Introduction to structural equation modeling and mixed models in

Day 9 – Part 1: SEM

Oksana Buzhdygan

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Outline

Introduction to Local Estimation in SEM

- ✓ Global vs. Local Estimations. Piecewise SEM
- ✓ Assessing Model Fit
- ✓ Model Comparison
- ✓ Categorical Data in Piecewise SEM

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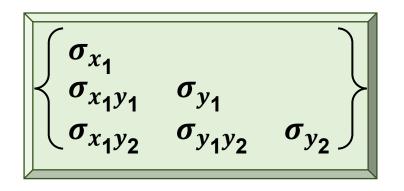
Global vs. Local Estimations

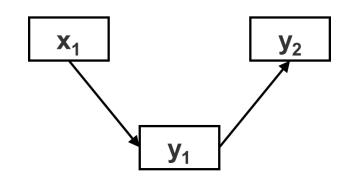
Two Paradigms for model estimation

Global Estimation

(Covariance-Based Estimation)

 reproduces a single variance-covariance matrix





Local Estimation

(piecewiseSEM)

- fit a model for each response
- strings together the inferences

$$y_1 = b_1 x + \zeta_1$$
 $y_2 = b_2 y_1 + \zeta_2$

Protocol for violated assumptions of covariance-based SEM

Violated assumptions	Steps for Corrections	
Non-normality of Residuals	Data transformation: e.g. log, square root	
	Local estimation with GLM: package piecewiseSEM	
Data are not multivariate normal	MLM estimation with robust SE & test statistic: library(lavaan) # Always report results for 'robust' test statistics sem(, estimator="MLM", se="robust" #or test="Satorra-Bentler")	
	Bootstapping: # Always report results for 'robust' test statistics library(lavaan) sem(, test="bollen.stine", se="bootstrap)	
Missing data	Full information maximum likelihood: library(lavaan) sem(, missing="fiml") #for normal data sem(, missing="fiml", estimator="MLR")#for non-normal data	
Positive definite S matrix	Check for multicolinearity in each single regression model: library(car) vif(m2) # vif \leq 2 (no collinearity)	
Dependant samples (hierarchical)	Local estimation with LMM or GLMM: package piecewiseSEM	
Not sufficient sample size	Local estimation: package piecewiseSEM	

Global vs. Local Estimations

STRUCTURAL EQUATION MODELING, 7(2), 206–218 Copyright © 2000, Lawrence Erlbaum Associates, Inc.

A New Inferential Test for Path Models Based on Directed Acyclic Graphs

Bill Shipley

Département de Biologie Université de Sherbrooke

This article introduces a new inferential test for acyclic structural equation models (SEM) without latent variables or correlated errors. The test is based on the independence relations predicted by the directed acyclic graph of the SEMs, as given by the concept of d-separation. A wide range of distributional assumptions and structural functions can be accommodated. No iterative fitting procedures are used, precluding problems involving convergence. Exact probability estimates can be obtained, thus permitting the testing of models with small data sets.

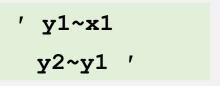
Local Estimation

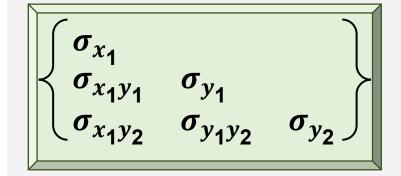
(piecewiseSEM)

- fit a model for each response
- strings together the inferences

$$y_1 = b_1 x + \zeta_1$$
 $y_2 = b_2 y_1 + \zeta_2$

Global vs. Local Estimations

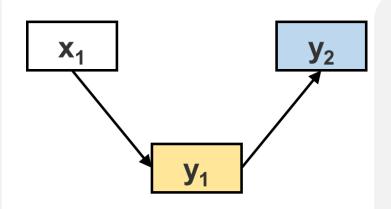


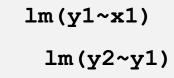


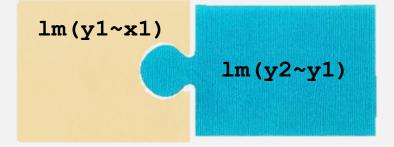
Global Estimation

(Covariance-Based Estimation)

 reproduces a single variance-covariance matrix







Local Estimation

(piecewiseSEM)

