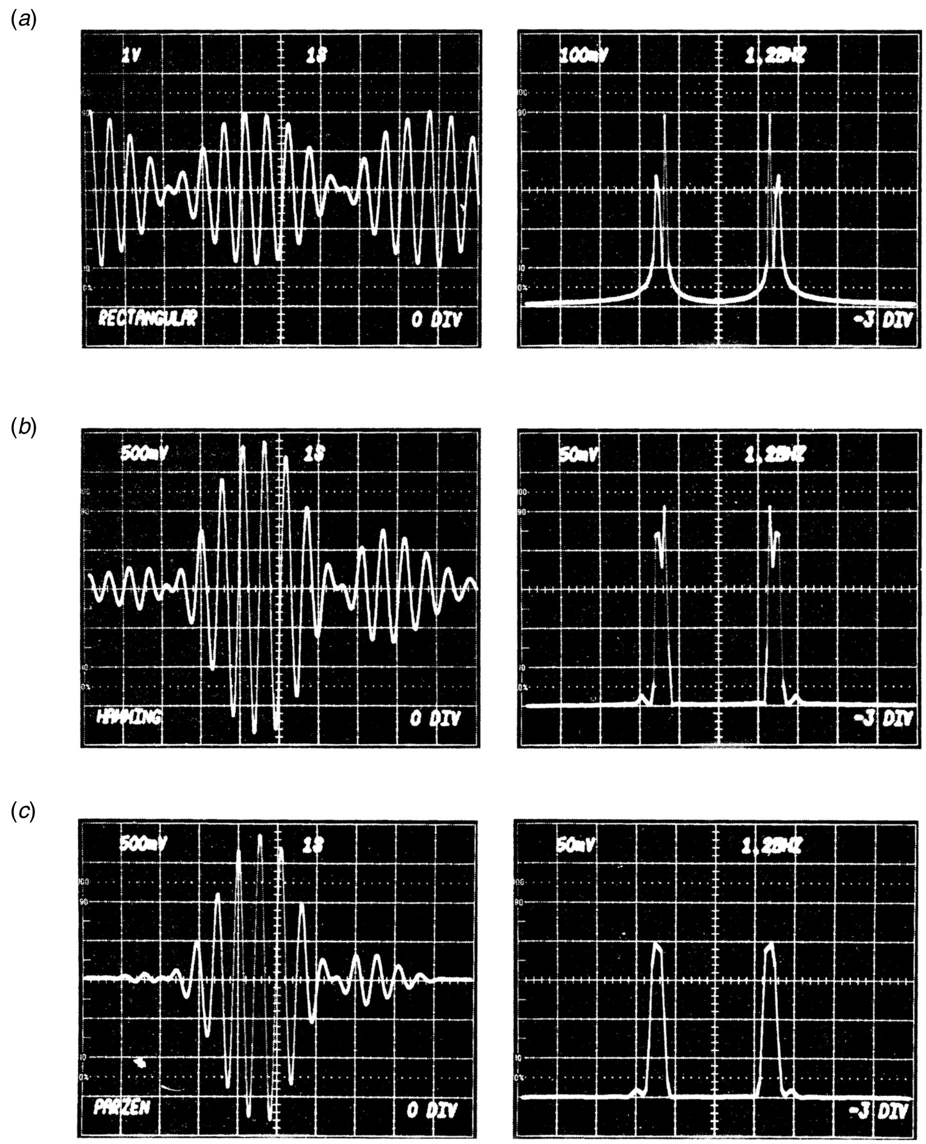


Figure 13.2.11 Window shapes trade off major-lobe bandwidth and side-lobe rejection. (*a*) An almost periodic waveform in the rectangular acquisition window. The FFT magnitude (expanded 4 times for detail) shows closely adjacent, nearly equal components; one has substantial leakage. Also, a small wrinkle at two divisions from center hints at a possible third component. (*b*) Multiplying the waveform by a Hamming window reduces side-lobe leakage and reveals a third low-frequency component in the FFT magnitude (expanded 4 times for detail). (*c*) A Parzen window offers more side-lobe reduction, but the increased bandwidth of the major lobe causes the two nearly equal components to merge completely into each other. (*After* [3].)

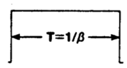
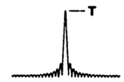
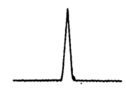
Unity-amplitude Window	Shape equation	Frequency-domain magnitude	Major-height	Highest side lobe dB	Bandwidth 3 dB	Theoretical roll-off, dB octave
Rectangle  $T=1/\beta$	$A = 1$ for $t = 0$ to T	 T	T	—13.2	0.86β	6
Hamming 	$A = 0.08 + 0.46\,(1 - \cos 2\pi t/T)$ for $t = 0$ to T		0.54 T	—41.9	1.26β	6 (beyond 5β)

Figure 13.2.12 Some common FFT data windows and their frequency-domain parameters. (*After* [3].)

gular window may be used. If not, a shaped window must still be used. However, if there is significant transient energy at the end of the window, the computed spectrum will not include the frequency contribution of that data.

Fourier-transform algorithms can be written for any number of data points. However, it is easiest if the number of points can be expressed as the product of two smaller numbers. In this case, the transform may be broken into the product of two smaller transforms, each of a length equal to the smaller numbers. The most convenient lengths for transforms, based on this scheme, are powers of 2. Common transform lengths are 512 points and 1024 points. These would ordinarily provide a spectrum with 256 and 512 components, respectively. However, because of errors at high frequencies due to aliasing and the rolloff introduced by windowing only 200 or 400 lines are displayed.

The transformation from the time domain to the frequency domain by using the FFT may be reversed to go from the frequency domain to the time domain. This allows signal spectra to be analyzed and filtered and the resulting effect on the time-domain response to be assessed. Other transformations may be applied to the data, yielding greater ability to separate the signal into its components.

13.2.5 References

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