- fit a model for each response
- strings together the inferences

piecewiseSEM: Piecewise Structural Equation Modeling in R

Jonathan S. Lefcheck

2020-12-09

- 1. An Introduction to Structural Equation Modeling
- 2. An Example using piecewiseSEM
 - 2.1 Worked example
 - 2.2 Standardized coefficients
 - 2.3 GLMs in pSEM
 - 2.4 Correlated errors
 - 2.5 Nested models and AIC
- 3. Comparing Package Versions
 - 3.1 Introduction to Shipley (2009)
 - 3.2 Comparing versions in evaluating the Shipley's SEM
 - 3.3 Additional functions
- 4. References

https://cran.rproject.org/web/packages/piecewiseSEM/vi gnettes/piecewiseSEM.html

Structural equation modeling (SEM) is among the fastest growing statistical techniques in ecology and evolution, and provides a new way to explore and quantify ecological systems. SEM unites multiple variables in a single causal network, thereby allowing simultaneous tests of multiple hypotheses. The idea of causality is central to SEM as the technique implicitly assumes that the relationships among variables represent causal links. Because

Lefcheck, J.S. "piecewiseSEM: Piecewise structural equation modelling in r for ecology, evolution, and systematics." Methods in Ecology and Evolution 7.5 (2016): 573-579.

1 Preface

- 2 Global Estimation
 - 2.1 What is (Co)variance?
 - 2.2 Regression Coefficients
 - 2.3 Variance-based Structural Eq...
 - 2.4 Model Identifiability
 - 2.5 Goodness-of-fit Measures
 - 2.6 Model Fitting Using lavaan
 - 2.7 References
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 - 3.2 Tests of directed separation
 - 3.3 A Log-Likelihood Approach to...
 - 3.4 Model fitting using piecewise...
 - 3.5 Extensions to Generalized Mi...
 - 3.6 Extensions to Non-linear Mod...
 - 3.7 A Special Case: Where Grap...

A i https://isle

https://jslefche.github.io/sem_book/

Jon Lefcheck

January 16, 2021

1 Preface

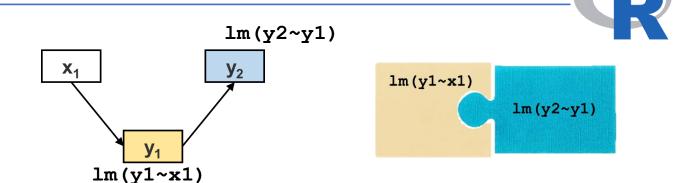
Structural equation modeling is among the fastest growing statistical techniques in the natural sciences, thanks in large part to new advances and software packages that make it broadly applicable and easy to use.

This book is meant to be an approachable and open-source guide to the theory, math, and application of SEM. It integrates code for the R software for statistical computing from popular packages such as *lavaan* and *piecewiseSEM*. Each chapter ends with worked examples from the published literature.

Moreover, as the author of the *piecewiseSEM* package, this format allows me to document newly-deployed functionality in the package, such as the addition of categorical variables, multigroup analysis and composite variables, new forms of coefficient standardization, and updates to model R²s.

Check back often, as this book is a "living resource:" as new functionality is added and bugs uncovered and fixed, they will be described in detail here (with worked examples where possible).

```
# Install the <u>development version</u> of
piecewiseSEM
devtools::install github("jslefche/piece
wiseSEM@devel")
library(piecewiseSEM)
# Check assumptions
m1 <- lm(y1 \sim x1, data = data1)
m2 < -lm(y2 \sim y1, data = data1)
plot(m1)
plot(m2)
# Model specification in piecewiseSEM
psem mod1 <- psem(</pre>
  lm(y1 \sim x1, data = data1),
  lm(y2 \sim y1, data = data1)
# or
psem mod1 <- psem (m1, m2)</pre>
```



```
# Install the development version of
piecewiseSEM
devtools::install github("jslefche/piece
wiseSEM@devel")
library(piecewiseSEM)
# Check assumptions
m1 < -lm(y1 \sim x1, data = data1)
m2 \leftarrow lm(y2 \sim y1, data = data1)
plot(m1)
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# Model specification in piecewiseSEM
psem mod1 <- psem (
  lm(y1 \sim x1, data = data1),
  lm(y2 \sim y1, data = data1)
# or
psem mod1 \leftarrow psem (m1, m2)
# Examine the psem object
psem mod1
# Plot the model
plot(psem mod1)
```

```
> psem mod1
Structural Equations of x :
lm: y1 \sim x1
lm: y2 \sim y1
Data: y1 x1
                           y2
1 0.46353960 0.8461242 0.6299346
2 0.48084442 0.6626756 1.2847942
3 0.03044414 0.3068555 0.8495422
4 0.78902095 0.9327949 1.0768962
5 0.25329331 1.0490908 0.7902750
6 0.61409128 0.7134605 1.4559118
...with 94 more rows
[1] "class(psem)"
```



Global vs. Local Estimations

Properties:	Covariance-based SEM	Piecewise SEM	
Estimation procedure	Single (global) variance-covariance matrix estimated	Variance-covariance matrices estimated separately for each endogenous variable	
Solutions from the estimation	Simultaneous solution (computationally intensive)	Multiple solutions (modularized)	
Data and residual distribution	Fit to normal distribution	Assumes constant variance an independence of errors for each regression equation. If violated, it icorporates various distributions (Poisson, Gamma, etc.)	
Sample size	Minimum requirement $n = p \times 5$, where n sample size, p number of path coefficients	Only enough data is needed to be able to fit and estimate each individual regression	
Independence of samples	Assumes independence of samples	Can model non-independence (blocked, temporal, spatial, etc.)	
Latent variables	Latent & composite variables	No latent or composite variables (yet*)	
Feedback-loops in a model	Non-recursive (cyclic) models are possible	Only for recursive (acyclic) models, i.e. no bidirectional relationships	

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Global vs. Local Estimations

Properties:	Covariance-based SEM	Piecewise SEM
	x_1 y_1 y_2 Non-recursive models • with bidirectional feedbacks	X ₁ X ₂ Y ₂ Y ₂ Recursive models • all causal effects are unidirectional
Feedback-loops in a model	Non-recursive (cyclic) models are possible	Only for recursive (acyclic) models, i.e. no bidirectional relationships

The concept of Goodness of Fit

Are we ignoring important links?

Links = Processes

When we are missing important paths:

- our parameter estimates may be incorrect
- our model is misspecified

Does the model fit the data?

Does the model represent the data well?

Are we missing important information?

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Tests of directed (d-)separation

Two variables are said to be *d-separated* if they are statistically **independent** conditional on their joint influences

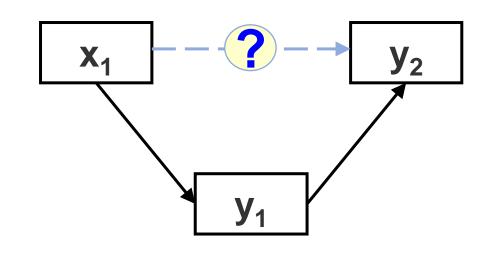
no directed path connecting these two variables

accounting for contributions from other variables through indirect effects

H₀: partial effect of x₁ on y₂ is not different from 0

of x_1 on y_2

effect of x₁ on y₂ not other way around or both



Independence claim

 $x_1|y_2(y_1)$ x_1 and y_2 are independent conditioned on y_1

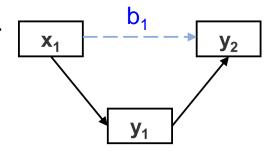
Thus, we test the partial effect of x_1 on y_2 given y_1

Basis set is the minimum number of independence claims derived from a path diagram.

Tests of directed (d-)separation

(of statistical independence):

Independence claim $x_1|y_2(y_1)$



Steps for each independence claim:

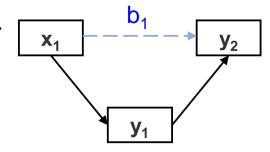
- 1. Fit sub-model including missing link
- 2. Extract *p*-value associated with that missing link
- 3. If p < 0.05 reject H₀ (that the missing effect is not different 0), suggesting model change

$$y_2 = a + b_1 x_1 + b_2 y_1$$

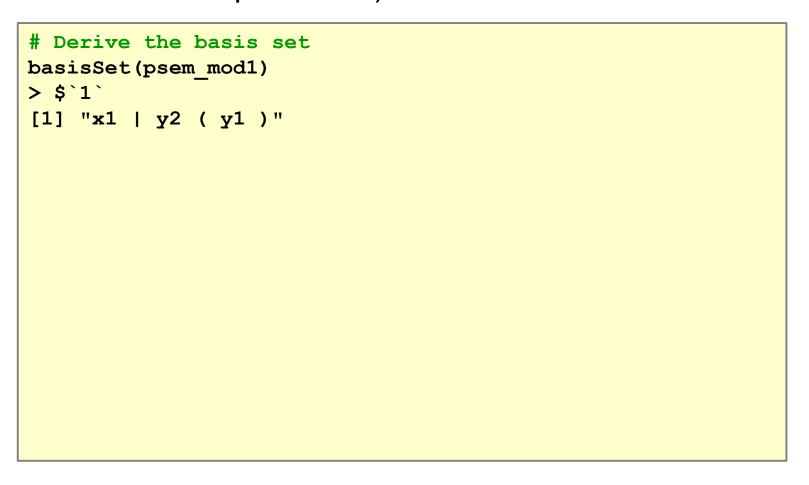
Tests of directed (d-)separation

(of statistical independence):

Independence claim $x_1|y_2(y_1)$



$$y_2 = a + b_1 x_1 + b_2 y_1$$



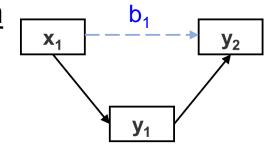
Fisher's C statistic

Tests of directed (d-)separation

(of statistical independence):

Independence claim

 $x_1|y_2(y_1)$

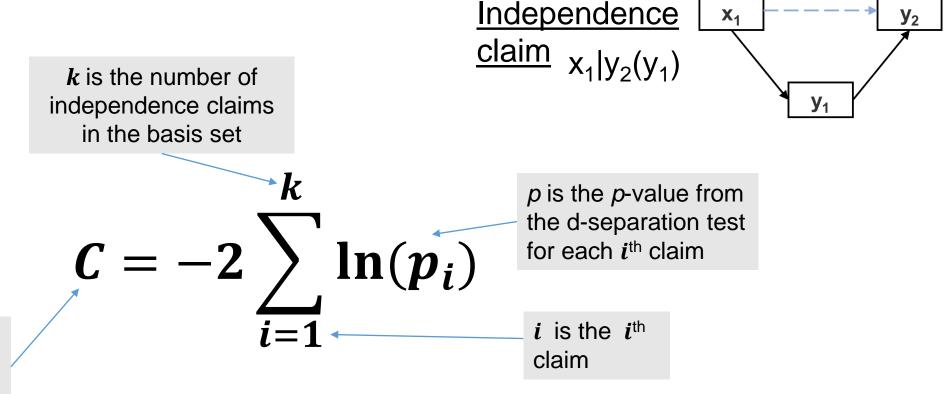


$$y_2 = a + b_1 x_1 + b_2 y_1$$

```
# Derive the basis set
basisSet(psem mod1)
> $`1`
[1] "x1 | y2 ( y1 )"
# Tests of directed separation
dSep (psem mod1)
 Independ.Claim Test.Type DF Crit.Value P.Value
1 \quad y2 \sim x1 + \dots coef 97 -1.018557 0.3109471
# Manually calculated:
summary(lm(y2 ~ y1+x1, data = data1))$coefficients[3, ]
 Estimate Std. Error t value Pr(>|t|)
-0.1164159 0.1142950
                        -1.0185570 0.3109471
```

missing effect is not different from 0

Fisher's C statistic

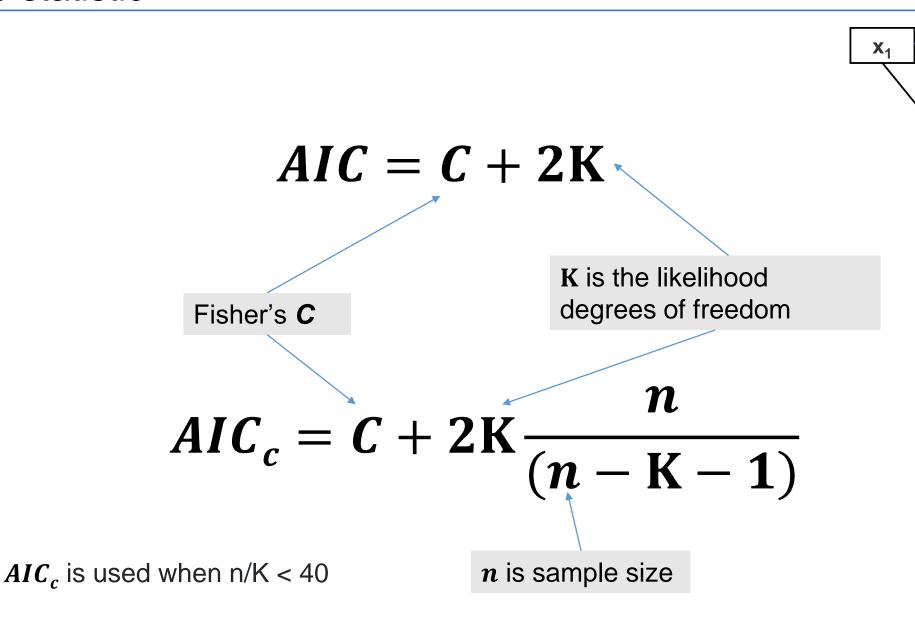


Model fit
Fisher's C statistic

 \boldsymbol{C} is χ^2 distributed with $2\boldsymbol{k}$ degrees of freedom (allows obtaining a model-wide P-value)

Good fit: P > 0.05 (model is supported by the data)

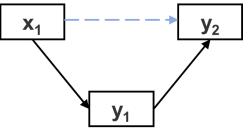
Fisher's C statistic



 $\mathbf{y_2}$

Fisher's C statistic

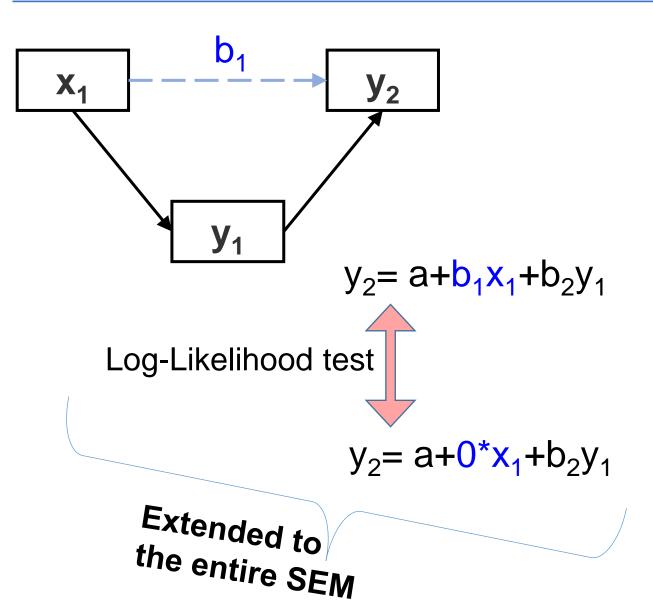
```
# Fisher's C statistic
fisherC(psem mod1)
   Fisher.C df P.Value
1 2.336 2 0.311
# Manually calculated:
C \leftarrow -2 * \log(summary(lm(y2 \sim y1+x1, data = data1))$coefficients[3, 4])
> C
[1] 2.336265
1-pchisq(C, 2) # 2 DF (DF=2k; k - number of independence claims)
[1] 0.3109471
# AIC value based on the Fisher's C statistic and the d-sep tests
AIC(psem mod1, AIC.type = "dsep", aicc = TRUE)
  AIC
      AICc K n
1 14.336 15.415 6 100
```



$$C = -2\sum_{i=1}^{k} \ln(p_i)$$

$$AIC = C + 2K$$

$$AIC_c = C + 2K \frac{n}{(n - K - 1)}$$



Log-Likelihood Approach

Compares the fitted sub-model to fully saturated sub-model

 χ^2 statistic

k is the number of sub-models in the SEM model

Log-likelihood of our (nested) *i*th sub-model **M**

Log-likelihood of the fully saturated *i*th sub-model *Ms*

 χ^2 of the SEM model

$$\chi^2 = -2\sum_{i=1}^{\infty} (\log(\mathbf{L}_{\mathbf{M}_i}) - \log(\mathbf{L}_{\mathbf{M}s_i}))$$

i is the *i*th sub-model in the SEM model

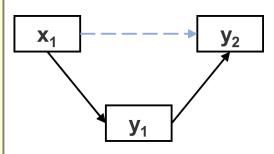
$$AIC = \sum_{i=1}^{k} AIC(M_i)$$

AIC of the SEM model

The same for the AIC_c

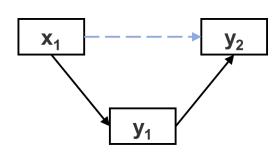
χ² statistic

```
# log-likelihood based x2 statistic
LLchisq(psem mod1)
                                       \chi^2 = -2\sum_{i=1}^{\infty} (\log(\mathbf{L}_{\mathbf{M}_i}) - \log(\mathbf{L}_{\mathbf{M}s_i}))
   Chisq df P. Value
1 1.064 1 0.302
# Manually calculated:
LL1 <- logLik(lm(y2 \sim y1, data=data1)) - logLik(lm(y2 \sim y1+x1, data=data1))
LL2 <- logLik(lm(y1 \sim x1, data=data1)) - logLik(lm(y1 \sim x1, data=data1))
ChiSq <- -2*sum(as.numeric(LL1), as.numeric(LL2))</pre>
> ChiSq
[1] 1.063866
DF <- 1 # one additional parameter estimated in the saturated model
1 - pchisq(ChiSq, DF)
                                                   AIC = \sum_{i=1}^{k} AIC(\mathbf{M}_i)
> [1] 0.3023352
# AIC value based on log-likelihood
AIC(psem mod1, aicc = TRUE)
> AIC AICc K n
1 -2.798 -2.298 6 100
```

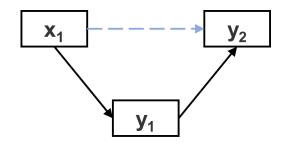


 χ^2 statistic

```
# log-likelihood based χ2 statistic piecewiseSEM
LLchisq(psem mod1)
   Chisq df P. Value
 1.064 1 0.302
# The same as from lavaan
library(lavaan)
sem mod1 <- '
            y1 \sim x1
            y2 \sim y1
sem_fit1 <- sem(sem_mod1, data = data1)</pre>
fit <- lavInspect(sem fit1, "fit")</pre>
fit["chisq"]; fit["pvalue"]
>
chisq
1.063866
pvalue
0.3023352
```



```
summary(psem mod1)
Call:
 y1 \sim x1
 y2 \sim y1
   AIC
 -2.798
Tests of directed separation:
 Independ.Claim
                     Test.Type DF
                                      Crit.Value
                                                   P.Value
  y2 \sim x1 + ...
                  coef
                                97
                                     -1.0186
                                                   0.3109
Global goodness-of-fit:
Chi-Squared = 1.064 with P-value = 0.302 and on 1 degrees of freedom
Fisher's C = 2.336 with P-value = 0.311 and on 2 degrees of freedom
Coefficients:
 Response Predictor Estimate Std.Error DF Crit.Value P.Value
                                                                 Std.Estimate
                       0.5171
                                 0.0548 98
                                               9.4291
                                                                     0.6897 ***
        y1
        y2
                  y1
                       1.1314
                                 0.1104 98
                                              10.2470
                                                                    0.7192 ***
  Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05
Individual R-squared:
 Response method R.squared
        y1
             none
                       0.48
        y2
                       0.52
             none
```



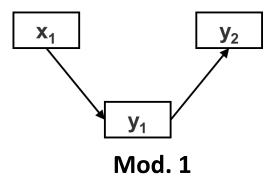
Outline

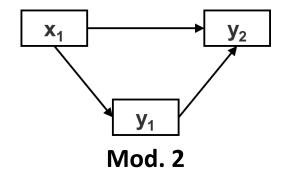
Introduction to Local Estimation in SEM

- ✓ Global vs. Local Estimations. Piecewise SEM
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- ✓ Model Comparison
- Categorical Data in Piecewise SEM

Model Comparison

```
# Model comparison
psem mod2 <- psem(</pre>
  lm(y1 \sim x1, data = data1),
 lm(y2 \sim y1+x1, data = data1))
# \chi2-difference test
anova(psem_mod2, psem_mod1) # arrange correct order manually
>Chi-square Difference Test
      Df AIC.AIC AIC.K AIC.n Chisq Chisq.diff Df.diff P.value
Mod.2 0 -1.862 7 100 0.000
Mod.1 1 -2.798 6 100 1.064 1.064 1 0.3023
# AIC comparison
aic <- AIC(psem mod1, psem mod2)</pre>
> AIC K n
1 -2.798 6 100
2 -1.862 7 100
d aic <- aic[1] - min(aic[1])</pre>
AIC
1 0.000
2 0.936
```





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Categorical Data in Piecewise SEM

Categorical Variables	Exogenous Categorical Variables	Endogenous Categorical Variables	
Binary variables yes/no; presence/absence; failure/success; dead/alive; male/female	1. Treat as numeric: set the values as 0 or 1 and model as numeric (yields a single path coefficient).	Endogenous categorical variables are not implemented in piecewiseSEM. Treat binary and ordinal variables as numerical (follow step 1 shown for	
	2. Create separate dummy variables for each factor levels with values 0, 1 each. Rule : for the factor with k levels use k-1 dummy variables (to avoid singularity).		
	3. Use as categorical variable (Marginal Means approach)		
Ordinal variables: small < medium < large; yang < middle < old	1. Treat as numeric: set the values depending on the order of the factor, e.g., small = 1 < medium = 2 < large = 3, and then model as numeric.		
	2. Create separate dummy variables for each factor levels with values 0, 1 each. Rule : for the factor with k levels use k-1 dummy variables (to avoid singularity).	'Endogenous Categorical Variables')	
	3. Use as categorical variable (Marginal Means approach)		
Nominal variables study sites ;countries; sampling campaigns	Use as categorical variable (Marginal Means approach)	Nominal endogenous categorical variables are not implemented in piecewiseSEM	

Marginal means are the expected average value of one predictor given the other co-variables in the model.

```
data3 <- read.csv("Data/SEMdata2.csv")</pre>
str(data3)
model1 <- lm(y ~ Group, data3)</pre>
summary(model1)
> Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 1.05644 0.05863 18.018 <2e-16 *** # the mean of y in group "A"
       -0.09223 0.08292 -1.112 0.269 # effect of "B" on y in the absence of "A".
GroupB
# marginal mean for A - expected average value of y in group "A" (no other covariates in mod 1)
predict(model1, data.frame(Group = "A"))
>
1.056437
```

```
the marginal mean is evaluated while
model2 <- lm(y \sim x+Group, data3)
                                        holding the covariate x at its mean value.
# the marginal mean:
predict(model2, data.frame(Group = "A", x = mean(data3$x)))
> 1.062983
predict(model2, data.frame(Group = "B", x = mean(data3$x)))
> 0.9576651
```

```
the marginal mean is evaluated while
model2 <- lm(y \sim x+Group, data3)
                                       holding the covariate x at its mean value.
# the marginal mean:
predict(model2, data.frame(Group = "A", x = mean(data3$x)))
> 1.062983
predict(model2, data.frame(Group = "B", x = mean(data3$x)))
> 0.9576651
                                    specs is the variable or list of variables
                                    whose means are to be estimated
library(emmeans)
emmeans (model2, specs = "Group")
Group emmean
                  SE df lower.CL upper.CL
        1.063 0.0419 97
                          0.980
                                        1.15
Α
        0.958 0.0419 97 0.875 1.04
В
```

```
the marginal mean is evaluated while
model2 <- lm(y \sim x+Group, data3)
                                      holding the covariate x at its mean value.
# the marginal mean:
predict(model2, data.frame(Group = "A", x = mean(data3$x)))
> 1.062983
predict(model2, data.frame(Group = "B", x = mean(data3$x)))
> 0.9576651
                                    specs is the variable or list of variables
                                    whose means are to be estimated
library(emmeans)
emmeans (model2, specs = "Group")
>
Group
       emmean SE df lower.CL upper.CL
        1.063 0.0419 97
                            0.980
                                       1.15
 Α
        0.958 0.0419 97 0.875 1.04
В
                                                   pairwise Tukey tests
emmeans(model2, list(pairwise ~ Group))
 > ...
       estimate
                               t.ratio
                                          p.value
                      SE df
 A - B 0.105
                               1.778
                 0.0592 97
                                          0.0786
```

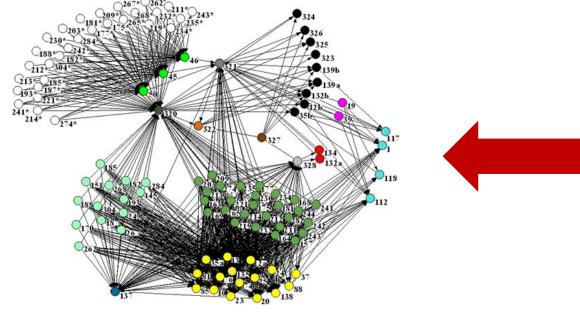
```
library(piecewiseSEM)
psem model <- psem(model2)</pre>
                                                            The significance test for
                                                            the effect of Group
coefs(psem model)
 >
Response Predictor Estimate Std.Error DF Crit.Value P.Value Std.Estimate
                                    0.108 97
                                                  9.7511
                                                           0.0000
                                                                        0.6994 ***
1
                        1.0529
                                                           0.0786
                                                  3.1604
               Group
                                                                              _ ***
                        0.9577
                                   0.0419 97
                                                 22.8640
                                                          0.0000
         y Group = B
                                                 25.3784
                                                          0.0000
         y Group = A
                         1.063
                                   0.0419 97
                                                                              _ ***
                   marginal means
                                                                               pairwise Tukey tests
summary(psem model)
```

Provides a measure of whether the path between the exogenous categorical variable and the response is significant as well as parameters for each level in the form of the model-estimated marginal means.

Categorical Data in Piecewise SEM

Categorical Variables	Exogenous Categorical Variables	Endogenous Categorical Variables	
Binary variables yes/no; presence/absence; failure/success; dead/alive; male/female	1. Treat as numeric: set the values as 0 or 1 and model as numeric (yields a single path coefficient).	Endogenous categorical variables are not implemented in piecewiseSEM. Treat binary and ordinal variables as numerical (follow step 1 shown for	
	2. Create separate dummy variables for each factor levels with values 0, 1 each. Rule : for the factor with k levels use k-1 dummy variables (to avoid singularity).		
	3. Use as categorical variable (Marginal Means approach)		
Ordinal variables: small < medium < large; yang < middle < old	1. Treat as numeric: set the values depending on the order of the factor, e.g., small = 1 < medium = 2 < large = 3, and then model as numeric.		
	2. Create separate dummy variables for each factor levels with values 0, 1 each. Rule : for the factor with k levels use k-1 dummy variables (to avoid singularity).	f'Endogenous Categorical Variables')	
	3. Use as categorical variable (Marginal Means approach)		
Nominal variables study sites ;countries; sampling campaigns	Use as categorical variable (Marginal Means approach)	Nominal endogenous categorical variables are not implemented in piecewiseSEM 37	

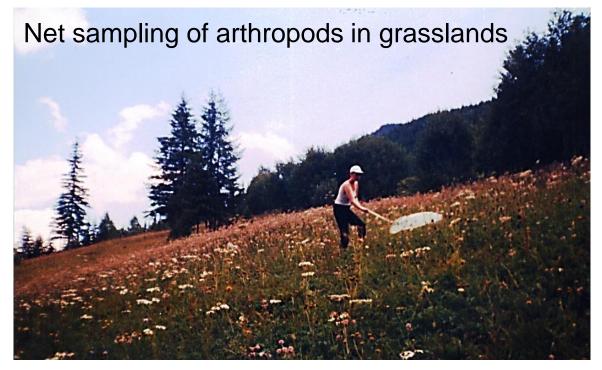
Day 9 Task 1



Food web length (1,2,3)

```
read.csv("Data/Food web data 2.csv")
'data.frame': 235 obs. of 5 variables:
 $ Gr type : chr "sheep" "sheep" "sheep" ...
  soil C : num 1.336 1.631 1.577 ...
 $ plant sr : num 5.07 28.39 24.52 ...
 $ plant biom: num 185 207 224 238 203 ...
 $ FW.length : int 1 1 1 1 1 1 1 1 1 ...
```

Effects of land use on arthropod food webs in grasslands



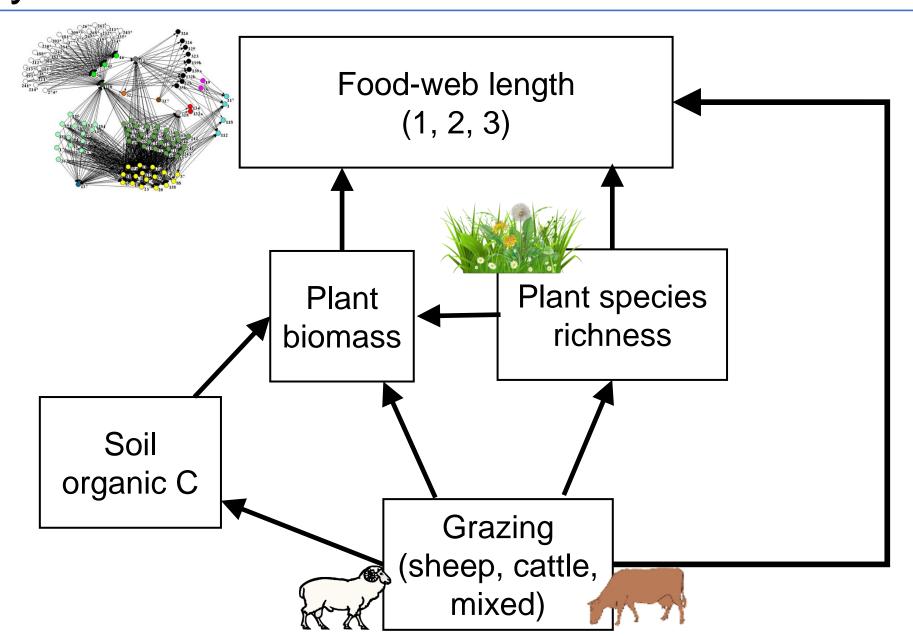
235 grasslands

Grazing type

("sheep", "cattle", or "mixed grazing")

Day 9 Task 1

Effects of land use on food webs in grasslands



Day 9 Task 1

Effects of land use on food webs in grasslands

Gr_type (grazing type) is your exogenous nominal categorical variables. Test **Gr_type** (as a part of the SEM model on fig. 1) using marginal means in **piecewiseSEM**.

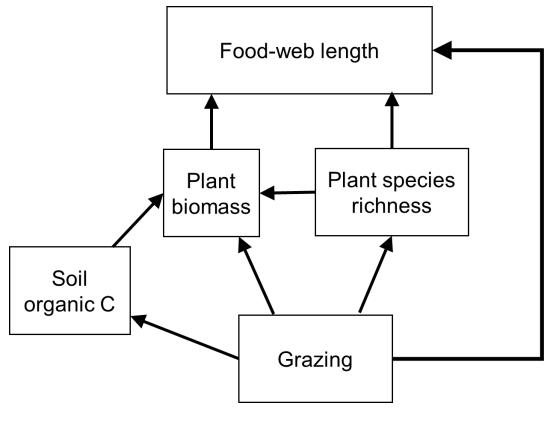


fig. 1